24

Wave Optics

Clicker Questions

Question P2.01

Description: Distinguishing several related optical phenomena, and relating them to everyday experience.

Question

A "rainbow" appears on an oil slick or soap bubble due to:

1. refraction of light.
2. transmission of light.
3. dispersion of light.
4. interference of light.
5. scattering of light.
6. reflection of light.
7. None of the above.

Commentary

Purpose: To distinguish and hone several optical concepts involved in the rainbow phenomenon, and identify the critical one.

Discussion: Rainbows are created because two light waves having different phases interfere, in such a way that constructive interference occurs at slightly different angles for different wavelengths (colors). When looking at one angle, you see one color because the two light waves are constructively interfering for that one color but not for others; look at a slightly different angle, you see a slightly different color. *Interference* (answer 4) is the crucial process for the rainbow effect.

Other listed processes, however, participate to enable interference to occur. The two light waves that interfere are one that *reflects* from the top surface of the oil or soap film, and another that *refracts* from the top surface, is *transmitted* through the oil or soap, *reflects* from the bottom surface, is *transmitted* back, and *refracts* again as it passes back through the top surface. The fact that light of different wavelengths refracts by a different amount when crossing the interface is responsible for the fact that constructive interference occurs in different directions for different colors.

Key Points:

- The rainbow effect is an interference phenomenon.
- Reflection, transmission, and wavelength-dependent refraction are all required for the proper interference situation to arise to create a rainbow.
- Being able to identify the critical process in a phenomenon, and distinguish it from subsidiary processes, is important for analyzing your experiences and distinguishing among similar phenomena.

For Instructors Only

Some students may reserve the term "rainbow" only for the atmospheric phenomenon; it is valuable to relate that case to other such as oil slick and soap bubble rainbows, so students can see that the underlying mechanism is the same.

This question provides a context for discussing various ways to produce a variegated color effect via interference.

Students may inquire why there is no rainbow produced with window glass, since reflection obviously occurs from both surfaces. If they do not raise the issue, the instructor should.

QUICK QUIZZES

1. (c). The fringes on the screen are equally spaced only at small angles where $\tan\theta \approx \sin\theta$ is a valid approximation.

2. (c). The screen locations of the dark fringes of order m are given by $(y_{\text{dark}})_m = (\lambda L/d)(m+\frac{1}{2})$, with $m = 0$ corresponding to the first dark fringe on either side of the central maximum. The width of the central maximum is then $2(y_{\text{dark}})_0 = 2(\lambda L/d)(\frac{1}{2}) = \lambda L/d$. Thus, doubling the distance d between the slits will cut the width of the central maximum in half.

3. (c). The screen locations of the bright fringes of order m are given by $(y_{\text{bright}})_m = (\lambda L/d)m$, and the distance between successive bright fringes for a given wavelength is

$$\Delta y_{\text{bright}} = \left(y_{\text{bright}}\right)_{m+1} - \left(y_{\text{bright}}\right)_m = \left(\lambda L/d\right)_{m+1} - \left(\lambda L/d\right)_m = \lambda L/d$$

Observe that this spacing is directly proportional to the wavelength. Thus, arranged from smallest to largest spacing between bright fringes, the order of the colors will be blue, green, red.

4. (b). The space between successive bright fringes is proportional to the wavelength of the light. Since the wavelength in water is less than that in air, the bright fringes are closer together in the second experiment.

5. (b). The outer edges of the central maximum occur where $\sin\theta = \pm\lambda/a$. Thus, as a, the width of the slit, becomes smaller, the width of the central maximum will increase.

6. The compact disc. The tracks of information on a compact disc are much closer together than on a phonograph record. As a result, the diffraction maxima from the compact disc will be farther apart than those from the record.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The bright fringe of order m occurs where $\delta = d\sin\theta = m\lambda$. For small angles, the sine of the angle is approximately equal to the angle expressed in radians. Thus, the angular position of the second order bright fringe in the case described is

$$\theta = 2\left(\frac{\lambda}{d}\right) = 2\left(\frac{5.0\times10^{-7}\text{ m}}{2.0\times10^{-5}\text{ m}}\right) = 0.050 \text{ radians}$$

making (a) the correct choice.

2. With phase reversals occurring in the reflections at both the air-oil boundary and the oil-water boundary, the condition for constructive interference in the reflected light is $2n_{oil}t = m\lambda$ where m is any integer. Thus, the minimum nonzero thickness of the oil which will strongly reflect the 530-nm light is $t_{min} = \lambda/2n_{oil} = (530 \text{ nm})/2(1.25) = 212 \text{ nm}$, and (d) is the proper choice.

3. In a single slit diffraction pattern, formed on a screen at distance L from the slit, dark fringes occur where $y_{dark}/L = \tan\theta_{dark} \approx \sin\theta_{dark} = m(\lambda/a)$ and m is any nonzero integer. Thus, the width of the slit, a, in the described situation must be

$$a = \frac{(1)\lambda L}{(y_{dark})_1} = \frac{(5.0\times10^{-7}\text{ m})(1.0\text{ m})}{5.0\times10^{-3}\text{ m}} = 1.0\times10^{-4}\text{ m} = 0.10\text{ mm}$$

and the correct answer is seen to be choice (b).

4. From Malus's law, the intensity of the light transmitted through a polarizer having its transmission axis oriented at angle θ to the plane of polarization of the incident polarized light is $I = I_0\cos^2\theta$. Therefore, the intensity transmitted through the first polarizer having $\theta = 45° - 0 = 45°$ is $I_1 = I_0\cos^2(45°) = 0.50I_0$, and the intensity passing through the second polarizer having $\theta = 90° - 45° = 45°$ is $I_2 = (0.50I_0)\cos^2(45°) = 0.25I_0$. The fraction of the original intensity making it through both polarizers is then $I_2/I_0 = 0.25$, which is choice (b).

5. The spacing between successive bright fringes in a double-slit interference pattern is given by $\Delta y_{bright} = (\lambda L/d)(m+1) - (\lambda L/d)m = \lambda L/d$, where d is the slit separation. As d decreases, the spacing between the bright fringes will increase and choice (b) is the correct answer.

6. As discussed in question 5 above, the fringe spacing in a double slit interference pattern is $\Delta y_{bright} = \lambda L/d$. Therefore, as the distance L between the screen and the plane of the slits is increased, the spacing between the bright fringes will increase, and (a) is the correct choice.

7. As discussed in question 5 above, the fringe spacing in a double slit interference pattern is $\Delta y_{bright} = \lambda L/d$. Note that this spacing is directly proportional to the wavelength of the light illuminating the slits. Thus, with $\lambda_2 > \lambda_1$, the spacing is greater in the second experiment and choice (c) gives the correct answer.

8. The bright colored patterns are the result of interference between light reflected from the upper surface of the oil and light reflected from the lower surface of the oil film. Thus, the best answer is choice (e).

9. The reflected light tends to be partially polarized, with the degree of polarization depending on the angle of incidence on the reflecting surface. Only if the angle of incidence equals the polarizing angle (or Brewster's angle) will the reflected light be completely polarized. The better answer for this question is choice (d).

10. In a single slit diffraction pattern, dark fringes occur where $y_{dark}/L \approx \sin\theta_{dark} = m(\lambda/a)$. The width of the central maximum is the distance between the locations of the first dark fringes on either side of the center (i.e., between the $m = \pm1$ dark fringes), giving

$$\text{width of central maximum} = (+1)\frac{\lambda}{a} - (-1)\frac{\lambda}{a} = \frac{2\lambda}{a}$$

Thus, if the width of the slit, a, is cut in half, the width of the central maximum will double, making (d) the correct choice.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The wavelength of light is extremely small in comparison to the dimensions of your hand, so the diffraction of light around obstacles the size of your hand is totally negligible. However, sound waves have wavelengths that are comparable to the dimensions of the hand or even larger. Therefore, significant diffraction of sound waves occurs around hand-sized obstacles.

4. The wavelength of light traveling in water would decrease, since the wavelength of light in a medium is given by $\lambda_n = \lambda/n$, where λ is the wavelength in vacuum and n is the index of refraction of the medium. Since the positions of the bright and dark fringes are proportional to the wavelength, the fringe separations would decrease.

6. Every color produces its own interference pattern, and we see them superimposed. The central maximum is white. The first maximum is a full spectrum with violet on the inside and red on the outside. The second maximum is also a full spectrum, with red in it overlapping with violet in the third maximum. At larger angles, the light soon starts mixing to white again.

8. The skin on the tip of a finger has a series of closely spaced ridges and swirls on it. When the finger touches a smooth surface, the oils from the skin will be deposited on the surface in the pattern of the closely spaced ridges. The clear spaces between the lines of deposited oil can serve as the slits in a crude diffraction grating and produce a colored spectrum of the light passing through or reflecting from the glass surface.

10. Suppose the index of refraction of the coating is intermediate between vacuum and the glass. When the coating is very thin, light reflected from its top and bottom surfaces will interfere constructively, so you see the surface white and brighter. Once the thickness reaches one-quarter of the wavelength of violet light in the coating, destructive interference for violet light will make the surface look red. Then other colors in spectral order (blue, green, yellow, orange, and red) will interfere destructively, making the surface look red, violet, and then blue. As the coating gets thicker, constructive interference is observed for violet light and then for other colors in spectral order. Even thicker coatings give constructive and destructive interference for several visible wavelengths, so the reflected light starts looking white again.

12. The reflected light is partially polarized, with the component parallel to the reflecting surface being the most intense. Therefore, the polarizing material should have its transmission axis oriented in the vertical direction in order to minimize the intensity of the reflected light from horizontal surfaces.

14. Due to gravity, the soap film tends to sag in its holder, being quite thin at the top and becoming thicker as one moves toward the bottom of the holding ring. Because light reflecting from the front surface of the film experiences a phase reversal, and light reflecting from the back surface of the film does not (see Figure 24.7 in the textbook), the film must be a minimum of a half wavelength thick before it can produce constructive interference in the reflected light. Thus, the light must be striking the film at some distance from the top of the ring before the thickness is sufficient to produce constructive interference for any wavelength in the visible portion of the spectrum.

PROBLEM SOLUTIONS

24.1 The location of the bright fringe of order *m* (measured from the position of the central maximum) is $(y_{\text{bright}})_m = (\lambda L/d)m,\ m = 0,\ \pm1,\ \pm2, \ldots$. Thus, the spacing between successive bright fringes is

$$\Delta y_{\text{bright}} = \left(y_{\text{bright}}\right)_{m+1} - \left(y_{\text{bright}}\right)_m = \left(\lambda L/d\right)\left(m+1\right) - \left(\lambda L/d\right)m = \lambda L/d$$

Thus, the wavelength of the laser light must be

$$\lambda = \frac{\left(\Delta y_{\text{bright}}\right)d}{L} = \frac{\left(1.58\times10^{-2}\ \text{cm}\right)\left(0.200\times10^{-3}\ \text{m}\right)}{5.00\ \text{m}} = 6.32\times10^{-7}\ \text{m} = \boxed{632\ \text{nm}}$$

24.2 (a) For a bright fringe of order *m*, the path difference is $\delta = m\lambda$, where $m = 0,\ 1,\ 2, \ldots$. At the location of the third order bright fringe, $m = 3$ and

$$\delta = 3\lambda = 3\left(589\ \text{nm}\right) = 1.77\times10^{3}\ \text{nm} = \boxed{1.77\ \mu\text{m}}$$

(b) For a dark fringe, the path difference is $\delta = \left(m + \frac{1}{2}\right)\lambda$, where $m = 0,\ 1,\ 2, \ldots$.

