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Induced Voltages and Inductance

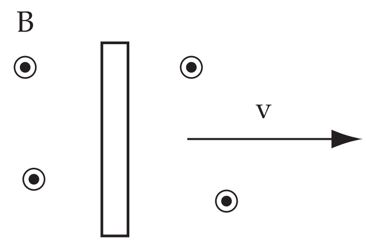
CLICKER QUESTIONS

Question M4.01

Description: Preparing for Faraday's law by exploring the Lorentz force law on a moving conductor.

Question

A long conducting bar moves with a constant velocity in a uniform magnetic field. The bar and its velocity are perpendicular to the magnetic field as shown. Which of the following statements are true?



A. At steady state there is an electric field in the bar.
B. At steady state there is a current in the bar.
C. At steady state there is a magnetic force on bar.

1. A only
2. B only
3. C only
4. A and B only
5. A and C only
6. B and C only
7. A, B, and C

Commentary

Purpose: To develop your understanding of the Lorentz force law and its application to a conducting material, in preparation for Faraday's law.

Discussion: A metal is a material composed of atoms held relatively rigidly in place, but with some "free" electrons that can flow from atom to atom through the material.

Consider the free electrons in the metal bar. As the bar moves through the magnetic field, the electrons are carried through it. Charges moving through a magnetic field experience a force according to the magnetic part of the Lorentz force law, $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$. According to the right-hand rule for the cross product, this means the negatively charged electrons experience a force towards the top of the diagram. (The magnetic field is shown as pointing out of the page.)

This force will push the electrons towards the top end of the bar. Since they cannot escape the isolated bar, they will "pile up" there, developing a negative static charge at that end of the bar. The other end of the bar will develop a positive static charge, since electrons are moving away from it and leaving positively charged metal ions behind. (The ions experience a force toward the bottom of the diagram, but because they are not free to move, they remain in place.)

As these static charges accumulate, they create an electric field within the bar, pointing from the positively charged bottom to the negatively charged top. According to the electric part of the Lorentz force law, $\mathbf{F}_E = q\mathbf{E}$, this causes a downward force on the electrons. As the electrons move and the static charge builds up, the strength of this electric force will grow until its magnitude is equal to the magnetic force on the electrons. Since it acts in the opposite direction of the magnetic force, this will result in a zero net force on the electrons, and they will cease to move. The static charge arrangement will not increase any more, and the electrons in the bar have reached "steady state": the electric force balances the magnetic force, and electrons no longer move.

Thus, statement A is true, and statement B is false.

Statement C is about the *net* magnetic force on the bar, not just on the electrons within it. Since the bar has no net charge, it contains as many positive protons as negative electrons. Once steady state is reached, all charges move with the same velocity, and the upward magnetic force on all the electrons is exactly balanced by the downward magnetic force on all the protons. So, there is no net magnetic force.

(Before steady state is reached, the situation is much more complicated. While current flows, the $\mathbf{F} = I\mathbf{L} \times \mathbf{B}$ form of the magnetic force law tells us that the bar will experience a magnetic force.)

Key Points:

- Conducting materials are composed of positive and negative charges. For metals, some of the negative charges can move while the rest of the negative charges and all of the positive charges are fixed in place.
- Analyzing current flow in "wires" from a microscopic perspective, by considering the Lorentz force on the charged particles, can be helpful.
- When charges move in an isolated conductor, they eventually build up a static charge distribution that prevents any further charge motion. (Motion around a circuit is different.)

For Instructors Only

This question can get exceedingly tacky, which makes it valuable for exploring and developing students' understanding. In particular, taking a microscopic perspective to explain how the bar can experience a nonzero magnetic force before steady state but not in steady state is *very* difficult.

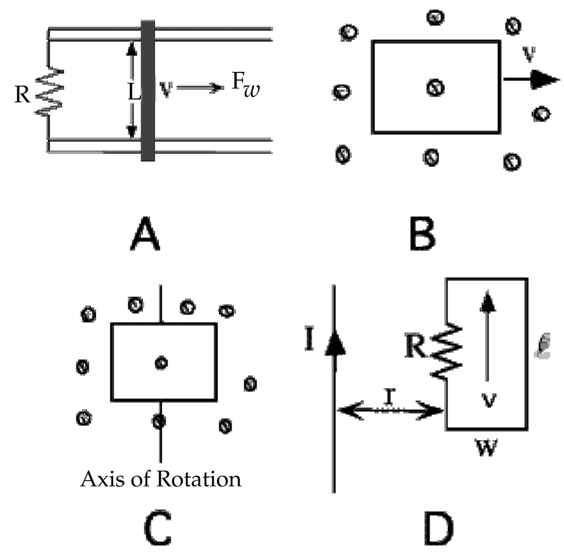
This question can be used to relate Faraday's law to the Lorentz force law. Another way to approach the situation is to make the bar part of an imaginary closed path, and apply Faraday's law. That can help explain why charge initially flows. Students must understand, however, that Faraday's law specifies the emf around any closed path — not just a *conducting* loop.

Question M4.02

Description: Understanding Faraday's law by identifying situations for which an emf is induced.

Question

For which of the following is an emf induced?



A. A conducting rod is pulled on conducting rails that are placed in a uniform magnetic field directed into the page.
B. A conducting loop moves through a uniform magnetic field directed into the page.
C. A conducting loop rotates in a uniform magnetic field directed into the page.
D. A conducting loop moves in a magnetic field produced by an infinite current-carrying wire.

1. A only
2. A and B only
3. A and C only
4. A and D only
5. B and C only
6. A, B, and C
7. A, C, and D
8. All of them
9. None of the above

Commentary

Purpose: To develop an intuitive understanding of when an emf is or is not induced, according to Faraday's law.

Discussion: According to Faraday's law, the magnitude of the emf induced around a closed loop is equal to the rate of change of the magnetic flux through the loop. For a uniform magnetic field **B** and a loop with area S all in one plane, the flux is $\Phi = \mathbf{B} \cdot \mathbf{S}$ where the direction of the vector **S** is normal to the loop.

There are three ways that the flux $\mathbf{B} \cdot \mathbf{S}$ can change: the magnetic field strength can change, the area of the loop can change, or the angle between them can change.

In situation A, a uniform magnetic field into the page (not shown in the figure, but described in statement A) causes a flux through the loop formed by the resistor, portions of the two rails, and the moving rod. As the rod moves, the area of this loop grows, so an emf is induced.

In situation B, the net flux through the loop is not changing: neither the field strength, nor the loop size, nor their relative orientation changes. The flux gained by the leading edge of the loop is lost by the trailing edge. So, no emf is induced.

In situation C, the angle between the field and the loop does change as the loop rotates, so an emf is generated. (This is the mechanism underlying an electric generator in, say, a hydroelectric power plant.)

In situation D, the current I in the infinite wire creates a magnetic field that circulates around it, so there is a nonzero flux through the moving loop. However, as the loop moves vertically the magnetic field it encounters does not change. Neither its size nor its orientation relative to the field change either, so no emf is induced. (If the loop were to move horizontally, away from the wire, the flux would decrease and an emf would be induced.)

Thus, answer 3 is best.

Key Points:

- An emf is induced around a closed path whenever the net magnetic flux through that path changes with time.
- There are three ways the flux through a path can change: the magnetic field strength can change, the area enclosed by the path can change, or the angle between them can change.
- Not all moving loops in magnetic fields experience an induced emf.

For Instructors Only

This problem is well suited to helping students develop an understanding of Faraday's law when they are first exposed to it.

To work well with Faraday's law, students need a firm, intuitive grasp of "magnetic flux" and the ability to visualize it.

