CHAPTER 17

Simple Harmonic Motion

IN THIS CHAPTER

Summary: An object whose position–time graph makes a sine or cosine function is in simple harmonic motion. The period of such motion can be calculated.

KEY IDEA

Key Ideas

- There are three conditions for something to be in simple harmonic motion. All are equivalent. Physics B students do not need to understand the third point, nor the mathematics behind the first:
  1. The object's position–time graph is a sine or cosine graph.
  2. The restoring force on the object is proportional to its displacement from equilibrium.
  3. The energy vs. position graph is parabolic, or nearly so.
- The mass on a spring is the most common example of simple harmonic motion.
- The pendulum is in simple harmonic motion for small amplitudes.

$F_s = Kx$

Relevant Equations

Period of a mass on a spring:

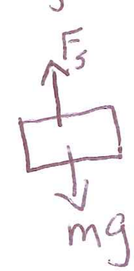
$$T = 2\pi\sqrt{\frac{m}{k}}$$

$K.E. = \frac{1}{2}mv^2$

$P.E._s = \frac{1}{2}Kx^2$

Period of a pendulum:

$$T = 2\pi\sqrt{\frac{L}{g}}$$



$F_s = F_w$

$Kx = mg$

$K = \frac{mg}{x}$

Relationship between period and frequency:

$$T = \frac{1}{f}$$

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Relevant Equations

Period of a mass on a spring:

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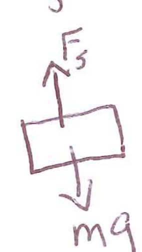

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What's so simple about simple harmonic motion (SHM)? Well, the name actually refers to a type of movement—regular, back and forth, and tick-tock tick-tock kind of motion. It's simple compared to, say, a system of twenty-five springs and masses and pendulums all tied to one another and waggling about chaotically.

The other reason SHM is simple is that, on the AP exam, there are only a limited number of situations in which you'll encounter it. Which means only a few formulas to memorize, and only a few types of problems to really master. Some of these problems can get a little tricky for the Physics C folks, and we've included a special section ("The Sinusoidal Nature of SHM") just for them. But regardless of which exam you're taking, we hope you'll agree that most of this material is, relatively, simple.

Amplitude, Period, and Frequency

Simple harmonic motion is the study of oscillations. An **oscillation** is motion of an object that regularly repeats itself over the same path. For example, a pendulum in a grandfather clock undergoes oscillation: it travels back and forth, back and forth, back and forth . . . Another term for oscillation is "periodic motion."

KEY IDEA

Objects undergo oscillation when they experience a **restoring force**. This is a force that restores an object to the equilibrium position. In the case of a grandfather clock, the pendulum's equilibrium position—the position where it would be if it weren't moving—is when it's hanging straight down. When it's swinging, gravity exerts a restoring force: as the pendulum swings up in its arc, the force of gravity pulls on the pendulum, so that it eventually swings back down and passes through its equilibrium position. Of course, it only remains in its equilibrium position for an instant, and then it swings back up the other way. A restoring force doesn't need to bring an object to rest in its equilibrium position; it just needs to make that object pass through an equilibrium position.

If you look back at the chapter on conservation of energy (Chapter 14), you'll find the equation for the force exerted by a spring, $F = kx$. This force is a restoring force: it tries to pull or push whatever is on the end of the spring back to the spring's equilibrium position. So if the spring is stretched out, the restoring force tries to squish it back in, and if the spring is compressed, the restoring force tries to stretch it back out. Some books present this equation as $F = -kx$. The negative sign simply signifies that this is a restoring force.

One repetition of periodic motion is called a **cycle**. For the pendulum of a grandfather clock, one cycle is equal to one back-and-forth swing.

The maximum displacement from the equilibrium position during a cycle is the **amplitude**. In Figure 17.1, the equilibrium position is denoted by "0," and the maximum displacement of the object on the end of the spring is denoted by "A."

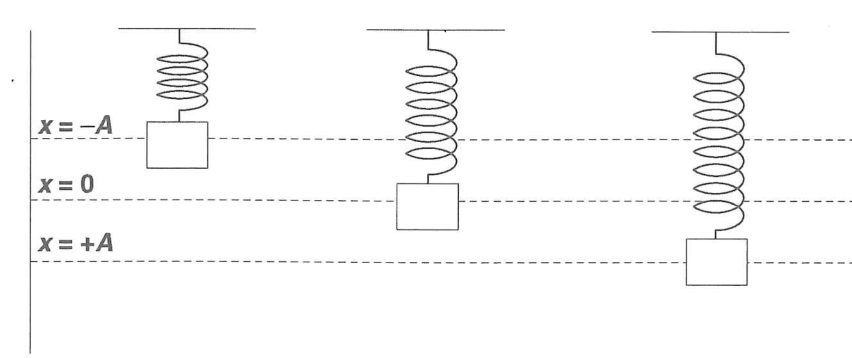

Figure 17.1 Periodic motion of a mass connected to a spring.