At the third dark fringe, $m = 2$ and

$$\delta = \left(2 + \frac{1}{2}\right)\lambda = \frac{5}{2}\left(589\ \text{nm}\right) = 1.47\times10^{3}\ \text{nm} = \boxed{1.47\ \mu\text{m}}$$

24.3 (a) The distance between the central maximum and the first order bright fringe is

$$\Delta y = y_{\text{bright}}\Big|_{m=1} - y_{\text{bright}}\Big|_{m=0} = \frac{\lambda L}{d},\ \text{or}$$

$$\Delta y = \frac{\lambda L}{d} = \frac{\left(546.1\times10^{-9}\ \text{m}\right)\left(1.20\ \text{m}\right)}{0.250\times10^{-3}\ \text{m}} = 2.62\times10^{-3}\ \text{m} = \boxed{2.62\ \text{mm}}$$

(b) The distance between the first and second dark bands is

$$\Delta y = y_{\text{dark}}\Big|_{m=1} - y_{\text{dark}}\Big|_{m=0} = \frac{\lambda L}{d} = \boxed{2.62\ \text{mm}}\ \text{as in (a) above.}$$

24.4 The location of the dark fringe of order *m* (measured from the position of the central maximum) is given by $(y_{\text{dark}})_m = (\lambda L/d)(m + \frac{1}{2})$, where $m = 0,\ \pm1,\ \pm2, \ldots$. Thus, the spacing between the first and second dark fringes will be

$$\Delta y = \left(y_{\text{dark}}\right)_{m=1} - \left(y_{\text{dark}}\right)_{m=0} = \left(\lambda L/d\right)\left(1 + \tfrac{1}{2}\right) - \left(\lambda L/d\right)\left(0 + \tfrac{1}{2}\right) = \lambda L/d$$

or $$\Delta y = \frac{\left(5.30\times10^{-7}\ \text{m}\right)\left(2.00\ \text{m}\right)}{0.300\times10^{-3}\ \text{m}} = 3.53\times10^{-3}\ \text{m} = \boxed{3.53\ \text{mm}}$$

24.5 (a) From $d\sin\theta = m\lambda$, the angle for the $m = 1$ maximum for the sound waves is

$$\theta = \sin^{-1}\left(\frac{m}{d}\lambda\right) = \sin^{-1}\left[\frac{m}{d}\left(\frac{v_{\text{sound}}}{f}\right)\right] = \sin^{-1}\left[\frac{1}{0.300\ \text{m}}\left(\frac{354\ \text{m/s}}{2\ 000\ \text{Hz}}\right)\right] = \boxed{36.2^\circ}$$

continued on next page

(b) For 3.00-cm microwaves, the required slit spacing is

$$d = \frac{m\lambda}{\sin\theta} = \frac{(1)(3.00\ \text{m})}{\sin(36.2^\circ)} = \boxed{5.08\ \text{cm}}$$

(c) The wavelength is $\lambda = d\sin\theta/m$; and if this is light, the frequency is

$$f = \frac{c}{\lambda} = \frac{m\,c}{d\sin\theta} = \frac{(1)(3.00\times10^{8}\ \text{m/s})}{(1.00\times10^{-6}\ \text{m})\sin 36.2^\circ} = \boxed{5.08\times10^{14}\ \text{Hz}}$$

24.6 (a) The location of the bright fringe of order m (measured from the position of the central maximum) is $(y_{\text{bright}})_m = (\lambda L/d)m,\ m = 0,\ \pm1,\ \pm2,\ldots$.

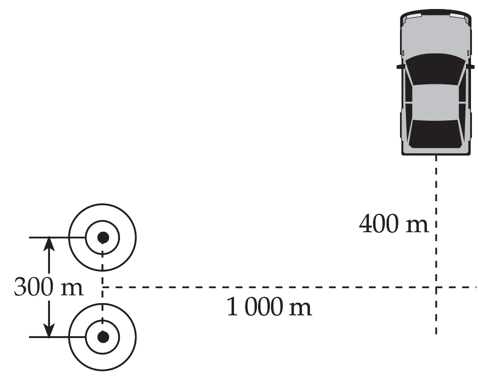
If $(y_{\text{bright}})_{m=3} - (y_{\text{bright}})_{m=0} = 5.30\ \text{cm}$, then $3(\lambda L/d) - 0 = 5.30\ \text{cm}$, and

$$\lambda = \frac{(5.30\ \text{cm})d}{3L} = \frac{(5.30\times10^{-2}\ \text{m})(0.050\,0\times10^{-3}\ \text{m})}{3(1.50\ \text{m})} = 5.89\times10^{-7}\ \text{m} = \boxed{589\ \text{nm}}$$

(b) The separation between adjacent bright fringes is $\Delta y_{\text{bright}} = \lambda L/d$, or

$$\Delta y_{\text{bright}} = \frac{(589\times10^{-9}\ \text{m})(1.50\ \text{m})}{0.050\,0\times10^{-3}\ \text{m}} = 1.77\times10^{-2}\ \text{m} = \boxed{1.77\ \text{cm}}$$

24.7 Note that, with the conditions given, the small angle approximation **does not work well.** That is, $\sin\theta$, $\tan\theta$, and θ are significantly different. The approach to be used is outlined below.



(a) At the $m = 2$ maximum, $\delta = d\sin\theta = 2\lambda$,

or $$\lambda = \frac{d}{2}\sin\theta = \frac{d}{2}\left(\frac{y}{\sqrt{L^2+y^2}}\right)$$

$$\lambda = \frac{(300\ \text{m})}{2}\left[\frac{400\ \text{m}}{\sqrt{(1\,000\ \text{m})^2 + (400\ \text{m})^2}}\right] = \boxed{55.7\ \text{m}}$$

(b) The next minimum encountered is the $m = 2$ minimum; and at that point,

$$\delta = d\sin\theta = \left(m + \frac{1}{2}\right)\lambda = \frac{5}{2}\lambda$$

or $$\theta = \sin^{-1}\left(\frac{5\lambda}{2d}\right) = \sin^{-1}\left(\frac{5(55.7\ \text{m})}{2(300\ \text{m})}\right) = 27.7^\circ$$

Then, $y = (1\,000\ \text{m})\tan 27.7^\circ = 524\ \text{m}$

so the car must travel an additional $\boxed{124\ \text{m}}$.

24.8 The angular position of the bright fringe of order m is given by $d\sin\theta = m\lambda$. Thus, if the $m = 1$ bright fringe is located at $\theta = 12^\circ$ when $\lambda = 6.0\times10^{2}\ \text{nm} = 6.0\times10^{-7}\ \text{m}$, the slit spacing is

$$d = \frac{m\lambda}{\sin\theta} = \frac{(1)(6.0\times10^{-7}\ \text{m})}{\sin 12^\circ} = 2.9\times10^{-6}\ \text{m} = \boxed{2.9\ \mu\text{m}}$$

24.9 The location of the bright fringe of order m (measured from the position of the central maximum) is $(y_{\text{bright}})_m = (\lambda L/d)m,\ m = 0,\ \pm 1,\ \pm 2, \ldots$. If the $m = 1$ bright fringe is located at $y = 3.40$ mm when $d = 0.500$ mm and $L = 3.30$ m, the wavelength of the light is

$$\lambda = \frac{(y_{\text{bright}})_m d}{mL} = \frac{(3.40\times10^{-3}\text{ m})(0.500\times10^{-3}\text{ m})}{(1)(3.30\text{ m})} = 5.15\times10^{-7}\text{ m} = \boxed{515\text{ nm}}$$

24.10 The angular deviation from the line of the central maximum is given by

$$\theta = \tan^{-1}\left(\frac{y}{L}\right) = \tan^{-1}\left(\frac{1.80\text{ cm}}{140\text{ cm}}\right) = 0.737^\circ$$

(a) The path difference is then

$$\delta = d\sin\theta = (0.150\text{ mm})\sin(0.737^\circ) = 1.93\times10^{-3}\text{ mm} = \boxed{1.93\ \mu\text{m}}$$

(b) $$\delta = (1.93\times10^{-6}\text{ m})\left(\frac{\lambda}{643\times10^{-9}\text{ m}}\right) = \boxed{3.00\,\lambda}$$

(c) Since the path difference for this position is a whole number of wavelengths, the waves interfere constructively and produce a $\boxed{\text{maximum}}$ at this spot.

24.11 The distance between the central maximum (position of A) and the first minimum is

$$y = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right)\Bigg|_{m=0} = \frac{\lambda L}{2d}$$

Thus, $d = \dfrac{\lambda L}{2y} = \dfrac{(3.00\text{ m})(150\text{ m})}{2(20.0\text{ m})} = \boxed{11.3\text{ m}}$

24.12 (a) The angular position of the bright fringe of order m is given by $d\sin\theta = m\lambda$, or $\theta_m = \sin^{-1}(m\lambda/d)$. If $d = 25\lambda$, the first three bright fringes are found at

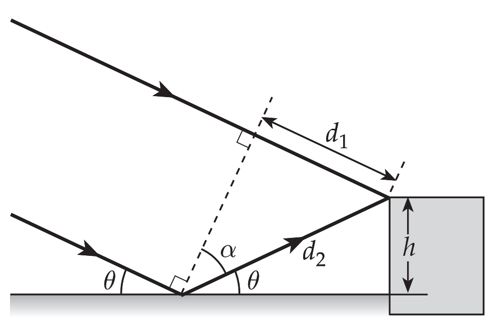
$$\theta_1 = \sin^{-1}\left(\frac{1}{25}\right) = \boxed{2.3^\circ},\quad \theta_2 = \sin^{-1}\left(\frac{2}{25}\right) = \boxed{4.6^\circ},\quad \text{and}\quad \theta_3 = \sin^{-1}\left(\frac{3}{25}\right) = \boxed{6.9^\circ}$$

(b) The angular position of the dark fringe of order m is given by $d\sin\theta = (m + \frac{1}{2})\lambda$, or $\theta_m = \sin^{-1}\left[(m + \frac{1}{2})\lambda/d\right],\ m = 0,\ \pm1,\ \pm2, \ldots$. If $d = 25\lambda$, the first three dark fringes are found at

$$\theta_0 = \sin^{-1}\left(\frac{1/2}{25}\right) = \boxed{1.1^\circ},\quad \theta_1 = \sin^{-1}\left(\frac{3/2}{25}\right) = \boxed{3.4^\circ},\quad \text{and}\quad \theta_2 = \sin^{-1}\left(\frac{5/2}{25}\right) = \boxed{5.7^\circ}$$

(c) The answers are evenly spaced because $\boxed{\text{the angles are small and } \theta \approx \sin\theta}$. At larger angles, the approximation breaks down and the spacing isn't so regular.