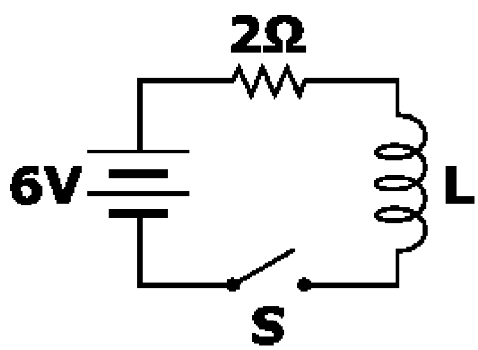
Discussing each case from a microscopic perspective, using the Lorentz force law, might help students understand *why* current flows in the loops experiencing a change in magnetic flux.

Question N4.01a

Description: Introducing inductors as circuit elements.

Question

Consider the circuit below. Switch S is closed at $t = 0$.



What is the current through the inductor L just after the switch is closed?

1. 0
2. 1 A
3. 2 A
4. 3 A
5. 4 A
6. 5 A
7. 6 A
8. 9 A
9. None of the above
10. Impossible to determine

Commentary

Purpose: To introduce the behavior of an inductor in a circuit.

Discussion: The voltage across an inductor is proportional to how quickly the current through it changes. An instantaneous change in current requires an infinite voltage, which is impossible even for an "ideal" circuit. This means that the current through an inductor must be a continuous function of time.

Note that the current through a resistor or capacitor *can* change instantaneously (in an ideal circuit).

Therefore, because the current through inductor L is zero just before the switch is closed, it must be zero just afterwards as well. (In other words, with the switch open, no current can flow through the circuit; it is an open circuit. Thus, the current everywhere in the circuit is zero immediately after the switch is closed.)

Key Points:

- The current through an inductor is a continuous function of time and cannot change instantaneously.
- The current through a resistor or a capacitor *can* change instantaneously.
- When current begins to flow through a circuit branch containing an inductor, it starts flowing very gradually and increases with time.

For Instructors Only

This is the first of three questions using this situation.

Students who choose answer (4), 3 A, are probably treating the inductor as a short circuit, as if it were a capacitor or wire. They may also be giving the steady-state value of the current rather than its immediate value.

Students might not appreciate how the current can ever change if it does not change when the switch is closed. The current is zero, but changing. This is analogous to dropping something: At the instant it is dropped, it is not moving, but it is accelerating, and that is how the object comes to have speed. Likewise, the current is zero, but changing, because there is a voltage across the inductor. (The voltage across the inductor is analogous to the net force on a dropped object. $\mathbf{F}_{\text{net}} = m\, d\mathbf{v}/dt$ and $V_L = L\, dI/dt$.)

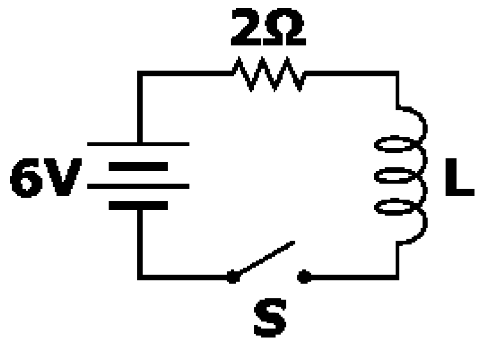
Unlike similar questions involving capacitors, there is no ambiguity here about the initial condition of the circuit before the switch is closed; the current is necessarily zero.

Question N4.01b

Description: Introducing inductors as circuit elements.

Question

Consider the circuit below. Switch S is closed at $t = 0$.



What is the voltage across inductor L just after the switch is closed?

1. 0
2. 1 V
3. 2 V
4. 3 V
5. 4 V
6. 5 V
7. 6 V
8. 9 V
9. None of the above
10. Impossible to determine

Commentary

Purpose: To introduce the behavior of an inductor in a circuit.

Discussion: Immediately after the switch is closed, no current flows through the circuit, so the voltage across the resistor is zero. Therefore, according to Kirchhoff's loop law, the voltage across the inductor must be 6 V.

Note that there is a voltage across L even though no current flows through it. For an inductor, a nonzero voltage means a nonzero *rate of change* of the current ($V_L = L\, dI/dt$). So, the instant after the switch is closed, the current is zero but rising. This is similar to the velocity of an object that is dropped from rest: at the instant the object is released, the object's velocity is zero, but changing (i.e., the acceleration is nonzero), because the net force is nonzero ($\mathbf{F}_{\text{net}} = m\, d\mathbf{v}/dt$). Since the voltage across the inductor is zero just before the switch is closed and is 6 V just after, the voltage across an inductor can change instantaneously (unlike for a capacitor); it is not a continuous function of time.

Key Points:

- The voltage across an inductor can be nonzero even when the current through it is zero.
- Nonzero voltage across an inductor means the current through it is changing.

- For an inductor, current is a continuous function of time, but voltage is not.
- We can say that an inductor with no current through it behaves like an "open circuit" for an instant.

For Instructors Only

This is the second of three questions using this situation.

Students who select answer (1) may be treating the inductor like a capacitor, for which the voltage cannot change instantaneously. Or, they may know that the current through the inductor is zero and then apply Ohm's law as if the inductor were a resistor to say that the voltage must also be zero.

Additional Questions: [instructor notes]

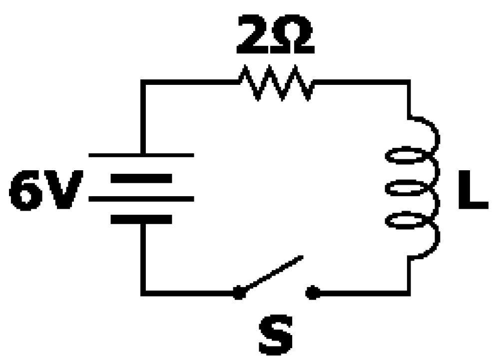
1. What is the voltage across the inductor just before the switch is closed? [The voltage across the inductor is zero, because the current is not changing. Kirchhoff's loop law is satisfied, because the voltage drop across the switch is 6 V.]
2. Sketch current vs. time through the inductor.
3. For $L = 25$ mH, what is the rate at which the current through the inductor is changing just after the switch is closed? That is, what is the slope of the current vs. time graph?

Question N4.01c

Description: Introducing inductors as circuit elements.

Question

Consider the circuit below. Switch S is closed at $t = 0$.



What is the current through inductor L long after the switch is closed?

1. 0
2. 1 A
3. 2 A
4. 3 A
5. 4 A
6. 5 A
7. 6 A
8. 9 A
9. None of the above
10. Impossible to determine

Commentary

Purpose: To introduce the behavior of an inductor in a circuit.

Discussion: The voltage across an inductor is proportional to the *rate of change* of the current through it. After a long time, the current through the inductor has reached a constant, steady-state value, so the voltage across it must be zero. In other words, an inductor in steady state behaves like a "short circuit."

Since the voltage drop across the inductor is zero, the voltage drop across the resistor must be 6 V according to Kirchhoff's loop law, so 3 A flows everywhere in the circuit.

Note that for resistors, a current flow indicates a nonzero voltage drop; for inductors, a *change* in current flow indicates a nonzero voltage drop.

Key Points:

- In steady state (i.e., for constant current), the voltage across an inductor is zero.
- An inductor may have current through it without any voltage drop across it.

For Instructors Only

This is the last of 3 questions using this situation.

It might be hard for students to understand how the voltage drop across something with current can be zero.

Students who say the current is zero (answer 1) may be treating the inductor as a capacitor, in which the current goes to zero asymptotically.

