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Math 111 Section 4
(Quiz 7 over sections 4.5 – 5.2)

The first sixteen problems are worth $\frac{1}{2}$ point apiece.

Evaluate the expression (example: leave 8 not 2^3)

1. a. $4^{-3} * 4^5$ b. $3^{-3} * 3^6$

$$\begin{array}{r} -3+5 \\ 4 \\ \hline 4^2 \end{array} \quad \begin{array}{r} -3+6 \\ 3 \\ \hline 3^3 \end{array}$$

answers: 1. a. 16 b. 27

2. a. $9(9)^{-1/2}$ b. $5(5)^{-1/2} = 5^{1-\frac{1}{2}} = 5^{\frac{1}{2}}$

$$9\left(\frac{1}{\sqrt{9}}\right) \quad \text{or} \quad \frac{5}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5}$$

answers: 2. a. 3 b. $\sqrt{5}$ or $5^{\frac{1}{2}}$

Simplify the equation for x:

3. a. $(64x^9)^{1/3}$ b. $(25x^3y^4)^{1/2}$

$$\begin{array}{r} \sqrt[3]{64} x^{\frac{9}{3}} \\ 4x^3 \end{array} \quad \begin{array}{r} \sqrt{25} x^{\frac{3}{2}} y^{\frac{4}{2}} \\ 5x^{\frac{3}{2}} y^2 \end{array}$$

Answers: 3. a. $4x^3$ b. $5x^{\frac{3}{2}}y^2$

4. a. $\frac{6a^{-4}}{3a^{-3}}$ b. $\frac{4b^{-4}}{12b^{-6}}$

$$\begin{array}{r} \frac{6}{3} a^{-4-(-3)} \\ = 2^1 a^{-4+3} \\ = 2^1 a^{-1} \end{array} \quad \begin{array}{r} \frac{1}{3} b^{-4-(-6)} \\ = 3^1 b^{-4+6} \\ = 3^1 b^2 \end{array}$$

Answers: 4. a. $\frac{2}{9}$ b. $b^2/3$

Problems 5-8 Solve the equation for x.

5. $6^{2x} = 6^6$

$$\begin{aligned} 2x &= 6 \\ x &= 3 \end{aligned}$$

Answer: 5. $x = 3$

Still solving the equation for x problems 6-8.

6. $3^{3x-4} = 3^5$

$$\begin{aligned} 3x-4 &= 5 \\ 3x &= 9 \end{aligned}$$

Answer(s): 6. $x = 3$

7. $3^{2x} - 12 * 3^x + 27 = 0$

let $y = 3^x$ Then $y^2 - 12y + 27 = 0$
 $(y-3)(y-9) = 0$ ~~so $y = 3$ or $y = 9$~~
 $y = 3$ or ~~$y = 9$~~ $3^x = 3$ $3^x = 9$

Answer(s): 7. $x = 1$; $x = 2$

8. $2^{2x} - 4 * 2^x + 4 = 0$

let $y = 2^x$ $y^2 - 4y + 4 = 0$
 $(y-2)(y-2) = 0$
 $y = 2$

Answer(s): 8. $x = 1$

Express each equation in logarithmic form.

9. $2^6 = 64$

Answer: 9. $\log_2 64 = 6$

10. $4^{-2} = \frac{1}{16}$

Answer: 10. $\log_4 \left(\frac{1}{16}\right) = -2$

11. $3^5 = 243$

Answer: 11. $\log_3 243 = 5$

Cont. expressing in logarithmic form.

12. $81^{\frac{3}{4}} = 27$

Answer: 12. $\log_{81} 27 = \frac{3}{4}$

Given that $\log 3 \approx 0.4771$ and $\log 4 \approx 0.6021$ find the value of the logarithm.

$$\begin{aligned}
 13. \quad \log \frac{1}{300} &= \log 1 - \log 300 \\
 &= 0 - \log(3)(100) \\
 &= -(\log 3 + \log 100) \\
 &\stackrel{!}{=} -(\log 3 + \log_{10} 10^2) \\
 &\stackrel{!}{=} -(\log 3 + 2 \log_{10} 10) \\
 &\stackrel{!}{=} -(0.4771 + 2(1)) \\
 &\stackrel{!}{=} -2.4771
 \end{aligned}$$

Answer: 13. - 2.4771

Write the expression as the logarithm of a single quantity.

$$14. \quad \frac{1}{2} \ln x + 2 \ln y - 3 \ln z$$

$$\ln x^{\frac{1}{2}} + \ln y^2 - \ln z^3$$

Answer: 14. $y^2 \sqrt{x} / z^3$ or $\frac{x^{\frac{1}{2}}}{z^3}$

Use the laws of logarithms to expand and simplify the expression.

$$15. \quad \log \frac{\sqrt{x+1}}{x^2+1}$$

$$\log \frac{(x+1)^{\frac{1}{2}}}{(x^2+1)}$$

$$\log(x+1)^{\frac{1}{2}} - \log(x^2+1)$$

Answer: 15. $\frac{1}{2} \log(x+1) - \log(x^2+1)$

Use logarithms to solve the equation for t. (use calculator)

$$16. \quad 5e^{-2t} = 6$$

$$\left. \begin{aligned}
 e^{-2t} &= \frac{6}{5} \\
 \ln e^{-2t} &= \ln \frac{6}{5} \\
 -2t \ln e &= \ln \frac{6}{5}
 \end{aligned} \right\} \rightarrow -2t(1) = \ln \frac{6}{5}$$

$$\begin{aligned}
 t &= -\frac{\ln \frac{6}{5}}{2} \\
 t &\stackrel{!}{=} -\frac{1.823}{2}
 \end{aligned}$$