24.13 As shown in the figure at the right, the path difference in the waves reaching the telescope is $\delta = d_2 - d_1 = d_2(1 - \sin\alpha)$. If the first minimum $(\delta = \lambda/2)$ occurs when $\theta = 25.0^\circ$, then

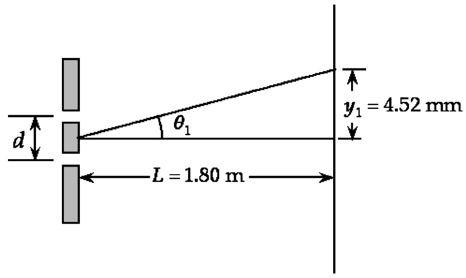


$$\alpha = 180^\circ - (\theta + 90.0^\circ + \theta) = 40.0^\circ, \text{ and}$$

$$d_2 = \frac{\delta}{1 - \sin\alpha} = \frac{(250\text{ m}/2)}{1 - \sin 40.0^\circ} = 350\text{ m}$$

Thus, $h = d_2 \sin 25.0^\circ = \boxed{148\text{ m}}$.

24.14 (a)



(b) $\tan\theta_1 = \dfrac{y_1}{L} = \dfrac{4.52\times10^{-3}\text{ m}}{1.80\text{ m}} = 2.51\times10^{-3} = \boxed{0.002\ 51}$

(c) $\theta_1 = \tan^{-1}\left(2.51\times10^{-3}\right) = \boxed{0.144°}$ and $\sin\theta_1 = \sin(0.144°) = \boxed{2.51\times10^{-3}}$
The sine and the tangent are very nearly the same, but only because the angle is small.

(d) From $\delta = d\sin\theta = m\lambda$ for the order m bright fringe,

$$\lambda = \frac{d\sin\theta_1}{1} = \frac{\left(2.40\times10^{-4}\text{ m}\right)\sin(0.144°)}{1} = 6.03\times10^{-7}\text{ m} = \boxed{603\text{ nm}}$$

(e) $\theta_5 = \sin^{-1}\left(\dfrac{5\lambda}{d}\right) = \sin^{-1}\left[\dfrac{5\left(6.03\times10^{-7}\text{ m}\right)}{2.40\times10^{-4}\text{ m}}\right] = \boxed{0.720°}$

(f) $y_5 = L\tan\theta_5 = (1.80\text{ m})\tan(0.720°) = 2.26\times10^{-2}\text{ m} = \boxed{2.26\text{ cm}}$

24.15 The path difference in the two waves received at the home is $\delta = 2d$, where d is the distance from the home to the mountain. Neglecting any phase change upon reflection, the condition for destructive interference is

$$\delta = \left(m+\frac{1}{2}\right)\lambda \text{ with } m = 0, 1, 2, \ldots$$

so $d_{\min} = \dfrac{\delta_{\min}}{2} = \left(0+\dfrac{1}{2}\right)\dfrac{\lambda}{2} = \dfrac{\lambda}{4} = \dfrac{300\text{ m}}{4} = \boxed{75.0\text{ m}}$

24.16 (a) With phase reversal in the reflection at the outer surface of the soap film and no reversal on reflection from the inner surface, the condition for constructive interference in the light reflected from the soap bubble is $2n_{\text{film}}t = (m+\frac{1}{2})\lambda$ where $m = 0, 1, 2, \ldots$. For the lowest order reflection ($m = 0$), and the wavelength is

$$\lambda = \frac{2n_{\text{water}}t}{(0+1/2)} = 4(1.333)(120\text{ nm}) = 6.4\times10^2\text{ nm} = \boxed{640\text{ nm}}$$

(b) To strongly reflect the same wavelength light, a thicker film will need to make use of a higher order reflection, i.e., $\boxed{\text{use a larger value of } m}$.

(c) The next greater thickness of soap film that can strongly reflect 640 nm light corresponds to $m = 1$, giving

$$t_1 = \frac{(1+1/2)\lambda}{2n_{\text{film}}} = \frac{3}{2}\left[\frac{640\text{ nm}}{2(1.333)}\right] = 3.6\times10^2\text{ nm} = \boxed{360\text{ nm}}$$

and the third such thickness (corresponding to $m = 2$) is

$$t_2 = \frac{(2+1/2)\lambda}{2n_{\text{film}}} = \frac{5}{2}\left[\frac{640\text{ nm}}{2(1.333)}\right] = 6.0\times10^2\text{ nm} = \boxed{600\text{ nm}}$$

24.17 Light reflecting from the first (glass-iodine) interface suffers a phase reversal, but light reflecting at the second (iodine-glass) interface does not have a phase reversal. Thus, the condition for constructive interference in the reflected light is $2n_{\text{film}}t = (m + \frac{1}{2})\lambda$ with $m = 0, 1, 2, \ldots$. The smallest film thickness capable of strongly reflecting the incident light is

$$t_{\min} = \frac{(0 + 1/2)\lambda}{2n_{\text{film}}} = \frac{6.00 \times 10^{2} \text{ nm}}{4(1.756)} = \boxed{85.4 \text{ nm}}$$

24.18 (a) With $n_{\text{air}} < n_{\text{oil}} < n_{\text{water}}$, phase reversals are experienced by light reflecting at both surfaces of the oil film, an upper air-oil interface and a lower oil-water interface. Under these conditions, the requirement for constructive interference is $2n_{\text{film}}t = m_1\lambda$ with $m_1 = 0, 1, 2, \ldots$, and the requirement for destructive interference is $2n_{\text{film}}t = (m_2 + \frac{1}{2})\lambda$ with $m_2 = 0, 1, 2, \ldots$. To have the thinnest film that produces simultaneous constructive interference of $\lambda_1 = 640$ nm and destructive interference of $\lambda_2 = 512$ nm, it is necessary that

$$2n_{\text{film}}t = m_1(640 \text{ nm}) = \left(m_2 + \frac{1}{2}\right)(512 \text{ nm})$$

where both m_1 and m_2 are the smallest integers for which this is true. It is found that $m_1 = m_2 = 2$ are the smallest integer values that will satisfy this condition, giving the minimum acceptable film thickness as

$$t_{\min} = \frac{m_1\lambda_1}{2n_{\text{film}}} = \frac{(m_2 + \frac{1}{2})\lambda_2}{2n_{\text{film}}} = \frac{2(640 \text{ nm})}{2(1.25)} = \frac{(2.5)(512 \text{ nm})}{2(1.25)} = \boxed{512 \text{ nm}}$$

(b) From the discussion above, it is seen that in order to have a film thickness that produces simultaneous constructive interference of 640-nm light and destructive interference of 512-nm light, it is necessary that

$$m_1(640 \text{ nm}) = \left(m_2 + \frac{1}{2}\right)(512 \text{ nm}) \quad \text{or} \quad m_1\left(\frac{640 \text{ nm}}{512 \text{ nm}}\right) = m_2 + \frac{1}{2}$$

This gives $\quad (1.25)m_1 = m_2 + \frac{1}{2} \quad$ or $\quad \boxed{2.5\, m_1 = 2m_2 + 1}$.

24.19 With $n_{\text{coating}} > n_{\text{air}}$ and $n_{\text{coating}} > n_{\text{lens}}$, light reflecting at the air-coating boundary experiences a phase reversal, but light reflecting from the coating-lens boundary does not. Therefore, the condition for destructive interference in the two reflected waves is

$$2n_{\text{coating}}t = m\lambda \qquad \text{where} \qquad m = 0, 1, 2, \ldots$$

For finite wavelengths, the lowest allowed value of m is $m = 1$. Then, if $t = 177.4$ nm and $n_{\text{coating}} = 1.55$, the wavelength associated with this lowest order destructive interference is

$$\lambda_1 = \frac{2n_{\text{coating}}t}{1} = 2(1.55)(177.4 \text{ nm}) = \boxed{550 \text{ nm}}$$

24.20 Since $n_{\text{air}} < n_{\text{oil}} < n_{\text{water}}$, light reflected from both top and bottom surfaces of the oil film experiences phase reversal, resulting in zero net phase difference due to reflections. Therefore, the condition for constructive interference in reflected light is

$$2t = m\lambda_n = m\frac{\lambda}{n_{\text{film}}}, \text{ or } t = m\left(\frac{\lambda}{2n_{\text{film}}}\right) \text{ where } m = 0, 1, 2, \ldots$$

Assuming that $m = 1$, the thickness of the oil slick is

$$t = (1)\frac{\lambda}{2n_{\text{film}}} = \frac{600 \text{ nm}}{2(1.29)} = \boxed{233 \text{ nm}}$$

24.21 There will be a phase reversal of the radar waves reflecting from both surfaces of the polymer, giving zero net phase change due to reflections. The requirement for destructive interference in the reflected waves is then

$$2t = \left(m + \frac{1}{2}\right)\lambda_n, \text{ or } t = (2m+1)\frac{\lambda}{4n_{\text{film}}} \text{ where } m = 0, 1, 2, \ldots$$

If the film is as thin as possible, then $m = 0$ and the needed thickness is

$$t = \frac{\lambda}{4n_{\text{film}}} = \frac{3.00 \text{ cm}}{4(1.50)} = \boxed{0.500 \text{ cm}}$$

This anti-reflectance coating could be easily countered by changing the wavelength of the radar — to 1.50 cm — now creating maximum reflection!

24.22 (a) With $n_{\text{air}} < n_{\text{water}} < n_{\text{oil}}$, reflections at the air-oil interface experience a phase reversal, while reflections at the oil-water interface experience no phase reversal. Thus, with one phase reversal at the surfaces, the condition for constructive interference in the light reflected by the film is $2n_{\text{film}}t = (m + \frac{1}{2})\lambda$, where m is any positive integer. Thus,

$$\lambda_m = \frac{2n_{\text{film}}t}{m + \frac{1}{2}} = \frac{2(1.45)(280 \text{ nm})}{m + \frac{1}{2}} = \frac{812 \text{ nm}}{m + \frac{1}{2}} \qquad m = 0, 1, 2, 3, \ldots$$

The possible wavelengths are: $\lambda_0 = 1.62 \times 10^3$ nm, $\lambda_1 = 541$ nm, $\lambda_2 = 325$nm, …

Of these, only $\boxed{\lambda_1 = 541 \text{ nm} \quad \text{(green)}}$ is in the visible portion of the spectrum.