Asking students to sketch the current through and voltage across the inductor vs. time, and relating the two graphs, may be helpful.

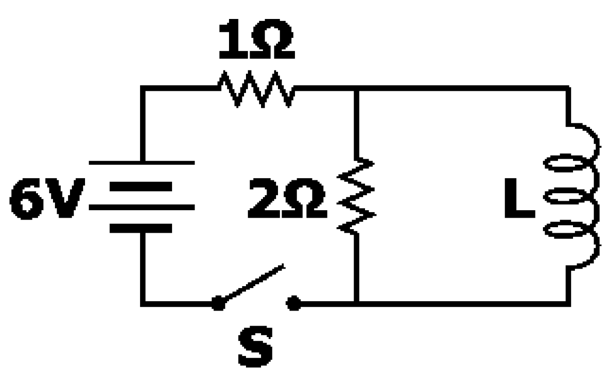
It can also help to redraw the circuit with the inductor replaced by a wire.

Question N4.02a

Description: Extending understanding of inductor behavior in DC circuits.

Question

Consider the circuit below. Switch S is closed at $t = 0$.



What is the current through the inductor *L* just after the switch is closed?

1. 0
2. 1 A
3. 2 A
4. 3 A
5. 4 A
6. 5 A
7. 6 A
8. 9 A
9. None of the above
10. Impossible to determine

Commentary

Purpose: To develop your understanding of how inductors behave in circuits with other elements.

Discussion: The voltage across an inductor is proportional to the rate at which the current through it changes. An instantaneous change requires an infinite voltage, which is impossible, even for an "ideal" circuit. This means the current through an inductor is a continuous function of time. (For resistors and capacitors, the current *can* change instantly and discontinuously in an ideal circuit.)

Therefore, assuming the current through the inductor is zero just before the switch is closed, it must be zero just afterwards as well.

Key Points:

- The voltage across an inductor is proportional to the rate of change of the current through it.
- The current through an inductor cannot change instantaneously, but is a continuous function of time.

For Instructors Only

This is the first of three questions using this situation.

Students who select 6 A (answer 7) may be providing the steady-state current through the inductor, treating the inductor as a short circuit, or providing the behavior for an uncharged capacitor instead of an inductor.

Students who select 2 A (answer 3) may be providing the current through the battery at $t = 0$, the correct answer to the wrong question. (This happens surprisingly often.)

Students may claim that the answer is impossible to determine (answer 10), on the grounds that the current through the inductor is not given just before the switch is closed, and may not be zero. This argument is defensible. However, if the system is in a steady state (the switch has been closed a "long time") before the switch is closed, the current must be zero. Thus, assuming zero initial current is reasonable.

Additional Question:

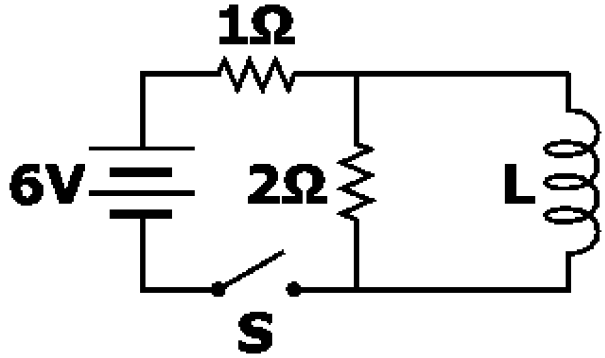
- What are the currents through the rest of the circuit elements at $t = 0$ (after the switch is closed)?

Question N4.02b

Description: Extending understanding of inductor behavior in DC circuits.

Question

Consider the circuit below. Switch S is closed at $t = 0$.



What is the voltage across inductor L just after the switch is closed?

1. 0
2. 1 V
3. 2 V
4. 3 V
5. 4 V
6. 5 V
7. 6 V
8. 9 V
9. None of the above
10. Impossible to determine

Commentary

Purpose: To develop your understanding of how inductors behave in circuits with other elements.

Discussion: We'll assume no current is flowing through the inductor just before the switch is closed. When the switch is closed, the current through the inductor is initially zero as discussed in the previous question, so all the current flows through the other three components. This means the two resistors and battery are in series with each other, and the current through them must be 2 A.

The voltage across the inductor and the 2 Ω resistor must always be equal. At $t = 0$, just after the switch is closed, the voltage across the 2 Ω resistor is 4 V, so this is the voltage across the inductor.

Note that although the current through an inductor cannot change instantaneously, the voltage can.

Key Points:

- An inductor can have zero current and nonzero voltage. Nonzero voltage means the current flow is *changing*.
- The voltage across an inductor is not necessarily a continuous function of time; it can change instantaneously.
- Kirchhoff's laws are always valid and often useful.

For Instructors Only

This is the second of three questions using this situation.

Students who answer zero might be treating the inductor as a capacitor, for which voltage and current are proportional. They may also be treating it as a capacitor, for which voltage cannot change instantaneously.

Students who answer 6 V may be erroneously generalizing from simpler circuits in which the voltage across an inductor jumps to the battery's voltage. This would be correct here if the 2 Ω resistor were removed, or if the 1 Ω resistor were replaced by wire.

It might be hard for students to realize that they need to focus on the 2 Ω resistor in order to find the voltage across the inductor. But this is a good opportunity to discuss problem-solving skills, since it is often fruitful to transform a hard question into an easier one. In this case, if you had asked students to tell you the voltage across the 2 Ω resistor just after the switch is closed, many more would be able to do so. (Not all, since they must also recognize that there is no current through L.)

This question is also a good opportunity to discuss the continued usefulness of Kirchhoff's laws.

As with the previous question, "impossible to determine" because the initial condition is unspecified is defensible but not laudable, since any current circulating through the inductor and 2 Ω resistor will damp out if the switch is open for significant time.

Additional Questions:

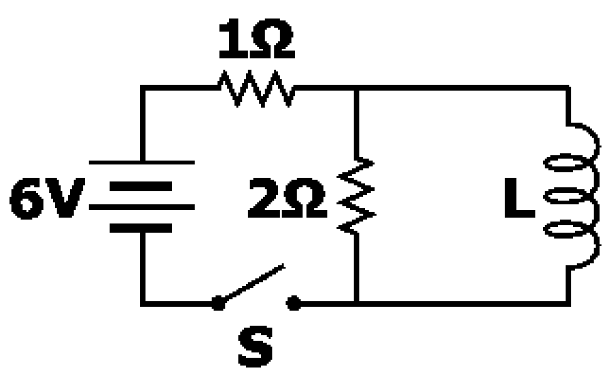
- What is the voltage across the inductor just before the switch is closed?
- Sketch the current through the inductor vs. time.
- For $L = 25$ mH, what is the rate at which the current through the inductor is changing just after the switch is closed? (What is the slope of current vs. time?)

Question N4.02c

Description: Extending understanding of inductor behavior in DC circuits.

Question

Consider the circuit below. Switch S is closed at $t = 0$.



What is the current through inductor L long after the switch is closed?

1. 0
2. 1 A
3. 2 A
4. 3 A
5. 4 A
6. 5 A
7. 6 A
8. 9 A
9. None of the above
10. Impossible to determine

Commentary

Purpose: To develop your understanding of how inductors behave in circuits with other elements.

