Name Key

Math 111 Section 4 Instructor: Paul Konichek

Quiz 9 (sections 5.6 – 6.1) (the Last Quiz)

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1. **Exponential Growth: Given that a quantity Q(t) is described by the exponential growth function**

$$Q(t) = 300\, e^{0.02\, t}$$

Where t is measured in minutes, answer the following questions:

a. What quantity is present initially? 300 (½ pt.)
b. What is the growth constant? 0.02 (½ pt.)
c. Complete the following table of values: (1 pt.)

t	0	10	30	50	70
Q	300	366.42	546.64	815.48	1216.6

***Choose either one of the following two problems 2 or 3: Please cross out the ungraded one.**

2. (3 pts.) Atmospheric Pressure: If the temperature is constant, then the atmospheric pressure P (in pounds per square inch) varies with the altitude above sea level h in accordance with the law

$$P(h) = p_0\, e^{-kh}$$

where p_0 is the atmospheric pressure at sea level and k is a constant. *If the atmospheric pressure is 15 lb/in² at sea level and 12.5 lb/in² at 4000 ft*, find k first, then secondly find the atmospheric pressure at an altitude of 14,000 ft. And thirdly, find how fast is the atmospheric pressure changing with respect to altitude t at an altitude of 14,000 ft? (show all work here)

$$P(0) = 15e^{-K(0)} = 15$$

given: $$P(4000) = 15e^{-4000K} = 12.5$$

$$\text{so } e^{-4000K} = \frac{12.5}{15} = .8\bar{3}$$

$$\ln e^{-4000K} = \ln .8\bar{3}$$

$$-4000K = \ln .8\bar{3}$$

$$\boxed{K = \frac{\ln 0.8\bar{3}}{-4000} = -4.558\times10^{-5} \text{ OR} \doteq -.0000456}$$

k ≐ $-.0000456$

P(14,000) = 7.92 lb/in²

P'(14,000) = $-.00036$ lb/in²/ft

$$P(14{,}000) = 15e^{14000(-.0000456)} \doteq \boxed{7.92\ \text{lb/in}^2}$$

$$P'(x) = +K\,P(x)$$

$$P'(14000) = (-.0000456)(7.92\ \text{lb/in}^2) = \boxed{-.00036\ \text{lb/in}^2/\text{ft}}$$

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Quiz 9 cont. You are doing either 2 or 3 and crossing out the one NOT to grade.

3. **(or 3 pts.) Growth of Bacteria: The growth rate of the bacterium *Escherichia coli*, a common bacterium found in the human intestine, is proportional to its size. Under ideal laboratory conditions, when this bacterium is grown in a nutrient broth medium, the <u>*number of cells in a culture doubles approximately every 20 min*</u>. Given that Growth of Bacteria under these conditions grows in accordance with the law $Q(t) = Q_0 e^{kt}$ where Q_0 denotes the number of bacteria initially present in the culture, k is a constant determined by the strain of bacteria under consideration and other factors, and t is the elapsed time measure in minutes. (show all work here)**

a). If the initial cell population at t=0 is 100 and <u>*number of cells in a culture doubles approximately every 20 min*</u> , find k for the bacterium *Escherichia coli* under the above conditions:

$Q(20) = 100 e^{20K} = 200$

$e^{20K} = \frac{200}{100} = 2$

$\ln e^{20K} = \ln 2$

$20K \ln e = \ln 2$

$20K(1) = \ln 2$

$20K = \ln 2$

$K = \frac{\ln 2}{20} \doteq 0.0347$

k = .0347

b). How long will it take for a colony of 100 cells to increase to a population of 500,000 cells?

$500{,}000 \doteq 100 e^{.0347t}$

$e^{.0347t} \doteq \frac{500{,}000}{100} \doteq 5{,}000$

$\ln e^{.0347t} = \ln 5{,}000$

$.0347t \ln e = \ln 5{,}000$

$.0347t = \ln 5{,}000$

$t = \frac{\ln 5{,}000}{.0347}$

$t \doteq 245.45225$

t = 246 min **(round up to the nearest minute)**

c). What is the rate of growth of the population at the end of 246 minutes knowing Q(246) = 500,000?

$Q'(t) = K\,Q(t)$

$Q'(246) = K\,Q(246) = .0347(500{,}000) = 17{,}350 \ \frac{\text{cells}}{\text{min}}$

Q'(246) = 17,350 $\frac{\text{cells}}{\text{minute}}$

The remaining five problems are 1 point each.

Verify directly that F is an antiderivative of f(x).

4. $F(x) = \frac{1}{3}x^3 + 2x^2 - x + 2$; $f(x) = x^2 + 4x - 1$

Answer: $F'(x) = x^2 + 4x - 1 = f(x)$

In 5-8, find the indefinite integral.

5. $\int x^{-4}\,dx$ $= \frac{1}{-4+1} x^{-4+1} + C = -\frac{1}{3} x^{-3} + C$

Answer: $-\frac{1}{3}x^{-3} + C$ or $-\frac{1}{3x^3} + C$

6. $\int \frac{2}{x^3}\,dx$ $= \int 2x^{-3}dx = 2\int x^{-3}dx = \frac{2}{-3+1}x^{-3+1} + C$

$= -\frac{2}{2}x^{-2} + C$

$= -1x^{-2} + C$

Answer: $-x^{-2} + C$ or $-\frac{1}{x^2} + C$

7. $\int 5e^x\,dx$ $= 5\int e^x dx = 5e^x + C$

Answer: $5e^x + C$

8. $\int (2t+1)(t-2)\,dt$ $= \int (2t^2 - 4t + t - 2)dt$

$\int (2t^2 - 3t - 2)dt$

$= \frac{2}{2+1}t^{2+1} - \frac{3}{1+1}t^{1+1} - \frac{2}{0+1}t^{0+1} + C$

$= \frac{2}{3}t^3 - \frac{3}{2}t^2 - 2t + C$

Answer: $\frac{2}{3}t^3 - \frac{3}{2}t^2 - 2t + C$