(b) The wavelengths that will be most strongly transmitted are those that suffer destructive interference in the reflected light. With one phase reversal at the surfaces, the condition for destructive interference in the light reflected by the film is $2n_{\text{film}}t = m\lambda$, where m is any positive, nonzero, integer. The possible wavelengths are

$$\lambda_m = \frac{2n_{\text{film}}t}{m} = \frac{2(1.45)(280 \text{ nm})}{m} = \frac{812 \text{ nm}}{m} \qquad m = 1, 2, 3, \ldots$$

or $\lambda_1 = 812$ nm, $\lambda_2 = 406$ nm, $\lambda_3 = 271$ nm, …

of which only $\boxed{\lambda_2 = 406 \text{ nm} \quad \text{(violet)}}$ is in the visible spectrum.

24.23 (a) For maximum transmission, we want destructive interference in the light reflected from the front and back surfaces of the film.

If the surrounding glass has refractive index greater than 1.378, light reflected from the front surface suffers no phase reversal, and light reflected from the back does undergo phase reversal. This effect by itself would produce destructive interference, so we want the distance down and back to be one whole wavelength in the film. Thus, we require that

$$2t = \lambda_n = \lambda/n_{\text{film}} \quad \text{or} \quad t = \frac{\lambda}{2n_{\text{film}}} = \frac{656.3\text{ nm}}{2(1.378)} = \boxed{238\text{ nm}}$$

(b) The filter will expand. As t increases in $2n_{\text{film}}\,t = \lambda$, so does $\boxed{\lambda\text{ increase}}$.

(c) Destructive interference for reflected light happens also for λ in $2t = 2\lambda/n_{\text{film}}$, or

$$\lambda = n_{\text{film}}t = (1.378)(238\text{ nm}) = \boxed{328\text{ nm}} \text{ (near ultraviolet)}$$

24.24 Light reflecting from the lower surface of the air layer experiences phase reversal, but light reflecting from the upper surface of the layer does not. The requirement for a dark fringe (destructive interference) is then

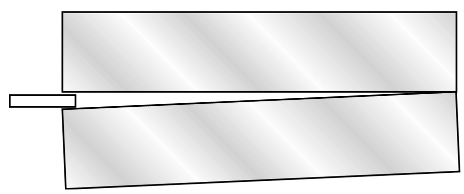
$$2t = m\lambda_n = m\left(\frac{\lambda}{n_{\text{air}}}\right) = m\lambda, \text{ where } m = 0, 1, 2, \ldots$$

At the thickest part of the film $(t = 2.00\ \mu\text{m})$, the order number is

$$m = \frac{2t}{\lambda} = \frac{2(2.00\times10^{-6}\text{ m})}{546.1\times10^{-9}\text{ m}} = 7.32$$

Since m must be an integer, $m = 7$ is the order of the last dark fringe seen. Counting the $m = 0$ order along the edge of contact, a total of $\boxed{8\text{ dark fringes}}$ will be seen.

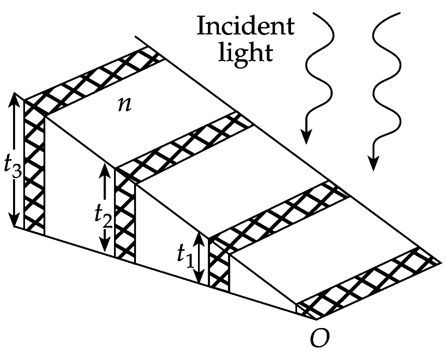
24.25 With a phase reversal upon reflection from the lower surface of the air layer and no phase change for reflection at the upper surface of the layer, the condition for destructive interference is



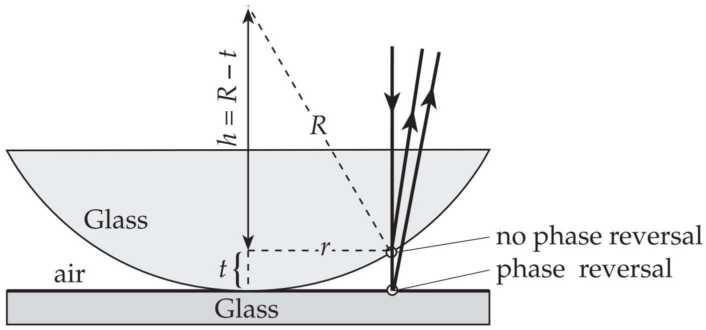
$$2t = m\lambda_n = m\left(\frac{\lambda}{n_{\text{air}}}\right) = m\lambda, \text{ where } m = 0, 1, 2, \ldots$$

Counting the zeroth order along the edge of contact, the order number of the thirtieth dark fringe observed is $m = 29$. The thickness of the air layer at this point is $t = 2r$, where r is the radius of the wire. Thus,

$$r = \frac{t}{2} = \frac{29\lambda}{4} = \frac{29(600\times10^{-9}\text{ m})}{4} = \boxed{4.35\ \mu\text{m}}$$



24.26



From the geometry shown in the figure, $R^2 = (R-t)^2 + r^2$, or

$$t = R - \sqrt{R^2 - r^2}$$

$$= 3.0\text{ m} - \sqrt{(3.0\text{ m})^2 - (9.8\times10^{-3}\text{ m})^2}$$

$$= 1.6\times10^{-5}\text{ m}$$

With a phase reversal upon reflection at the lower surface of the air layer, but no reversal with reflection from the upper surface, the condition for a bright fringe is

$$2t = \left(m+\frac{1}{2}\right)\lambda_n = \left(m+\frac{1}{2}\right)\frac{\lambda}{n_{\text{air}}} = \left(m+\frac{1}{2}\right)\lambda, \text{ where } m = 0, 1, 2, \ldots$$

At the 50^{th} bright fringe, $m = 49$, and the wavelength is found to be

$$\lambda = \frac{2t}{m+1/2} = \frac{2(1.6\times10^{-5}\text{ m})}{49.5} = 6.5\times10^{-7}\text{ m} = \boxed{6.5\times10^{2}\text{ nm}}$$

24.27 There is a phase reversal due to reflection at the bottom of the air film but not at the top of the film. The requirement for a dark fringe is then

$$2t = m\lambda_n = m\frac{\lambda}{n_{\text{air}}} = m\lambda, \text{ where } m = 0, 1, 2, \ldots$$

At the 19^{th} dark ring (in addition to the dark center spot), the order number is $m = 19$, and the thickness of the film is

$$t = \frac{m\lambda}{2} = \frac{19(500\times10^{-9}\text{ m})}{2} = 4.75\times10^{-6}\text{ m} = \boxed{4.75\ \mu\text{m}}$$

24.28 With a phase reversal due to reflection at each surface of the magnesium fluoride layer, there is zero net phase difference caused by reflections. The condition for destructive interference is then

$$2t = \left(m+\frac{1}{2}\right)\lambda_n = \left(m+\frac{1}{2}\right)\frac{\lambda}{n_{\text{film}}}, \text{ where } m = 0, 1, 2, \ldots$$

For minimum thickness, $m = 0$, and the thickness is

$$t = (2m+1)\frac{\lambda}{4n_{\text{film}}} = (1)\frac{(550\times10^{-9}\text{ m})}{4(1.38)} = 9.96\times10^{-8}\text{ m} = \boxed{99.6\text{ nm}}$$

24.29 There is a phase reversal upon reflection at each surface of the film and hence zero net phase difference due to reflections. The requirement for constructive interference in the reflected light is then

$$2t = m\lambda_n = m\frac{\lambda}{n_{\text{film}}}\text{, where } m = 1,\ 2,\ 3, \ldots$$

With $t = 1.00\times10^{-5}$ cm $= 100$ nm, and $n_{\text{film}} = 1.38$, the wavelengths intensified in the reflected light are

$$\lambda = \frac{2\,n_{\text{film}}\,t}{m} = \frac{2(1.38)(100\text{ nm})}{m}\text{, with } m = 1,\ 2,\ 3, \ldots$$

Thus, $\lambda = \boxed{276\text{ nm, }138\text{ nm, }92.0\text{ nm }\ldots}$

and $\boxed{\text{none of these wavelengths are in the visible spectrum}}$.

24.30 The transmitted light is brightest when the reflected light is a minimum (that is, the same conditions that produce destructive interference in the reflected light will produce constructive interference in the transmitted light). As light enters the air layer from glass, any light reflected at this surface has zero phase change. Light reflected from the other surface of the air layer (where light is going from air into glass) does have a phase reversal. Thus, the condition for destructive interference in the light reflected from the air film is $2t = m\lambda_n$, $m = 0, 1, 2, \ldots$.

Since $\lambda_n = \dfrac{\lambda}{n_{\text{film}}} = \dfrac{\lambda}{1.00} = \lambda$, the minimum nonzero plate separation satisfying this condition is

$$d = t = (1)\frac{\lambda}{2} = \frac{580\text{ nm}}{2} = \boxed{290\text{ nm}}$$

24.31 In a single slit diffraction pattern, with the slit having width *a*, the dark fringe of order *m* occurs at angle θ_m, where $\sin\theta_m = m(\lambda/a)$ and $m = \pm1,\ \pm2,\ \pm3, \ldots$. The location, on a screen located distance *L* from the slit, of the dark fringe of order *m* (measured from $y = 0$ at the center of the central maximum) is $(y_{\text{dark}})_m = L\tan\theta_m \approx L\sin\theta_m = m\lambda(L/a)$.

(a) The central maximum extends from the $m = -1$ dark fringe to the $m = +1$ dark fringe, so the width of this central maximum is

$$\text{Central max. width} = (y_{\text{dark}})_1 - (y_{\text{dark}})_{-1} = 1\left(\frac{\lambda L}{a}\right) - (-1)\left(\frac{\lambda L}{a}\right) = \frac{2\lambda L}{a}$$

$$= \frac{2(5.40\times10^{-7}\text{ m})(1.50\text{ m})}{0.200\times10^{-3}\text{ m}} = 8.10\times10^{-3}\text{ m} = \boxed{8.10\text{ mm}}$$

(b) The first order bright fringe extends from the $m = 1$ dark fringe to the $m = 2$ dark fringe, or

$$(\Delta y_{\text{bright}})_1 = (y_{\text{dark}})_2 - (y_{\text{dark}})_1 = 2\left(\frac{\lambda L}{a}\right) - 1\left(\frac{\lambda L}{a}\right) = \frac{\lambda L}{a}$$

$$= \frac{(5.40\times10^{-7}\text{ m})(1.50\text{ m})}{0.200\times10^{-3}\text{ m}} = 4.05\times10^{-3}\text{ m} = \boxed{4.05\text{ mm}}$$

Note that the width of the first order bright fringe is exactly one-half the width of the central maximum.