Discussion: As discussed in the previous question, the inductor has zero current through it but nonzero voltage across it after the switch is closed. Nonzero voltage means that the current is changing, so although the current is in initially zero, it gradually increases. The current cannot increase infinitely, however; eventually it must reach or approach some steady-state value. When the current is no longer changing, the voltage across the inductor must be zero. Thus, according to Kirchhoff's loop law for the loop with the battery and inductor, the voltage drop across the 1 Ω resistor must equal the voltage rise across the battery. But if this is true, Kirchhoff's loop law for the loop containing both resistors indicates that the voltage drop across the 2 Ω resistor is zero, so no current flows through that resistor.

Thus, once the circuit has reached steady state, all the current in the circuit flows through the inductor and none goes through the 2 Ω resistor. If the voltage drop across the 1 Ω resistor is equal to the battery voltage, we can deduce that the current through it and the inductor must be 6 A.

Key Points:

- In steady state, the voltage across an inductor is zero, because its current is not changing.
- In steady state, an inductor behaves like a "short circuit": a zero-resistance wire.
- There can be current through an inductor without any voltage drop across it.
- Kirchhoff's laws are useful for circuits with inductors and capacitors as well as resistors.

For Instructors Only

This is the last of three questions using this situation.

Students who answer "zero" may be treating the inductor as a capacitor, in which current (rather than voltage and rate of change of current) must be zero in steady-state.

Some students may find it counterintuitive that no current at all flows through the 2 Ω resistor. Redrawing the diagram with a short circuit in place of the inductor, representing the steady-state solution, may help.

Additional Questions:

- When the voltage drop across the inductor is 2 V, determine the currents through all four circuit elements.
- What is the voltage across the inductor in steady state?

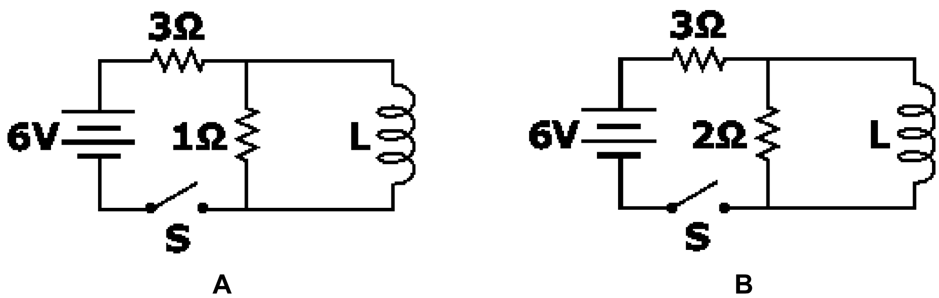
- After steady state is reached, the switch is opened again. Just after the switch is opened, determine the current through the inductor, the current through the 2 Ω resistor, the current through the 1 Ω resistor, and the voltage across the inductor.

Question N4.03

Description: Linking energy conservation ideas to inductor circuits.

Question

Consider the following circuits. Two identical batteries are connected to two identical inductors as shown. The switches are closed for a very long time.



At $t = 0$, both switches are opened. Which resistor dissipates more total energy after the switches are opened?

1. The 1 Ω resistor in A dissipates more energy.
2. The 2 Ω resistor in B dissipates more energy.
3. Both resistors dissipate the same amount of energy.
4. The amounts of energy dissipated cannot be compared.

Commentary

Purpose: To link *conservation of energy* ideas to an inductive circuit.

Discussion: In steady state, inductors act like short circuits. After a long time, therefore, 2 A flows through the inductor L and the 3 Ω resistor in each circuit. No current is flowing through the 1 Ω and 2 Ω resistors just before the switch is opened. Since the currents through the inductors are the same, the amounts of energy stored in them are the same also: $U = \frac{1}{2}LI^2$. When the switch is opened again, no current flows through the battery or 3 Ω resistor of either circuit. The current through an inductor cannot change instantaneously, so current flows through the 1 Ω and 2 Ω resistors, gradually decreasing until it reaches zero. When the current is zero, no energy is stored in the inductors. The only mechanism for the originally stored energy to "escape" the circuit is to be dissipated through the 1 Ω or 2 Ω resistor. So, each of these resistors must dissipate exactly $\frac{1}{2}LI^2$ of energy: the amounts of energy dissipated by each resistor are therefore the same.

Note that the powers dissipated by the 1 Ω and 2 Ω resistors are not the same. Power is the *rate* at which energy is transferred. The circuit with the smaller resistance (A) takes longer to dissipate the energy, but the total amount dissipated is the same as B.

Key Points:

- The energy U stored in an inductor L with current I flowing through it is $U = \frac{1}{2}LI^2$.
- An inductor acts like a short circuit in a steady-state situation.
- The current through an inductor cannot change instantaneously. (The current through a resistor, however, can.)

For Instructors Only

Students might not realize that the current in each inductor is the same just before the switches are opened. They might think that the current at steady state depends on the value of the resistance in parallel with L (1 Ω or 2 Ω).

Students might think that they need to know L to determine the current in steady state.

Students choosing answer (1) may be thinking that more energy is dissipated in circuit A because the current takes more time to damp out (having a longer L/R time constant).

Students choosing answer (2) may be thinking that more energy is dissipated in circuit B either because the initial voltage across the 2 Ω resistor is larger or because its resistance (and therefore power dissipation) is larger.

Students choosing answer (4) may think they need a value for L so they can perform actual calculations.

QUICK QUIZZES

1. *b*, *c*, *a*. At each instant, the magnitude of the induced emf is proportional to the rate of change of the magnetic field (hence, proportional to the slope of the curve shown on the graph).

2. (a). All charged particles within the metal bar move straight downward with the bar. According to right-hand rule #1, positive changes moving downward through a magnetic field that is directed northward will experience magnetic forces toward the east. This means that the free electrons (negative charges) within the metal will experience westward forces and will drift toward the west end of the bar, leaving the east end with a net positive charge.

3. (b). According to Equation (20.3), because B and v are constant, the emf depends only on the length of the wire moving in the magnetic field. Thus, you want the long dimension moving through the magnetic field lines so that it is perpendicular to the velocity vector. In this case, the short dimension is parallel to the velocity vector. From a more conceptual point of view, you want the rate of change of area in the magnetic field to be the largest, which you do by thrusting the long dimension into the field.

4. (c). In order to oppose the approach of the north pole, the magnetic field generated by the induced current must be directed upward. An induced current directed counterclockwise around the loop will produce a field with this orientation along the axis of the loop.

5. (b). As the counterclockwise current in the left-hand loop increases, it produces an increasing downward flux through the area enclosed by the right-hand loop. This changing flux will induce a counterclockwise current in the right-hand loop in order to oppose the increasing flux from the left-hand loop.

6. (b). When the iron rod is inserted into the solenoid, the inductance of the coil increases. As a result, more potential difference appears across the coil than before. Consequently, less potential difference appears across the bulb, and its brightness decreases.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. Since the magnetic field is directed at a 30.0° angle with respect to the plane of the loop, the angle between the field and the normal to the plane of the loop is $\theta = 60.0°$. Thus, the flux through the rectangular area is

$$\Phi_B = BA\cos\theta = (3.0\ \text{T})\left[(2.0\ \text{m})(1.0\ \text{m})\right]\cos 60.0° = 3.0\ \text{T}\cdot\text{m}^2 = 3.0\ \text{Wb}$$

and (c) is the correct choice.

2. Choose the positive z-direction to be the reference direction ($\theta = 0°$) for the normal to the plane of the coil. Then, the change in flux through the coil is

$$\Delta\Phi_B = \left(B_f\cos\theta_f - B_i\cos\theta_i\right)A = \left[(3.0\ \text{T})\cos 0° - (1.0\ \text{T})\cos 180°\right](0.5\ \text{m})^2 = 1.0\ \text{Wb}$$

and the magnitude of the induced emf is

$$|\mathcal{E}| = N\frac{|\Delta\Phi_B|}{\Delta t} = (10)\left(\frac{1.0\ \text{Wb}}{2.0\ \text{s}}\right) = 5.0\ \text{V}$$

The correct answer is choice (b).

