The time it takes for an object to pass through one cycle is the period, abbreviated "T." Going back to the grandfather clock example, the period of the pendulum is the time it takes to go back and forth once: one second. Period is related to frequency, which is the number of cycles per second. The frequency of the pendulum of the grandfather clock is $f = 1$ cycle/s, where "f" is the standard abbreviation for frequency; the unit of frequency, the cycle per second, is called a hertz, abbreviated Hz. Period and frequency are related by this equation:

$$\boxed{T = \frac{1}{f}}$$

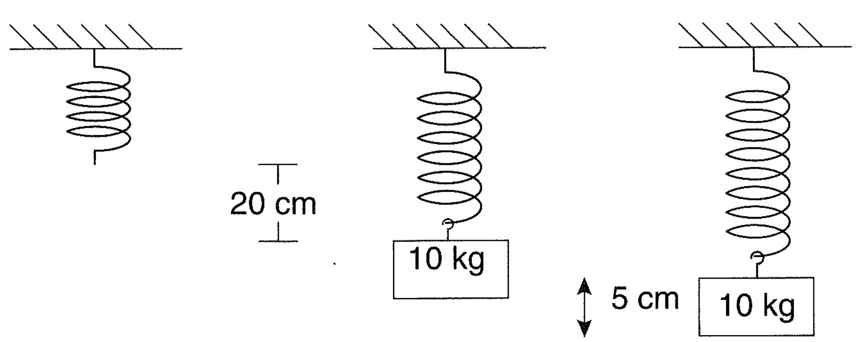
Vibrating Mass on a Spring

A mass attached to the end of a spring will oscillate in simple harmonic motion. The period of the oscillation is found by this equation:

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

In this equation, m is the mass of the object on the spring, and k is the "spring constant." As far as equations go, this is one of the more difficult ones to memorize, but once you have committed it to memory, it becomes very simple to use.

> A block with a mass of 10 kg is placed on the end of a spring that is hung from the ceiling. When the block is attached to the spring, the spring is stretched out 20 cm from its rest position. The block is then pulled down an additional 5 cm and released. What is the block's period of oscillation, and what is the speed of the block when it passes through its rest position?



Let's think about how to solve this problem methodically. We need to find two values, a period and a speed. Period should be pretty easy—all we need to know is the mass of the block (which we're given) and the spring constant, and then we can plug into the formula. What about the speed? That's going to be a conservation of energy problem—potential energy in the stretched-out spring gets converted to kinetic energy—and here again, to calculate the potential energy, we need to know the spring constant. So let's start by calculating that.

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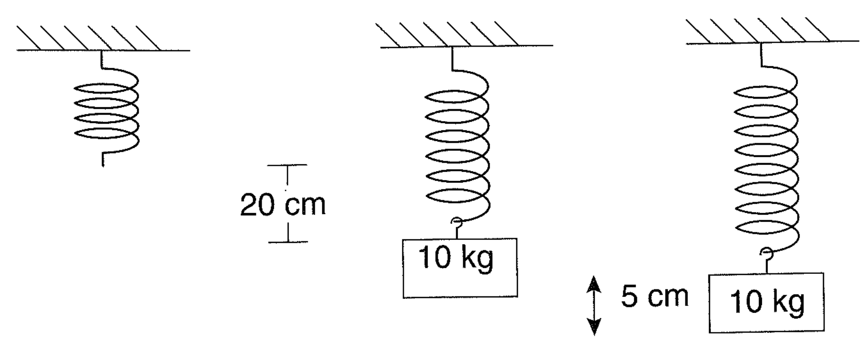

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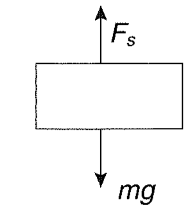
First, we draw our free-body diagram of the block.



We'll call "up" the positive direction. Before the mass is oscillating, the block is in equilibrium, so we can set F_s equal to mg. (Remember to convert centimeters to meters!)

$$kx = mg$$
$$k\,(0.20\text{ m}) = (10\text{ kg})(10\text{ m/s}^2)$$
$$k = 500\text{ N/m}$$

Now that we have solved for k, we can go on to the rest of the problem. The period of oscillation can be found be plugging into our formula.

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$T = 2\pi\sqrt{\frac{10\text{ kg}}{500\text{ N/m}}}$$
$$T = 0.89\text{ s}$$

To compute the velocity at the equilibrium position, we can now use conservation of energy.

$$\text{KE}_a + \text{PE}_a = \text{KE}_b + \text{PE}_b$$

STRATEGY

When dealing with a vertical spring, it is best to define the rest position as $x = 0$ in the equation for potential energy of the spring. If we do this, then gravitational potential energy can be ignored. Yes, gravity still acts on the mass, and the mass changes gravitational potential energy. So what we're really doing is taking gravity into account in the spring potential energy formula by redefining the $x = 0$ position, where the spring is stretched out, as the resting spot rather than where the spring is unstretched.

In the equation above, we have used a subscript "a" to represent values when the spring is stretched out the extra 5 cm, and "b" to represent values at the rest position.

TIP

When the spring is stretched out the extra 5 cm, the block has no kinetic energy because it is being held in place. So, the KE term on the left side of the equation will equal 0. At this point, all of the block's energy is entirely in the form of potential energy. (The equation for the PE of a spring is $\frac{1}{2}kx^2$, remember?) And at the equilibrium position, the block's energy will be entirely in the form of kinetic energy. Solving, we have

$$\tfrac{1}{2}mv_a^2 + \tfrac{1}{2}kx_a^2 = \tfrac{1}{2}mv_b^2 + \tfrac{1}{2}kx_b^2.$$
$$0 + \tfrac{1}{2}\,(500\text{ N/m})\,(0.05\text{ m})^2 = \tfrac{1}{2}\,(10\text{ kg})v^2 + 0.$$
$$v = 0.35\text{ m/s}.$$

Simple Pendulums

Problems that involve simple pendulums—in other words, basic, run-of-the-mill, grandfather clock-style pendulums—are actually really similar to problems that involve springs. For example, the formula for the period of a simple pendulum is this:

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$