24.32 (a) Dark bands occur where $\sin\theta = m(\lambda/a)$. At the first dark band, $m = 1$, and the distance from the center of the central maximum is

$$y_1 = L\tan\theta \approx L\sin\theta = L\left(\frac{\lambda}{a}\right)$$

$$= (1.5\ \text{m})\left(\frac{600\times10^{-9}\ \text{m}}{0.40\times10^{-3}\ \text{m}}\right) = 2.25\times10^{-3}\ \text{m} = \boxed{2.3\ \text{mm}}$$

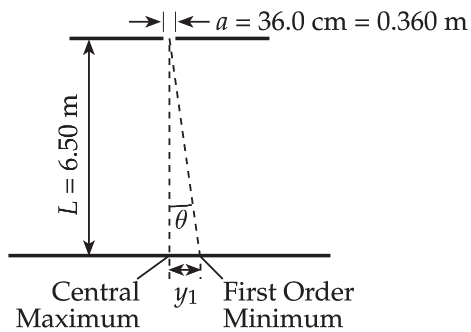
(b) The width of the central maximum is $2y_1 = 2(2.25\ \text{mm}) = \boxed{4.5\ \text{mm}}$

24.33 (a) Dark bands (minima) occur where $\sin\theta = m(\lambda/a)$. For the first minimum, $m = 1$ and the distance from the center of the central maximum is $y_1 = L\tan\theta \approx L\sin\theta = L(\lambda/a)$. Thus, the needed distance to the screen is

$$L = y_1\left(\frac{a}{\lambda}\right) = (0.85\times10^{-3}\ \text{m})\left(\frac{0.75\times10^{-3}\ \text{m}}{587.5\times10^{-9}\ \text{m}}\right) = \boxed{1.1\ \text{m}}$$

(b) The width of the central maximum is $2y_1 = 2(0.85\ \text{mm}) = \boxed{1.7\ \text{mm}}$.

24.34 **Note:** The small angle approximation does not work well in this situation. Rather, you should proceed as follows.



At the first order minimum, $\sin\theta_1 = (1)\lambda/a$ or

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{a}\right) = \sin^{-1}\left(\frac{5.00\ \text{cm}}{36.0\ \text{cm}}\right) = 7.98°$$

Then, $y_1 = L\tan\theta_1 = (6.50\ \text{m})\tan 7.98° = 0.912\ \text{m} = \boxed{91.2\ \text{cm}}$

24.35 With the screen locations of the dark fringe of order m at

$$(y_{\text{dark}})_m = L\tan\theta_m \approx L\sin\theta_m = m(\lambda L/a) \quad \text{for } m = \pm1, \pm2, \pm3, \ldots$$

the width of the central maximum is $\Delta y_{\substack{\text{central}\\\text{maximum}}} = (y_{\text{dark}})_{m=+1} - (y_{\text{dark}})_{m=-1} = 2(\lambda L/a)$, so

$$\lambda = \frac{a\left(\Delta y_{\substack{\text{central}\\\text{maximum}}}\right)}{2L} = \frac{(0.600\times10^{-3}\ \text{m})(2.00\times10^{-3}\ \text{m})}{2(1.30\ \text{m})} = 4.62\times10^{-7}\ \text{m} = \boxed{462\ \text{nm}}$$

24.36 At the positions of the minima, $\sin\theta_m = m(\lambda/a)$ and

$$y_m = L\tan\theta_m \approx L\sin\theta_m = m\left[L(\lambda/a)\right]$$

Thus, $\tan\theta_p = n_2/n_1$

and $$a = \frac{2L\lambda}{y_3 - y_1} = \frac{2(0.500\ \text{m})(680\times10^{-9}\ \text{m})}{3.00\times10^{-3}\ \text{m}} = 2.27\times10^{-4}\ \text{m} = \boxed{0.227\ \text{mm}}$$

24.37 The locations of the dark fringes (minima) mark the edges of the maxima, and the widths of the maxima equals the spacing between successive minima.

At the locations of the minima, $\sin\theta_m = m(\lambda/a)$ and

$$y_m = L\tan\theta_m \approx L\sin\theta_m = m\left[L(\lambda/a)\right]$$
$$= m\left[(1.20\text{ m})\left(\frac{500\times10^{-9}\text{ m}}{0.500\times10^{-3}\text{ m}}\right)\right] = m(1.20\text{ mm})$$

Then, $\Delta y = \Delta m(1.20\text{ mm})$ and for successive minima, $\Delta m = 1$.

Therefore, the width of each maxima, *other than the central maximum,* in this interference pattern is

$$\text{width} = \Delta y = (1)(1.20\text{ mm}) = \boxed{1.20\text{ mm}}$$

24.38 For diffraction by a grating, the angle at which the maximum of order m occurs is given by $d\sin\theta_m = m\lambda$, where d is the spacing between adjacent slits on the grating. Thus,

$$d = \frac{m\lambda}{\sin\theta_m} = \frac{(1)(632.8\times10^{-9}\text{ m})}{\sin 20.5°} = 1.81\times10^{-6}\text{ m} = \boxed{1.81\ \mu\text{m}}$$

24.39 The grating spacing is $d = (1/3\,660)\text{cm} = (1/3.66\times10^5)\text{ m}$ and $d\sin\theta = m\lambda$.

(a) The wavelength observed in the first-order spectrum is $\lambda = d\sin\theta$, or

$$\lambda = \left(\frac{1\text{ m}}{3.66\times10^5}\right)\left(\frac{10^9\text{ nm}}{1\text{ m}}\right)\sin\theta = \left(\frac{10^4\text{ nm}}{3.66}\right)\sin\theta$$

This yields: at 10.1°, $\lambda = \boxed{479\text{ nm}}$; at 13.7°, $\lambda = \boxed{647\text{ nm}}$;

and at 14.8°, $\lambda = \boxed{698\text{ nm}}$

(b) In the second order, $m = 2$. The second order images for the above wavelengths will be found at angles $\theta_2 = \sin^{-1}(2\lambda/d) = \sin^{-1}[2\sin\theta_1]$.

This yields: for $\lambda = 479$ nm, $\theta_2 = \boxed{20.5°}$; for $\lambda = 647$ nm, $\theta_2 = \boxed{28.3°}$;
and for $\lambda = 698$ nm, $\theta_2 = \boxed{30.7°}$

24.40 (a) The longest wavelength in the visible spectrum is 700 nm, and the grating spacing is $d = 1\text{ mm}/600 = 1.67\times10^{-3}\text{ mm} = 1.67\times10^{-6}\text{ m}$.

Thus, $$m_{\text{max}} = \frac{d\sin 90.0°}{\lambda_{\text{red}}} = \frac{\left(1.67\times10^{-6}\text{ m}\right)\sin 90.0°}{700\times10^{-9}\text{ m}} = 2.38$$

so $\boxed{\text{2 complete orders}}$ will be observed.

(b) From $\lambda = d\sin\theta$, the angular separation of the red and violet edges in the first order will be

$$\Delta\theta = \sin^{-1}\left[\frac{\lambda_{\text{red}}}{d}\right] - \sin^{-1}\left[\frac{\lambda_{\text{violet}}}{d}\right] = \sin^{-1}\left[\frac{700\times10^{-9}\text{ m}}{1.67\times10^{-6}\text{ m}}\right] - \sin^{-1}\left[\frac{400\times10^{-9}\text{ m}}{1.67\times10^{-6}\text{ m}}\right]$$

or $\Delta\theta = \boxed{10.9°}$

24.41 The grating spacing is $d = \dfrac{1\text{ cm}}{4\ 500} = \dfrac{1\text{ m}}{4.50\times10^{5}}$. From $d\sin\theta = m\lambda$, the angular separation between the given spectral lines will be

$$\Delta\theta = \sin^{-1}\left[\frac{m\,\lambda_{\text{red}}}{d}\right] - \sin^{-1}\left[\frac{m\,\lambda_{\text{violet}}}{d}\right]$$

or

$$\Delta\theta = \sin^{-1}\left[\frac{m\left(656\times10^{-9}\text{ m}\right)\left(4.50\times10^{5}\right)}{1\text{ m}}\right] - \sin^{-1}\left[\frac{m\left(434\times10^{-9}\text{ m}\right)\left(4.50\times10^{5}\right)}{1\text{ m}}\right]$$

The results obtained are: for $m = 1$, $\Delta\theta = \boxed{5.91°}$; for $m = 2$, $\Delta\theta = \boxed{13.2°}$;
and for $m = 3$, $\Delta\theta = \boxed{26.5°}$. Complete orders for $m \geq 4$ are not visible.

24.42 (a) If $d = \dfrac{1\text{ cm}}{1\ 500} = 6.67\times10^{-4}\text{ cm} = 6.67\times10^{-6}\text{ m}$, the highest order of $\lambda = 500\text{ nm}$ that can be observed will be

$$m_{\text{max}} = \frac{d\sin 90°}{\lambda} = \frac{\left(6.67\times10^{-6}\text{ m}\right)(1)}{500\times10^{-9}\text{ m}} = 13.3 \text{ or } \boxed{\text{13 orders}}$$

(b) If $d = \dfrac{1\text{ cm}}{15\ 000} = 6.67\times10^{-5}\text{ cm} = 6.67\times10^{-7}\text{ m}$, then

$$m_{\text{max}} = \frac{d\sin 90°}{\lambda} = \frac{\left(6.67\times10^{-7}\text{ m}\right)(1)}{500\times10^{-9}\text{ m}} = 1.33 \text{ or } \boxed{\text{1 order}}$$

24.43 The grating spacing is $d = \dfrac{1\text{ cm}}{5\,000} = 2.00\times10^{-4}\text{ cm} = 2.00\times10^{-6}\text{ m}$, and $d\sin\theta = m\lambda$ gives the angular position of a second order spectral line as

$$\sin\theta = \frac{2\lambda}{d} \quad\text{or}\quad \theta = \sin^{-1}\left(\frac{2\lambda}{d}\right)$$

For the given wavelengths, the angular positions are

$$\theta_1 = \sin^{-1}\left[\frac{2\left(610\times10^{-9}\text{ m}\right)}{2.00\times10^{-6}\text{ m}}\right] = 37.6^\circ \quad\text{and}\quad \theta_2 = \sin^{-1}\left[\frac{2\left(480\times10^{-9}\text{ m}\right)}{2.00\times10^{-6}\text{ m}}\right] = 28.7^\circ$$

If L is the distance from the grating to the screen, the distance on the screen from the central maximum to a second order bright line is $y = L\tan\theta$. Therefore, for the two given wavelengths, the screen separation is

$$\Delta y = L\left[\tan\theta_1 - \tan\theta_2\right]$$

$$= (2.00\text{ m})\left[\tan(37.6^\circ) - \tan(28.7^\circ)\right] = 0.445\text{ m} = \boxed{44.5\text{ cm}}$$

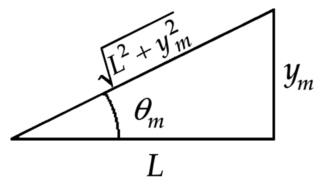
24.44 With 2 000 lines per centimeter, the grating spacing is

$$d = \frac{1}{2\,000}\text{ cm} = 5.00\times10^{-4}\text{ cm} = 5.00\times10^{-6}\text{ m}$$

Then, from $d\sin\theta = m\lambda$, the location of the first order for the red light is