3. The motional emf induced in a conductor of length ℓ moving at speed v through a magnetic field of magnitude B is $\Delta V = B_\perp \ell v = (B\sin\theta)\ell v$, where $B_\perp = B\sin\theta$ is the component of the field perpendicular to the velocity of the conductor. In the described situation,

$$\Delta V = B_\perp \ell v = (B\sin\theta)\ell v = \left[(60.0\times10^{-6}\ \text{T})\sin 60.0°\right](12\ \text{m})(60.0\ \text{m/s}) = 3.7\times10^{-2}\ \text{V} = 37\ \text{mV}$$

and the correct choice is (c).

4. The angular velocity of the rotating coil is $\omega = 10.0\ \text{rev/s} = 20\pi\ \text{rad/s}$ and the maximum emf induced in the coil is

$$\mathcal{E}_{\text{max}} = NBA\omega = (100)(0.050\ 0\ \text{T})(0.100\ \text{m}^2)(20\pi\ \text{rad/s}) = 31.4\ \text{V}$$

showing the correct choice to be (a).

5. The magnitude of the voltage drop across the inductor is

$$|\mathcal{E}_L| = L\left|\frac{\Delta I}{\Delta t}\right| = (5.00\ \text{H})(2.00\ \text{A/s}) = 10.0\ \text{V}$$

and (d) is the correct choice.

6. From $i = \dfrac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) = I_{\text{max}}\left(1 - e^{-t/\tau}\right)$, the time when $i = I_{\text{max}}/2$ is given by $e^{-t/\tau} = 1/2$ or $e^{t/\tau} = 2$, yielding $t = \tau\ln 2$. Since the time constant of the circuit is $\tau = \dfrac{L}{R} = \dfrac{5.0\ \text{H}}{2.5\ \Omega} = 2.0$ s, the desired time is $t = (2.0\ \text{s})\ln 2 = 1.4\ \text{s}$ and the answer closest to this result is choice (d).

7. The energy stored in an inductor of inductance L and carrying current I is $PE_L = \frac{1}{2}LI^2$. Thus, if the current is doubled while the inductance is constant, the stored energy increases by a factor of 4 and the correct choice is (d).

8. An emf is induced in the coil by any action which causes a change in the flux through the coil. The actions described in choices (a), (b), (d), and (e) all change the flux through the coil and induce an emf. However, moving the coil through the constant field without changing its orientation with respect to the field will not cause a change of flux. Thus, choice (c) is the correct answer.

9. With the current in long wire flowing in the direction shown in Figure MCQ20.9, the magnetic flux through the rectangular loop is directed into the page. If this current is decreasing in time, the *change* in the flux is directed opposite to the flux itself (or out of the page). The induced current will then flow clockwise around the loop, producing a flux directed into the page through the loop and opposing the change in flux due to the decreasing current in the long wire. The correct choice is for this question is (b).

10. A current flowing counterclockwise in the outer loop of Figure MCQ20.10 produces a magnetic flux through the inner loop that is directed out of the page. If this current is increasing in time, the *change* in the flux is in the same direction as the flux itself (or out of the page). The induced current in the inner loop will then flow clockwise around the loop, producing a flux through the loop directed into the page, opposing the change in flux due to the increasing current in the outer loop. The correct answer is choice (b).

11. <u>As the bar magnet approaches the loop from above</u>, with its south end downward as shown in Figure MCQ20.11, magnetic flux through the area enclosed by the loop is directed upward and increasing in magnitude. To oppose this increasing upward flux, the induced current in the loop will flow clockwise, as seen from above, producing a flux directed downward through the area enclosed by the loop. <u>After the bar magnet has passed through the plane of the loop</u>, and is departing with its north end upward, a decreasing flux is directed upward through the loop. To oppose this decreasing upward flux, the induced current in the loop flows counterclockwise as seen from above, producing flux directed upward through the area enclosed by the loop. From this analysis, we see that (a) is the only true statement among the listed choices.

12. With the magnetic field perpendicular to the plane of the page in Figure MCQ20.12, the flux through the closed loop to the left of the bar is given by $\Phi_B = BA$, where B is the magnitude of the field and A is the area enclosed by the loop. Any action which produces a change in this product, BA, will induce a current in the loop and cause the bulb to light. Such actions include increasing or decreasing the magnitude of the field (B), and moving the bar to the right or left and changing the enclosed area A. Thus, the bulb will light during all of the actions in choices (a), (b), (c), and (d).

ANSWERS TO CONCEPTUAL QUESTIONS

2. Consider the copper tube to be a large set of rings stacked one on top of the other. As the magnet falls toward or falls away from each ring, a current is induced in the ring. Thus, there is a current in the copper tube around its circumference.

4. The flux is calculated as $\Phi_B = BA\cos\theta = B_\perp A$. The flux is therefore maximum when the magnetic field vector is perpendicular to the plane of the loop. We may also deduce that the flux is zero when there is no component of the magnetic field that is perpendicular to the loop.

6. No. Once the bar is in motion and the charges are separated, no external force is necessary to maintain the motion. An applied force in the x-direction will cause the bar to accelerate in that direction.

8. As water falls, it gains velocity and kinetic energy. It then pushes against the blades of a turbine, transferring this energy to the rotor or coil of a large alternating current generator. The rotor moves in a strong external magnetic field and a voltage is induced in the coil. This induced emf is the voltage source for the current in our electric power lines.

10. Let us assume the north pole of the magnet faces the ring. As the bar magnet falls toward the conducting ring, a magnetic field is induced in the ring pointing upward. This upward directed field will oppose the motion of the magnet preventing it from moving as a freely-falling body. Try it for yourself to show that an upward force also acts on the falling magnet if the south end faces the ring.

12. A constant induced emf requires a magnetic field that is changing at a constant rate in one direction — for example, always increasing or always decreasing. It is impossible for a magnetic field to increase forever, both in terms of energy considerations and technological concerns. In the case of a decreasing field, once it reaches zero and then reverses direction, we again face the problem with the field increasing without bounds in the opposite direction.

14. As the magnet moves at high speed past the fixed coil, the magnetic flux through the coil changes very rapidly, increasing as the magnet approaches the coil and decreasing as the magnet moves away. The rapid change in flux through the coil induces a large emf, large enough to cause a spark across the gap in the spark plug.

PROBLEM SOLUTIONS

20.1 The angle between the direction of the constant field and the normal to the plane of the loop is $\theta = 0°$, so

$$\Phi_B = BA\cos\theta = (0.50\ \text{T})\left[(8.0\times10^{-2}\ \text{m})(12\times10^{-2}\ \text{m})\right]\cos 0° = \boxed{4.8\times10^{-3}\ \text{T}\cdot\text{m}^2} = \boxed{4.8\ \text{mWb}}$$

20.2 The magnetic flux through the loop is given by $\Phi_B = BA\cos\theta$, where B is the magnitude of the magnetic field, A is the area enclosed by the loop, and θ is the angle the magnetic field makes with the normal to the plane of the loop. Thus,

$$\Phi_B = BA\cos\theta = (5.00\times10^{-5}\ \text{T})\left[20.0\ \text{cm}^2\left(\frac{10^{-2}\ \text{m}}{1\ \text{cm}}\right)^2\right]\cos\theta = (1.00\times10^{-7}\ \text{T}\cdot\text{m}^2)\cos\theta$$

(a) When $\vec{\mathbf{B}}$ is perpendicular to the plane of the loop, $\theta = 0°$ and $\Phi_B = \boxed{1.00\times10^{-7}\ \text{T}\cdot\text{m}^2}$

(b) If $\theta = 30.0°$, then $\Phi_B = (1.00\times10^{-7}\ \text{T}\cdot\text{m}^2)\cos 30.0° = \boxed{8.66\times10^{-8}\ \text{T}\cdot\text{m}^2}$

(c) If $\theta = 90.0°$, then $\Phi_B = (1.00\times10^{-7}\ \text{T}\cdot\text{m}^2)\cos 90.0° = \boxed{0}$

20.3 $\Phi_B = BA\cos\theta = B(\pi r^2)\cos\theta$ where θ is the angle between the direction of the field and the normal to the plane of the loop.