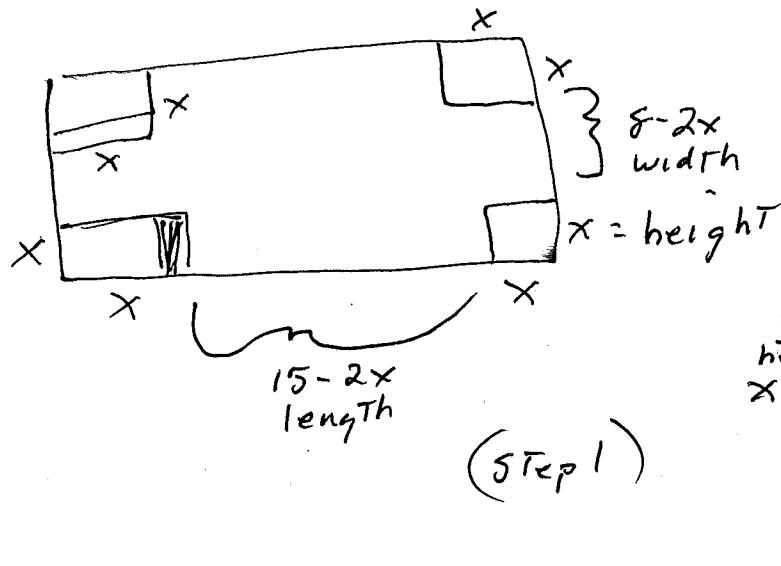
$$t \stackrel{!}{=} -.09115$$

Answer: 16. $\stackrel{!}{=} -.09115$

The final two are worth a point apiece.

Your choice: Do number 17 or 18 worth 2 points. Only do one of these and cross out the one I don't grade.

- 17. Packaging** By cutting away identical squares from each corner of a rectangular piece of cardboard and folding up the resulting flaps, an open box may be made. If the cardboard is 15 in. long and 8 in. wide, find the dimensions of the box that will yield the maximum volume.



$$\begin{array}{lll}
 \text{height} & \text{length} & \text{width} \\
 x \geq 0 & 15 - 2x \geq 0 & 8 - 2x \geq 0 \\
 x \leq 7.5 & \text{AND } x \leq 4 & \\
 [0, 4]
 \end{array}$$

$$\text{Step 2} \quad V = (15 - 2x)(8 - 2x)(x)$$

$$V = f(x) = 4x^3 - 46x^2 + 120x$$

$$f'(x) = 12x^2 - 92x + 120$$

$$f'(x) = 4(3x^2 - 23x + 30) = 0$$

$$4(3x^2 - 23x + 30) = 0$$

$$x = 5/3 \quad x = 6 \text{ outside } [0, 4]$$

$$f(0) = 0$$

$$f(4) = 4(4)^3 - 46(4)^2 + 120(4) = 256 - 236 + 480 = 0$$

$f(5/3) > 0$ so $x = 5/3$ is an absolute maximum

$$\text{height} = x = 5/3, \text{ length } 15 - \frac{10}{3} = \frac{35}{3} \text{ inches, width } 8 - \frac{10}{3} = \frac{14}{3}$$

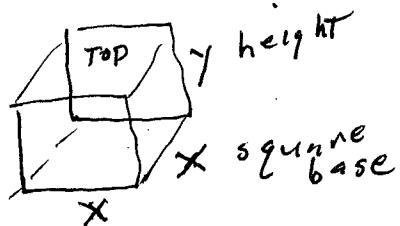
maximum volume when
dimensions are $\frac{35}{3} \times \frac{14}{3} \times \frac{5}{3}$

$$\text{Answer: 17. } \frac{35}{3} \times \frac{14}{3} \times \frac{5}{3}$$

$$\text{OR } 11\frac{2}{3} \times 4\frac{2}{3} \times 1\frac{2}{3}$$

$14/3$

18. Minimizing Costs A pencil cup with a capacity of 36 in.³ is to be constructed in the shape of a rectangular box with a square base and an open top. If the material for the sides costs 15¢ / in² and the material for the base cost 40 ¢ / in.², what should the dimensions of the cup be to minimize the construction cost?



$$V = x^2 y$$

$$36 = x^2 y \quad \text{so} \quad y = \frac{36}{x^2}$$

$$C(x) = \underset{\text{base}}{40x^2} + \underset{\text{area of sides}}{15(4xy)} \text{ cents}$$

$$C(x) = 40x^2 + 60xy \text{ cents}$$

~~$$C(x) = 40x^2 + 60x\left(\frac{36}{x^2}\right)$$~~

$$C(x) = 40x^2 + \frac{2160}{x}$$

$$C'(x) = 80x - \frac{2160}{x^2} = 0$$

$$80x^3 - 2160 = 0$$

$$80x^3 = 2160$$

$$x^3 = 27$$

$$x = 3$$

$$C''(x) = 80 + \frac{4320}{x^3} \quad \boxed{x=3} \quad \begin{array}{l} > 0 \\ \text{gives minimum} \end{array}$$

required dimensions

Answer: 18. $3'' \times 3'' \times 4''$

$$y = \frac{36}{x^2} = \frac{36}{(3)^2} = \frac{36}{9} = 4$$