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{1\left(640\times10^{-9}\text{ m}\right)}{5.00\times10^{-6}\text{ m}}\right) = \boxed{7.35^\circ}$$

24.45 The spacing between adjacent slits on the grating is

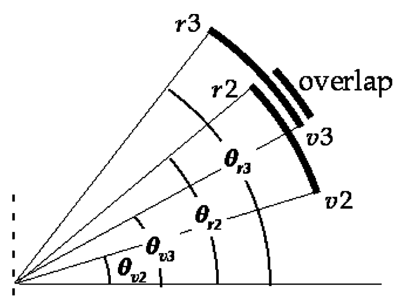


$$d = \frac{1\text{ cm}}{5\,310\text{ slits}} = \frac{10^{-2}\text{ m}}{5\,310}$$

The maximum of order m is located where $d\sin\theta_m = m\lambda$, so

$$\lambda = \frac{d}{m}\sin\theta_m = \frac{d}{m}\left(\frac{y_m}{\sqrt{L^2+y_m^2}}\right) = \frac{10^{-2}\text{ m}}{(1)(5\,310)}\left(\frac{0.488\text{ m}}{\sqrt{(1.72\text{ m})^2+(0.488\text{ m})^2}}\right) = 5.14\times10^{-7}\text{ m} = \boxed{514\text{ nm}}$$

24.46 From $d\sin\theta_m = m\lambda$, or $\sin\theta_m = m\lambda/d$, we see that



$$\sin\theta_{v2} = \frac{2\lambda_{\text{violet}}}{d} = \frac{800\text{ nm}}{d} \qquad \sin\theta_{v3} = \frac{3\lambda_{\text{violet}}}{d} = \frac{1\,200\text{ nm}}{d}$$

$$\sin\theta_{r2} = \frac{2\lambda_{\text{red}}}{d} = \frac{1\,400\text{ nm}}{d} \qquad \sin\theta_{r3} = \frac{3\lambda_{\text{red}}}{d} = \frac{2\,100\text{ nm}}{d}$$

Since, for $0^\circ \le \theta \le 90^\circ$, θ increases as $\sin\theta$ increases, we have that

$$\theta_{v2} < \theta_{v3} < \theta_{r2} < \theta_{r3}$$

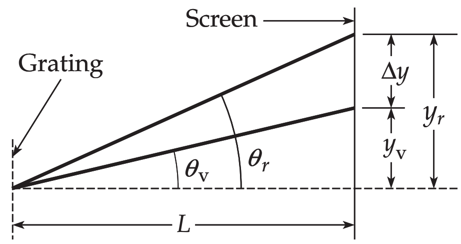
so $\boxed{\text{the second and third order spectra overlap in the range } \theta_{v3} \le \theta \le \theta_{r2}}$.

24.47 The grating spacing is $d = \dfrac{1\text{ cm}}{2\,750} = \dfrac{10^{-2}\text{ m}}{2\,750} = 3.636\times10^{-6}\text{ m}$. From $d\sin\theta = m\lambda$, or $\theta = \sin^{-1}(m\lambda/d)$, the angular positions of the red and violet edges of the second-order spectrum are found to be

$$\theta_{\mathrm{r}} = \sin^{-1}\left(\frac{2\lambda_{\mathrm{red}}}{d}\right) = \sin^{-1}\left(\frac{2\left(700\times10^{-9}\text{ m}\right)}{3.636\times10^{-6}\text{ m}}\right) = 22.65°$$

and $$\theta_{\mathrm{v}} = \sin^{-1}\left(\frac{2\lambda_{\mathrm{violet}}}{d}\right) = \sin^{-1}\left(\frac{2\left(400\times10^{-9}\text{ m}\right)}{3.636\times10^{-6}\text{ m}}\right) = 12.71°$$

Note from the sketch at the right that $y_{\mathrm{r}} = L\tan\theta_{\mathrm{r}}$ and $y_{\mathrm{v}} = L\tan\theta_{\mathrm{v}}$, so the width of the spectrum on the screen is $\Delta y = L\left(\tan\theta_{\mathrm{r}} - \tan\theta_{\mathrm{v}}\right)$.



Since it is given that $\Delta y = 1.75$ cm, the distance from the grating to the screen must be

$$L = \frac{\Delta y}{\tan\theta_{\mathrm{r}} - \tan\theta_{\mathrm{v}}} = \frac{1.75\text{ cm}}{\tan(22.65°) - \tan(12.71°)}$$

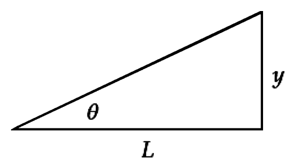
or $$L = \boxed{9.13\text{ cm}}$$

24.48 (a) $$d = \frac{1\text{ cm}}{4.200\times10^{3}\text{ slits}} = 2.381\times10^{-4}\text{ cm} = \boxed{2.381\times10^{-6}\text{ m}}$$

(b) $$d\sin\theta_m = m\lambda \quad\Rightarrow\quad \theta_2 = \sin^{-1}\left(\frac{2\lambda}{d}\right) = \sin^{-1}\left[\frac{2\left(589.0\times10^{-9}\text{ m}\right)}{2.381\times10^{-6}\text{ m}}\right] = \boxed{29.65°}$$

(c) $$y_2 = L\tan\theta_2 = (2.000\text{ m})\tan(29.65°) = \boxed{1.138\text{ m}}$$

(d) $$\theta_2' = \sin^{-1}\left(\frac{2\lambda'}{d}\right) = \sin^{-1}\left[\frac{2\left(589.6\times10^{-9}\text{ m}\right)}{2.381\times10^{-6}\text{ m}}\right] = \boxed{29.69°}$$



$$y_2' = L\tan\theta_2' = (2.000\text{ m})\tan(29.69°) = \boxed{1.140\text{ m}}$$

(e) $$\Delta y = y_2' - y_2 = 1.140\text{ m} - 1.138\text{ m} = \boxed{2\times10^{-3}\text{ m}}$$

(f) $$\Delta y = y_2' - y_2 = L\left[\tan\left(\sin^{-1}\left(\frac{2\lambda'}{d}\right)\right) - \tan\left(\sin^{-1}\left(\frac{2\lambda}{d}\right)\right)\right]$$

$$= (2.000\text{ m})\left[\tan\left(\sin^{-1}\left(\frac{2\left(589.6\times10^{-9}\text{ m}\right)}{10^{-2}\text{ m}/4.200\times10^{3}}\right)\right) - \tan\left(\sin^{-1}\left(\frac{2\left(589.0\times10^{-9}\text{ m}\right)}{10^{-2}\text{ m}/4.200\times10^{3}}\right)\right)\right]$$

$$= \boxed{1.537\times10^{-3}\text{ m}}$$

The two answers agree to only one significant figure. The calculation is sensitive to rounding at intermediate steps.

24.49 The grating spacing is $d = \frac{1.00\text{ mm}}{400} = 2.50\times10^{-3}\text{ mm} = 2.50\times10^{-6}\text{ m}$.

From $d\sin\theta = m\lambda$, the angle of the second-order diffracted ray is $\theta = \sin^{-1}(2\lambda/d)$.

(a) When the grating is surrounded by air, the wavelength is $\lambda_{\text{air}} = \lambda/n_{\text{air}} \approx \lambda$ and

$$\theta_a = \sin^{-1}\left(\frac{2\lambda_{\text{air}}}{d}\right) = \sin^{-1}\left(\frac{2\left(541\times10^{-9}\text{ m}\right)}{2.50\times10^{-6}\text{ m}}\right) = \boxed{25.6^\circ}$$

(b) If the grating is immersed in water,

then $\lambda_n = \lambda_{\text{water}} = \lambda/n_{\text{water}} = \lambda/1.333$, yielding

$$\theta_b = \sin^{-1}\left(\frac{2\lambda_{\text{water}}}{d}\right) = \sin^{-1}\left[\frac{2\left(541\times10^{-9}\text{ m}\right)}{\left(2.50\times10^{-6}\text{ m}\right)(1.333)}\right] = \boxed{19.0^\circ}$$

(c) From $\sin\theta = \frac{m\lambda_n}{d} = \frac{m(\lambda/n)}{d}$, we have that $n\sin\theta = \frac{m\lambda}{d} = \textit{constant}$ when m is kept constant. Therefore $\boxed{n_{\text{air}}\sin\theta_a = n_{\text{water}}\sin\theta_b}$, or the angles of parts (a) and (b) satisfy Snell's law.

24.50 The grating spacing is $d = \frac{1\text{ cm}}{1\,200} = 8.33\times10^{-4}\text{ cm} = 8.33\times10^{-6}\text{ m}$.

Using $\sin\theta = \frac{m\lambda}{d}$ and the small angle approximation, the distance from the central maximum to the maximum of order m for wavelength λ is $y_m = L\tan\theta \approx L\sin\theta = (\lambda L/d)m$. Therefore, the spacing between successive maxima is $\Delta y = y_{m+1} - y_m = \lambda L/d$.

The longer wavelength in the light is found to be

$$\lambda_{\text{long}} = \frac{(\Delta y)d}{L} = \frac{\left(8.44\times10^{-3}\text{ m}\right)\left(8.33\times10^{-6}\text{ m}\right)}{0.150\text{ m}} = \boxed{469\text{ nm}}$$

Since the third order maximum of the shorter wavelength falls halfway between the central maximum and the first order maximum of the longer wavelength, we have

$$\frac{3\lambda_{\text{short}}L}{d} = \left(\frac{0+1}{2}\right)\frac{\lambda_{\text{long}}L}{d} \quad\text{or}\quad \lambda_{\text{short}} = \left(\frac{1}{6}\right)(469\text{ nm}) = \boxed{78.1\text{ nm}}$$

24.51 (a) From Brewster's law, the index of refraction is

$$n_2 = \tan\theta_p = \tan(48.0^\circ) = \boxed{1.11}$$

(b) From Snell's law, $n_2\sin\theta_2 = n_1\sin\theta_1$, we obtain when $\theta_1 = \theta_p$

$$\theta_2 = \sin^{-1}\left(\frac{n_1\sin\theta_p}{n_2}\right) = \sin^{-1}\left(\frac{(1.00)\sin 48.0^\circ}{1.11}\right) = \boxed{42.0^\circ}$$

Note that when $\theta_1 = \theta_p$, $\theta_2 = 90.0^\circ - \theta_p$ as it should.

24.52 (a) From Malus's law, the fraction of the incident intensity of the unpolarized light that is transmitted by the polarizer is

$$I' = I_0\left(\cos^2\theta\right)_{\text{av}} = I_0\left(0.500\right)$$

The fraction of this intensity incident on the analyzer that will be transmitted is

$$I = I'\cos^2\left(35.0°\right) = I'\left(0.671\right) = I_0\left(0.500\right)\left(0.671\right) = 0.336 I_0$$

Thus, the fraction of the incident unpolarized light transmitted is $I/I_0 = \boxed{0.336}$.