(a) If the field is perpendicular to the plane of the loop, $\theta = 0°$, and

$$B = \frac{\Phi_B}{(\pi r^2)\cos\theta} = \frac{8.00\times10^{-3}\ \text{T}\cdot\text{m}^2}{\pi(0.12\ \text{m})^2\cos 0°} = \boxed{0.177\ \text{T}}$$

(b) If the field is directed parallel to the plane of the loop, $\theta = 90°$, and

$$\Phi_B = BA\cos\theta = BA\cos 90° = \boxed{0}$$

20.4 The magnetic field lines are tangent to the surface of the cylinder, so that no magnetic field lines penetrate the cylindrical surface. The total flux through the cylinder is $\boxed{\text{zero}}$.

20.5 (a) Every field line that comes up through the area A on one side of the wire goes back down through area A on the other side of the wire. Thus, the net flux through the coil is $\boxed{\text{zero}}$.

(b) The magnetic field is parallel to the plane of the coil, so $\theta = 90.0°$. Therefore,

$$\Phi_B = BA\cos\theta = BA\cos 90.0° = \boxed{0}$$

20.6 (a) The magnitude of the field inside the solenoid is

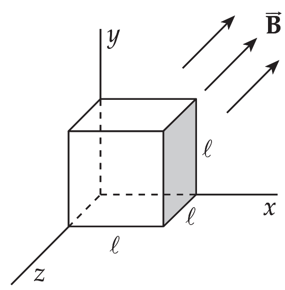
$$B = \mu_0 nI = \mu_0\left(\frac{N}{\ell}\right)I = \left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(\frac{400}{0.360\ \mathrm{m}}\right)(5.00\ \mathrm{A}) = 6.98\times10^{-3}\ \mathrm{T} = \boxed{6.98\ \mathrm{mT}}$$

(b) The field inside a solenoid is directed perpendicular to the cross-sectional area, so $\theta = 0°$ and the flux through a loop of the solenoid is

$$\Phi_B = BA\cos\theta = B\left(\pi r^2\right)\cos\theta$$

$$= \left(6.98\times10^{-3}\ \mathrm{T}\right)\pi\left(3.00\times10^{-2}\ \mathrm{m}\right)^2\cos 0° = \boxed{1.97\times10^{-5}\ \mathrm{T\cdot m^2}}$$

20.7 (a) The magnetic flux through an area A may be written as



$$\Phi_B = (B\cos\theta)A$$

$$= (\text{component of } B \text{ perpendicular to } A)\cdot A$$

Thus, the flux through the shaded side of the cube is

$$\Phi_B = B_x\cdot A = (5.0\ \mathrm{T})\cdot\left(2.5\times10^{-2}\ \mathrm{m}\right)^2 = \boxed{3.1\times10^{-3}\ \mathrm{T\cdot m^2}}$$

(b) Unlike electric field lines, magnetic field lines always form closed loops, without beginning or end. Therefore, no magnetic field lines originate or terminate within the cube and any line entering the cube at one point must emerge from the cube at some other point. The net flux through the cube, and indeed through any *closed* surface, is $\boxed{\text{zero}}$.

20.8 $$|\mathcal{E}| = \frac{|\Delta\Phi_B|}{\Delta t} = \frac{(\Delta B)A\cos\theta}{\Delta t} = \frac{(1.5\ \mathrm{T}-0)\left[\pi\left(1.6\times10^{-3}\ \mathrm{m}\right)^2\right]\cos 0°}{120\times10^{-3}\ \mathrm{s}} = 1.0\times10^{-4}\ \mathrm{V} = \boxed{0.10\ \mathrm{mV}}$$

20.9 With the constant field directed perpendicular to the plane of the coil, the flux through the coil is $\Phi_B = BA\cos 0° = BA$. As the enclosed area increases, the magnitude of the induced emf in the coil is

$$|\mathcal{E}| = \frac{|\Delta\Phi_B|}{\Delta t} = B\left(\frac{\Delta A}{\Delta t}\right) = (0.30\ \mathrm{T})\left(5.0\times10^{-3}\ \mathrm{m^2/s}\right) = 1.5\times10^{-3}\ \mathrm{V} = \boxed{1.5\ \mathrm{mV}}$$

20.10 $$|\mathcal{E}| = \frac{\Delta\Phi_B}{\Delta t} = \frac{B(\Delta A)\cos\theta}{\Delta t}$$

$$= \frac{(0.15\ \mathrm{T})\left[\pi(0.12\ \mathrm{m})^2 - 0\right]\cos 0°}{0.20\ \mathrm{s}} = 3.4\times10^{-2}\ \mathrm{V} = \boxed{34\ \mathrm{mV}}$$

20.11 The magnitude of the induced emf is $|\mathcal{E}| = \dfrac{|\Delta\Phi_B|}{\Delta t} = \dfrac{|\Delta(B\cos\theta)|A}{\Delta t}$

If the normal to the plane of the loop is considered to point in the original direction of the magnetic field, then $\theta_i = 0°$ and $\theta_f = 180°$. Thus, we find

$$|\mathcal{E}| = \frac{|(0.20\text{ T})\cos 180° - (0.30\text{ T})\cos 0°|\,\pi(0.30\text{ m})^2}{1.5\text{ s}} = 9.4\times10^{-2}\text{ V} = \boxed{94\text{ mV}}$$

20.12 With the field directed perpendicular to the plane of the coil, the flux through the coil is $\Phi_B = BA\cos 0° = BA$. As the magnitude of the field increases, the magnitude of the induced emf in the coil is

$$|\mathcal{E}| = \frac{|\Delta\Phi_B|}{\Delta t} = \left(\frac{\Delta B}{\Delta t}\right)A = (0.050\,0\text{ T/s})\left[\pi(0.120\text{ m})^2\right] = 2.26\times10^{-3}\text{ V} = \boxed{2.26\text{ mV}}$$

20.13 The required induced emf is $|\mathcal{E}| = IR = (0.10\text{ A})(8.0\ \Omega) = 0.80\text{ V}.$

From $|\mathcal{E}| = \dfrac{\Delta\Phi_B}{\Delta t} = \left(\dfrac{\Delta B}{\Delta t}\right)NA\cos\theta$

$$\frac{\Delta B}{\Delta t} = \frac{|\mathcal{E}|}{NA\cos\theta} = \frac{0.80\text{ V}}{(75)\left[(0.050\text{ m})(0.080\text{ m})\right]\cos 0°} = \boxed{2.7\text{ T/s}}$$

20.14 The initial magnetic field inside the solenoid is

$$B = \mu_0 nI = (4\pi\times10^{-7}\text{ T}\cdot\text{m/A})\left(\frac{100}{0.200\text{ m}}\right)(3.00\text{ A}) = 1.88\times10^{-3}\text{ T}$$