(b) The fraction of the original incident light absorbed by the analyzer is

$$\frac{I' - I}{I_0} = \frac{0.500 I_0 - 0.336 I_0}{I_0} = \boxed{0.164}$$

24.53 The more general expression for Brewster's angle is (see Problem 57)

$$\tan\theta_p = n_2/n_1$$

(a) When $n_1 = 1.00$ and $n_2 = 1.52$, $\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.52}{1.00}\right) = \boxed{56.7°}$

(b) When $n_1 = 1.333$ and $n_2 = 1.52$, $\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.52}{1.333}\right) = \boxed{48.8°}$

24.54 The polarizing angle for light in air striking a water surface is

$$\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.333}{1.00}\right) = 53.1°$$

This is the angle of incidence for the incoming sunlight (that is, the angle between the incident light and the normal to the surface). The altitude of the Sun is the angle between the incident light and the water surface. Thus, the altitude of the Sun is

$$\alpha = 90.0° - \theta_p = 90.0° - 53.1° = \boxed{36.9°}$$

24.55 (a) Brewster's angle (or the polarizing angle) is

$$\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{n_{\text{quartz}}}{n_{\text{air}}}\right) = \tan^{-1}\left(\frac{1.458}{1.000}\right) = \boxed{55.6°}$$

(b) When the angle of incidence is the polarizing angle, θ_p, the angle of refraction of the transmitted light is $\theta_2 = 90.0° - \theta_p$. Hence, $\theta_2 = 90.0° - 55.6° = \boxed{34.4°}$.

24.56 The critical angle for total reflection is $\theta_c = \sin^{-1}\left(n_2/n_1\right)$. Thus, if $\theta_c = 34.4°$ as light attempts to go from sapphire into air, the index of refraction of sapphire is

$$n_{\text{sapphire}} = n_1 = \frac{n_2}{\sin\theta_c} = \frac{1.00}{\sin 34.4°} = 1.77$$

Then, when light is incident on sapphire from air, the Brewster angle is

$$\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.77}{1.00}\right) = \boxed{60.5°}$$

24.57 From Snell's law, the angles of incidence and refraction are related by $n_1 \sin\theta_1 = n_2 \sin\theta_2$.

If the angle of incidence is the polarizing angle (that is, $\theta_1 = \theta_p$), the refracted ray is perpendicular to the reflected ray (see Figure 24.28 in the textbook), and the angles of incidence and refraction are also related by

$$\theta_p + \theta_2 + 90° = 180°, \quad \text{or} \quad \theta_2 = 90° - \theta_p$$

Substitution into Snell's law then gives

$$n_1 \sin\theta_p = n_2 \sin\left(90° - \theta_p\right) = n_2 \cos\theta_p \text{ or } \sin\theta_p / \cos\theta_p = \boxed{\tan\theta_p = n_2/n_1}$$

24.58 $I = I_0 \cos^2\theta \quad \Rightarrow \quad \theta = \cos^{-1}\left(\sqrt{\dfrac{I}{I_0}}\right)$

(a) $\dfrac{I}{I_0} = \dfrac{1}{2.00} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\sqrt{\dfrac{1}{2.00}}\right) = \boxed{45.0°}$

(b) $\dfrac{I}{I_0} = \dfrac{1}{4.00} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\sqrt{\dfrac{1}{4.00}}\right) = \boxed{60.0°}$

(c) $\dfrac{I}{I_0} = \dfrac{1}{6.00} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\sqrt{\dfrac{1}{6.00}}\right) = \boxed{65.9°}$

24.59 From Malus's law, the intensity of the light transmitted by the first polarizer is $I_1 = I_i \cos^2\theta_1$. The plane of polarization of this light is parallel to the axis of the first plate and is incident on the second plate. Malus's law gives the intensity transmitted by the second plate as $I_2 = I_1 \cos^2\left(\theta_2 - \theta_1\right) = I_i \cos^2\theta_1 \cos^2\left(\theta_2 - \theta_1\right)$. This light is polarized parallel to the axis of the second plate and is incident upon the third plate. A final application of Malus's law gives the transmitted intensity as

$$I_f = I_2 \cos^2\left(\theta_3 - \theta_2\right) = I_i \cos^2\theta_1 \cos^2\left(\theta_2 - \theta_1\right) \cos^2\left(\theta_3 - \theta_2\right)$$

With $\theta_1 = 20.0°$, $\theta_2 = 40.0°$, and $\theta_3 = 60.0°$, this result yields

$$I_f = \left(10.0 \text{ units}\right) \cos^2\left(20.0°\right) \cos^2\left(20.0°\right) \cos^2\left(20.0°\right) = \boxed{6.89 \text{ units}}$$

24.60 (a) Using Malus's law, the intensity of the transmitted light is found to be

$$I = I_0 \cos^2\left(45°\right) = I_0 \left(1/\sqrt{2}\right)^2, \text{ or } \boxed{I/I_0 = 1/2}$$

(b) From Malus's law, $I/I_0 = \cos^2\theta$. Thus, if $I/I_0 = 1/3$ we obtain

$$\cos^2\theta = 1/3 \quad \text{or} \quad \theta = \cos^{-1}\left(1/\sqrt{3}\right) = \boxed{54.7°}$$

24.61 (a) If light has wavelength λ in vacuum, its wavelength in a medium of refractive index n is $\lambda_n = \lambda/n$. Thus, the wavelengths of the two components in the specimen are

$$\lambda_{n_1} = \frac{\lambda}{n_1} = \frac{546.1 \text{ nm}}{1.320} = \boxed{413.7 \text{ nm}}$$

and $$\lambda_{n_2} = \frac{\lambda}{n_2} = \frac{546.1 \text{ nm}}{1.333} = \boxed{409.7 \text{ nm}}$$

continued on next page

(b) The number of cycles of vibration each component completes while passing through the specimen are

$$N_1 = \frac{t}{\lambda_{n_1}} = \frac{1.000 \times 10^{-6}\text{ m}}{413.7 \times 10^{-9}\text{ m}} = 2.417$$

and $$N_2 = \frac{t}{\lambda_{n_2}} = \frac{1.000 \times 10^{-6}\text{ m}}{409.7 \times 10^{-9}\text{ m}} = 2.441$$

Thus, when they emerge, the two components are out of phase by $N_2 - N_1 = 0.024$ cycles. Since each cycle represents a phase angle of 360°, they emerge with a phase difference of

$$\Delta\phi = (0.024\text{ cycles})(360°/\text{cycle}) = \boxed{8.6°}$$

24.62 In a single slit diffraction pattern, the first dark fringe occurs where $\sin(\theta_{\text{dark}})_1 = (1)\lambda/a$. If no diffraction minima are to be observed, it is necessary that no real solutions exist for this equation defining the location of the first minimum. Thus, it is necessary that $\sin(\theta_{\text{dark}})_1 = (1)\lambda/a > 1.00$, or $\lambda/a > 1.00$ and $a < \lambda$. We then see that the maximum width the slit can have before this condition will fail and diffraction minima will start being visible is $a_{\max} = \lambda = \boxed{632.8\text{ nm}}$.

24.63 The light has passed through $\boxed{\text{a single slit}}$ since the central maximum is twice the width of other maxima (the space between the centers of successive dark fringes. In a double slit pattern, the central maximum has the same width as all other maxima (compare Active Figures 24.1(b) and 24.16(b) in the textbook).

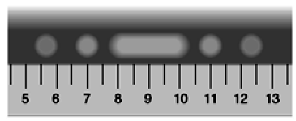


Figure P24.63

In single slit diffraction the width of the central maximum on the screen is given by

$$\Delta y_{\substack{\text{central}\\\text{maximum}}} = 2L\tan(\theta_{\text{dark}})_{m=1} \approx 2L\sin(\theta_{\text{dark}})_{m=1} = 2L\left(\frac{(1)\lambda}{a}\right) = 2\left(\frac{\lambda L}{a}\right)$$

The width of the slit is then

$$a = \frac{2\lambda L}{\Delta y_{\substack{\text{central}\\\text{maximum}}}} = \frac{2(632.8 \times 10^{-9}\text{ m})(2.60\text{ m})}{(10.3 - 7.6)\text{ cm}}\left(\frac{10^2\text{ cm}}{1\text{ m}}\right) = 1.2 \times 10^{-4}\text{ m} = \boxed{0.12\text{ mm}}$$

24.64 (a) In a double slit interference pattern, bright fringes on the screen occur where $(y_{\text{bright}})_m = m(\lambda L/d)$. Thus, if bright fringes of the wavelengths $\lambda_1 = 540$ nm and $\lambda_2 = 450$ nm are to coincide, it is necessary that

$$m_1\frac{(540\text{ nm})\cancel{L}}{\cancel{d}} = m_2\frac{(450\text{ nm})\cancel{L}}{\cancel{d}}$$

or, dividing both sides by 90 nm, $\boxed{6m_1 = 5m_2}$, where both m_1 and m_2 are integers.

(b) The condition found above may be written as $m_2 = (6/5)m_1$. Trial and error reveals that the smallest nonzero integer value of m_1 that will yield an integer value for m_2 is $\boxed{m_1 = 5}$, yielding $\boxed{m_2 = 6}$. Thus, the first overlap of bright fringes for the two given wavelengths occurs at the screen position (measured from the central maximum)

$$y_{\text{bright}} = \frac{5(540 \times 10^{-9}\text{ m})(1.40\text{ m})}{0.150 \times 10^{-3}\text{ m}} = \frac{6(450 \times 10^{-9}\text{ m})(1.40\text{ m})}{0.150 \times 10^{-3}\text{ m}} = 2.52 \times 10^{-2}\text{ m} = \boxed{2.52\text{ cm}}$$

24.65 Dark fringes (destructive interference) occur where $d\sin\theta = (m+1/2)\lambda$ for $m = 0, 1, 2, \ldots$. Thus, if the second dark fringe $(m=1)$ occurs at

$$\theta = (18.0 \text{ min})\left(\frac{1.00°}{60.0 \text{ min}}\right) = 0.300°,$$

the slit spacing is

$$d = \left(m+\frac{1}{2}\right)\frac{\lambda}{\sin\theta} = \left(\frac{3}{2}\right)\frac{(546\times10^{-9} \text{ m})}{\sin(0.300°)} = 1.56\times10^{-4} \text{ m} = \boxed{0.156 \text{ mm}}$$

24.66 The wavelength is $\lambda = \frac{v_{\text{sound}}}{f} = \frac{340 \text{ m/s}}{2\,000 \text{ Hz}} = 0.170 \text{ m}$.

Maxima occur where $d\sin\theta = m\lambda$, or $\theta = \sin^{-1}\left[m(\lambda/d)\right]$ for $m = 0, 1, 2, \ldots$.

Since $d = 0.350$ m, $\lambda/d = 0.486$, which gives $\theta = \sin^{-1}(0.486\,m)$.

For $m = 0$, 1, and 2, this yields $\boxed{\text{maxima at } 0°, 29.1°, \text{ and } 76.3°}$.

No solutions exist for $m \geq 3$ since that would imply $\sin\theta > 1$.