(a) $\Phi_B = BA\cos\theta = (1.88\times10^{-3}\text{ T})(1.00\times10^{-2}\text{ m})^2\cos 0°$

$$= \boxed{1.88\times10^{-7}\text{ T}\cdot\text{m}^2}$$

(b) When the current is zero, the flux through the loop is $\Phi_B = 0$ and the average induced emf has been

$$|\mathcal{E}| = \frac{|\Delta\Phi_B|}{\Delta t} = \frac{|0 - 1.88\times10^{-7}\text{ T}\cdot\text{m}^2|}{3.00\text{ s}} = \boxed{6.28\times10^{-8}\text{ V}}$$

20.15 (a) The initial field inside the solenoid is

$$B_i = \mu_0 nI_i = (4\pi\times10^{-7}\text{ T}\cdot\text{m/A})\left(\frac{300}{0.200\text{ m}}\right)(2.00\text{ A}) = \boxed{3.77\times10^{-3}\text{ T}}$$

(b) The final field inside the solenoid is

$$B_f = \mu_0 nI_f = (4\pi\times10^{-7}\text{ T}\cdot\text{m/A})\left(\frac{300}{0.200\text{ m}}\right)(5.00\text{ A}) = \boxed{9.42\times10^{-3}\text{ T}}$$

(c) The 4-turn coil encloses an area $A = \pi r^2 = \pi(1.50\times10^{-2}\text{ m})^2 = \boxed{7.07\times10^{-4}\text{ m}^2}$

(d) The change in flux through each turn of the 4-turn coil during the 0.900-s period is

$$\Delta\Phi_B = (\Delta B)A = (9.42\times10^{-3}\text{ T} - 3.77\times10^{-3}\text{ T})(7.07\times10^{-4}\text{ m}^2) = \boxed{3.99\times10^{-6}\text{ Wb}}$$

continued on next page

(e) The average induced emf in the 4-turn coil is

$$\mathcal{E} = N_2\left(\frac{\Delta\Phi_B}{\Delta t}\right) = 4\left(\frac{3.99\times10^{-6}\ \text{Wb}}{0.900\ \text{s}}\right) = \boxed{1.77\times10^{-5}\ \text{V}}$$

Since the current increases at a constant rate during this time interval, the induced emf at any instant during the interval is the same as the average value given above.

(f) The induced emf is small, so the current in the 4-turn coil will also be very small. $\boxed{\text{This means that the magnetic field generated by this current will be negligibly small in comparison to the field generated by the solenoid.}}$

20.16 The magnitude of the average emf is

$$|\mathcal{E}| = \frac{N(\Delta\Phi_B)}{\Delta t} = \frac{NBA[\Delta(\cos\theta)]}{\Delta t}$$

$$= \frac{200(1.1\ \text{T})(100\times10^{-4}\ \text{m}^2)(\cos 0° - \cos 180°)}{0.10\ \text{s}} = 44\ \text{V}$$

Therefore, the average induced current is $I = \frac{|\mathcal{E}|}{R} = \frac{44\ \text{V}}{5.0\ \Omega} = \boxed{8.8\ \text{A}}$

20.17 If the magnetic field makes an angle of 28.0° with the plane of the coil, the angle it makes with the normal to the plane of the coil is $\theta = 62.0°$. Thus,

$$|\mathcal{E}| = \frac{N(\Delta\Phi_B)}{\Delta t} = \frac{NB(\Delta A)\cos\theta}{\Delta t}$$

$$= \frac{200(50.0\times10^{-6}\ \text{T})\left[(39.0\ \text{cm}^2)(1\ \text{m}^2/10^4\ \text{cm}^2)\right]\cos 62.0°}{1.80\ \text{s}} = 1.02\times10^{-5}\ \text{V} = \boxed{10.2\ \mu\text{V}}$$

20.18 With the magnetic field perpendicular to the plane of the coil, the flux through each turn of the coil is $\Phi_B = BA = B(\pi r^2)$. Since the area remains constant, the change in flux due to the changing magnitude of the magnetic field is $\Delta\Phi_B = (\Delta B)\pi r^2$.

(a) The induced emf is: $\mathcal{E} = -N\left(\frac{\Delta\Phi}{\Delta t}\right) = -N\left[\frac{(B_0 - 0)\pi r^2}{t - 0}\right] = \boxed{-\frac{NB_0\pi r^2}{t}}$

(b) When looking down on the coil from a location on the positive z-axis, the magnetic field (in the positive z-direction) is directed up toward you and increasing in magnitude. This means the change in the flux through the coil is directed upward. In order to oppose this change in flux, the current must produce a flux directed downward through the area enclosed by the coil. Thus, the current must flow $\boxed{\text{clockwise}}$ as seen from your viewing location.

(c) Since the turns of the coil are connected in series, the total resistance of the coil is $R_{eq} = NR$. Thus, the magnitude of the induced current is

$$I = \frac{|\mathcal{E}|}{R_{eq}} = \frac{NB_0\pi r^2/t}{NR} = \boxed{\frac{B_0\pi r^2}{tR}}$$

20.19 The vertical component of the Earth's magnetic field is perpendicular to the horizontal velocity of the metallic truck body. Thus, the motional emf induced across the width of the truck is

$$\mathcal{E} = B_{\perp}\ell v = \left(35\times10^{-6}\ \text{T}\right)\left[\left(79.8\ \text{in}\right)\left(\frac{1\ \text{m}}{39.37\ \text{in}}\right)\right]\left(37\ \text{m/s}\right) = 2.6\times10^{-3}\ \text{V} = \boxed{2.6\ \text{mV}}$$

20.20 The vertical component of the Earth's magnetic field is perpendicular to the horizontal velocity of the wire. Thus, the magnitude of the motional emf induced in the wire is

$$\mathcal{E} = B_{\perp}\ell v = \left(40.0\times10^{-6}\ \text{T}\right)\left(2.00\ \text{m}\right)\left(15.0\ \text{m/s}\right) = 1.20\times10^{-3}\ \text{V} = \boxed{1.20\ \text{mV}}$$

Imagine holding your right hand horizontal with the fingers pointing north (the direction of the wire's velocity), such that when you close your hand the fingers curl downward (in the direction of $B_{\perp}$). Your thumb will then be pointing westward. By right-hand rule 1, the magnetic force on charges in the wire would tend to move positive charges westward. Thus, $\boxed{\text{the west end of the wire will be positive relative to the east end}}$.

20.21 (a) Observe that only the horizontal component, B_h, of Earth's magnetic field is effective in exerting a vertical force on charged particles in the antenna. For the magnetic force, $F_m = qvB_h\sin\theta$, on positive charges in the antenna to be directed upward and have maximum magnitude (when $\theta = 90°$), the car should move $\boxed{\text{toward the east}}$ through the northward horizontal component of the magnetic field.

(b) $\mathcal{E} = B_h\ell v$, where B_h is the horizontal component of the magnetic field.

$$\mathcal{E} = \left[\left(50.0\times10^{-6}\ \text{T}\right)\cos 65.0°\right]\left(1.20\ \text{m}\right)\left[\left(65.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{0.278\ \text{m/s}}{1\ \text{km/h}}\right)\right]$$

$$= \boxed{4.58\times10^{-4}\ \text{V}}$$

20.22 (a) Since $\mathcal{E} = B_{\perp}\ell v$, the magnitude of the vertical component of the Earth's magnetic field at this location is

$$B_{\text{vertical}} = B_{\perp} = \frac{\mathcal{E}}{\ell v} = \frac{0.45\ \text{V}}{\left(25\ \text{m}\right)\left(3.0\times10^{3}\ \text{m/s}\right)} = 6.0\times10^{-6}\ \text{T} = \boxed{6.0\ \mu\text{T}}$$

(b) $\boxed{\text{Yes.}}$ The magnitude and direction of the Earth's field varies from one location to the other, so the induced voltage in the wire changes. Further, the voltage will change if the tether cord changes its orientation relative to the Earth's field.