Minima occur where $d\sin\theta = (m+1/2)\lambda$ or $\theta = \sin^{-1}\left[(2m+1)\frac{\lambda}{2d}\right]$ for $m = 0, 1, 2, \ldots$.

With $\lambda/d = 0.486$, this becomes $\theta = \sin^{-1}\left[(2m+1)(0.243)\right]$.

For $m = 0$ and 1, we find $\boxed{\text{minima at } 14.1° \text{ and } 46.8°}$.

No solutions exist for $m \geq 2$ since that would imply $\sin\theta > 1$.

24.67 The source and its image, located 1.00 cm below the mirror, act as a pair of coherent sources. This situation may be treated as double-slit interference, with the slits separated by 2.00 cm, if it is remembered that the light undergoes a phase reversal upon reflection from the mirror. The existence of this phase change causes the conditions for constructive and destructive interference to be reversed. Therefore, dark bands (destructive interference) occur where

$$y = m(\lambda L/d) \text{ for } m = 0, 1, 2, \ldots$$

The $m = 0$ dark band occurs at $y = 0$ (that is, at mirror level). The first dark band above the mirror corresponds to $m = 1$ and is located at

$$y = (1)\left(\frac{\lambda L}{d}\right) = \frac{(500\times10^{-9} \text{ m})(100 \text{ m})}{2.00\times10^{-2} \text{ m}} = 2.50\times10^{-3} \text{ m} = \boxed{2.50 \text{ mm}}$$

24.68 Assuming the glass plates have refractive indices greater than that of both air and water, there will be a phase reversal at the reflection from the lower surface of the film but no reversal from reflection at the top of the film. Therefore, the condition for a dark fringe is

$$2t = m\lambda_n = m\left(\lambda/n_{\text{film}}\right) \text{ for } m = 0, 1, 2, \ldots$$

If the highest order dark band observed is $m = 84$ (a total of 85 dark bands counting the $m = 0$ order at the edge of contact), the maximum thickness of the wedge is

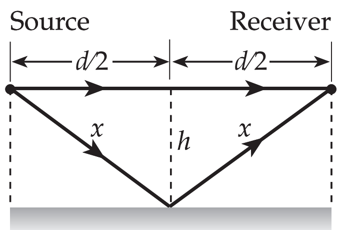
$$t_{\max} = \frac{m_{\max}}{2}\left(\frac{\lambda}{n_{\text{film}}}\right) = \frac{84}{2}\left(\frac{\lambda}{1.00}\right) = 42\,\lambda$$

When the film consists of water, the highest order dark fringe appearing will be

$$m_{\max} = 2\,t_{\max}\left(\frac{n_{\text{film}}}{\lambda}\right) = 2\left(42\,\lambda\right)\left(\frac{1.333}{\lambda}\right) = 112$$

Counting the $m = 0$ order, a total of $\boxed{113}$ dark fringes are now observed.

24.69 In the figure at the right, observe that the path difference between the direct and the indirect paths is



$$\delta = 2x - d = 2\sqrt{h^2 + \left(d/2\right)^2} - d$$

With a phase reversal (equivalent to a half-wavelength shift) occurring on the reflection at the ground, the condition for constructive interference is $\delta = \left(m + 1/2\right)\lambda$, and the condition for destructive interference is $\delta = m\,\lambda$. In both cases, the possible values of the order number are $m = 0, 1, 2, \ldots$.

(a) The wavelengths that will interfere constructively are $\lambda = \dfrac{\delta}{m + 1/2}$. The longest of these is for the $m = 0$ case and has a value of

$$\lambda = 2\delta = 4\sqrt{h^2 + \left(d/2\right)^2} - 2d$$

$$= 4\sqrt{\left(50.0\text{ m}\right)^2 + \left(300\text{ m}\right)^2} - 2\left(600\text{ m}\right) = \boxed{16.6\text{ m}}$$

(b) The wavelengths that will interfere destructively are $\lambda = \delta/m$, and the largest finite one of these is for the $m = 1$ case. That wavelength is

$$\lambda = \delta = 2\sqrt{h^2 + \left(d/2\right)^2} - d = 2\sqrt{\left(50.0\text{ m}\right)^2 + \left(300\text{ m}\right)^2} - 600\text{ m} = \boxed{8.28\text{ m}}$$

24.70 From Malus's law, the intensity of the light transmitted by the first polarizer is $I_1 = I_i \cos^2\theta_1$. The plane of polarization of this light is parallel to the axis of the first plate and is incident on the second plate. Malus's law gives the intensity transmitted by the second plate as $I_2 = I_1\cos^2\left(\theta_2 - \theta_1\right) = I_i\cos^2\theta_1\cos^2\left(\theta_2 - \theta_1\right)$. This light is polarized parallel to the axis of the second plate and is incident upon the third plate. A final application of Malus's law gives the transmitted intensity as

$$I_f = I_2\cos^2\left(\theta_3 - \theta_2\right) = I_i\cos^2\theta_1\cos^2\left(\theta_2 - \theta_1\right)\cos^2\left(\theta_3 - \theta_2\right)$$

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(a) If $\theta_1 = 45°$, $\theta_2 = 90°$, and $\theta_3 = 0°$, then

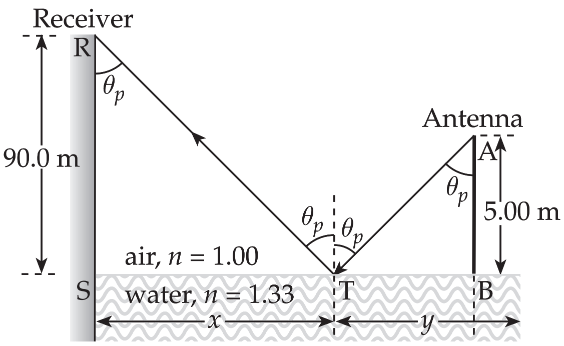
$$I_f/I_i = \cos^2 45° \cos^2(90° - 45°) \cos^2(0° - 90°) = \boxed{0}$$

(b) If $\theta_1 = 0°$, $\theta_2 = 45°$, and $\theta_3 = 90°$, then

$$I_f/I_i = \cos^2 0° \cos^2(45° - 0°) \cos^2(90° - 45°) = \boxed{0.25}$$

24.71 If the signal from the antenna to the receiver station is to be completely polarized by reflection from the water, the angle of incidence where it strikes the water must equal the polarizing angle from Brewster's law. This is given by

$$\theta_p = \tan^{-1}\left(\frac{n_{\text{water}}}{n_{\text{air}}}\right) = \tan^{-1}(1.33) = 53.1°$$



From the triangle RST in the sketch, the horizontal distance from the point of refection, T, to shore is given by

$$x = (90.0 \text{ m})\tan\theta_p = (90.0 \text{ m})(1.33) = 120 \text{ m}$$

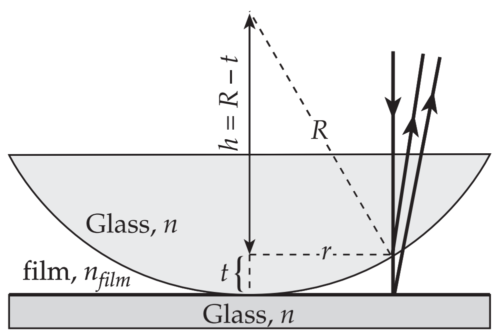
and from triangle ABT, the horizontal distance from the antenna to this point is

$$y = (5.00 \text{ m})\tan\theta_p = (5.00 \text{ m})(1.33) = 6.65 \text{ m}$$

The total horizontal distance from ship to shore is then $x + y = 120 \text{ m} + 6.65 \text{ m} = \boxed{127 \text{ m}}$.

24.72 There will be a phase reversal associated with the reflection at one surface of the film but no reversal at the other surface of the film. Therefore, the condition for a dark fringe (destructive interference) is

$$2t = m\lambda_n = m\left(\frac{\lambda}{n_{\text{film}}}\right) \qquad m = 0, 1, 2, \ldots$$



From the figure, note that $R^2 = r^2 + (R-t)^2 = r^2 + R^2 - 2Rt + t^2$, which reduces to $r^2 = 2Rt - t^2$. Since t will be very small in comparison to either r or R, we may neglect the term t^2, leaving $r \approx \sqrt{2Rt}$.

For a dark fringe, $t = \dfrac{m\lambda}{2n_{\text{film}}}$ so the radii of the dark rings will be

$$r \approx \sqrt{2R\left(\frac{m\lambda}{2n_{\text{film}}}\right)} = \boxed{\sqrt{\frac{m\lambda R}{n_{\text{film}}}}} \qquad \text{for} \quad m = 0, 1, 2, \ldots$$

24.73 In the single slit diffraction pattern, destructive interference (or minima) occur where $\sin\theta = m(\lambda/a)$ for $m = 0, \pm 1, \pm 2, \ldots$. The screen locations, measured from the center of the central maximum, of these minima are at

$$y_m = L\tan\theta_m \approx L\sin\theta_m = m(\lambda L/a)$$

If we assume the first-order maximum is halfway between the first- and second-order minima, then its location is

$$y = \frac{y_1 + y_2}{2} = \frac{(1+2)(\lambda L/a)}{2} = \frac{3\lambda L}{2a}$$

and the slit width is

$$a = \frac{3\lambda L}{2y} = \frac{3(500\times10^{-9}\text{ m})(1.40\text{ m})}{2(3.00\times10^{-3}\text{ m})} = 3.50\times10^{-4}\text{ m} = \boxed{0.350\text{ mm}}$$

24.74 As light emerging from the glass reflects from the top of the air layer, there is no phase reversal produced. However, the light reflecting from the end of the metal rod at the bottom of the air layer does experience phase reversal. Thus, the condition for constructive interference in the reflected light is $2t = \left(m+\frac{1}{2}\right)\lambda_{\text{air}}$.

As the metal rod expands, the thickness of the air layer decreases. The increase in the length of the rod is given by

$$\Delta L = |\Delta t| = \left(m_i + \tfrac{1}{2}\right)\frac{\lambda_{\text{air}}}{2} - \left(m_f + \tfrac{1}{2}\right)\frac{\lambda_{\text{air}}}{2} = |\Delta m|\frac{\lambda_{\text{air}}}{2}$$

The order number changes by one each time the film changes from bright to dark and back to bright. Thus, during the expansion, the measured change in the length of the rod is

$$\Delta L = (200)\frac{\lambda_{\text{air}}}{2} = (200)\frac{(500\times10^{-9}\text{ m})}{2} = 5.00\times10^{-5}\text{ m}$$

From $\Delta L = L_0\alpha(\Delta T)$, the coefficient of linear expansion of the rod is

$$\alpha = \frac{\Delta L}{L_0(\Delta T)} = \frac{5.00\times10^{-5}\text{ m}}{(0.100\text{ m})(25.0°\text{C})} = \boxed{20.0\times10^{-6}\ (°\text{C})^{-1}}$$