20.23 $\mathcal{E} = B_{\perp}\ell v$, where $B_{\perp}$ is the component of the magnetic field perpendicular to the velocity $\vec{\mathbf{v}}$. Thus,

$$\mathcal{E} = \left[\left(50.0\times10^{-6}\ \text{T}\right)\sin 58.0°\right]\left(60.0\ \text{m}\right)\left(300\ \text{m/s}\right) = \boxed{0.763\ \text{V}}$$

20.24 From $\mathcal{E} = B\ell v$, the required speed is

$$v = \frac{\mathcal{E}}{B\ell} = \frac{IR}{B\ell} = \frac{\left(0.500\ \text{A}\right)\left(6.00\ \Omega\right)}{\left(2.50\ \text{T}\right)\left(1.20\ \text{m}\right)} = \boxed{1.00\ \text{m/s}}$$

20.25 (a) To oppose the motion of the magnet, the magnetic field generated by the induced current should be directed to the right along the axis of the coil. The current must then be $\boxed{\text{left to right}}$ through the resistor.

(b) The magnetic field produced by the current should be directed to the left along the axis of the coil, so the current must be $\boxed{\text{right to left}}$ through the resistor.

20.26 When the switch is closed, the magnetic field due to the current from the battery will be directed to the left along the axis of the cylinder. To oppose this increasing leftward flux, the induced current in the other loop must produce a field directed to the right through the area it encloses. Thus, the induced current is $\boxed{\text{left to right}}$ through the resistor.

20.27 Since the magnetic force, $F_m = qvB\sin\theta$, on a positive charge is directed toward the top of the bar when the velocity is to the right, the right hand rule says that the magnetic field is directed $\boxed{\text{into the page}}$.

20.28 When the switch is closed, the current from the battery produces a magnetic field directed toward the right along the axis of both coils.

(a) As the battery current is growing in magnitude, the induced current in the rightmost coil opposes the increasing rightward directed field by generating a field toward to the left along the axis. Thus, the induced current must be $\boxed{\text{left to right}}$ through the resistor.

(b) Once the battery current, and the field it produces, have stabilized, the flux through the rightmost coil is constant and there is $\boxed{\text{no induced current}}$.

(c) As the switch is opened, the battery current and the field it produces rapidly decrease in magnitude. To oppose this decrease in the rightward directed field, the induced current must produce a field toward the right along the axis, so the induced current is $\boxed{\text{right to left}}$ through the resistor.

20.29 When the switch is closed, the current from the battery produces a magnetic field directed toward the left along the axis of both coils.

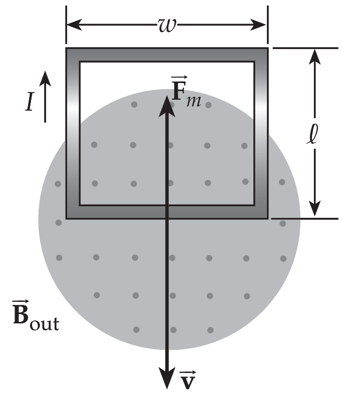
(a) As the current from the battery, and the leftward field it produces, increase in magnitude, the induced current in the leftmost coil opposes the increased leftward field by flowing $\boxed{\text{right to left}}$ through R and producing a field directed toward the right along the axis.

(b) As the variable resistance is decreased, the battery current and the leftward field generated by it increase in magnitude. To oppose this, the induced current is $\boxed{\text{right to left}}$ through R, producing a field directed toward the right along the axis.

(c) Moving the circuit containing R to the left decreases the leftward field (due to the battery current) along its axis. To oppose this decrease, the induced current is $\boxed{\text{left to right}}$ through R, producing an additional field directed toward the left along the axis.

(d) As the switch is opened, the battery current and the leftward field it produces decrease rapidly in magnitude. To oppose this decrease, the induced current is $\boxed{\text{left to right}}$ through R, generating additional magnetic field directed toward the left along the axis.

20.30 (a) As the bottom conductor of the loop falls, it cuts across the magnetic field lines coming out of the page. This induces an emf of magnitude $|\mathcal{E}| = Bwv$ in this conductor, with the left end at the higher potential. As a result, an induced current of magnitude



$$I = \frac{|\mathcal{E}|}{R} = \frac{Bwv}{R}$$

flows clockwise around the loop. The field then exerts an upward force of magnitude

$$F_m = BIw = B\left(\frac{Bwv}{R}\right)w = \frac{B^2w^2v}{R}$$

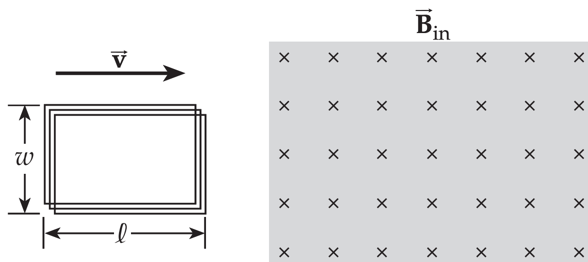
on this current-carrying conductor forming the bottom of the loop. If the loop is falling at terminal speed, the magnitude of this force must equal the downward gravitational force acting on the loop. That is, when $v = v_T$, we must have

$$\frac{B^2w^2v_t}{R} = Mg \qquad \text{or} \qquad v_T = \boxed{\frac{MgR}{B^2w^2}}$$

(b) A larger resistance would make the current smaller, so the loop must reach higher speed before the magnitude of the magnetic force will equal the gravitational force.

(c) The magnetic force is proportional to the product of the field and the current, while the current itself is proportional to the field. If B is cut in half, the speed must become four times larger to compensate and yield a magnetic force with magnitude equal to the that of the gravitational force.

20.31 (a) After the right end of the coil has entered the field, but the left end has not, the flux through the area enclosed by the coil is directed into the page and is increasing in magnitude. This increasing flux induces an emf of magnitude



$$|\mathcal{E}| = \frac{\Delta\Phi_B}{\Delta t} = \frac{NB(\Delta A)}{\Delta t} = NBwv$$

in the loop. Note that in the above equation, ΔA is the area enclosed by the coil that enters the field in time Δt. This emf produces a counterclockwise current in the loop to oppose the increasing inward flux. The magnitude of this current is $I = |\mathcal{E}|/R = NBwv/R$. The right end of the loop is now a conductor, of length Nw, carrying a current toward the top of the page through a field directed into the page. The field exerts a magnetic force of magnitude

$$F = BI(Nw) = B\left(\frac{NBwv}{R}\right)(Nw) = \boxed{\frac{N^2B^2w^2v}{R}} \quad \text{directed} \quad \boxed{\text{toward the left}}$$

on this conductor, and hence, on the loop.

(b) When the loop is entirely within the magnetic field, the flux through the area enclosed by the loop is constant. Hence, there is no induced emf or current in the loop, and the field exerts $\boxed{\text{zero}}$ force on the loop.

continued on next page

(c) After the right end of the loop emerges from the field, and before the left end emerges, the flux through the loop is directed into the page and decreasing. This decreasing flux induces an emf of magnitude $|\mathcal{E}| = NBwv$ in the loop, which produces an induced current directed clockwise around the loop so as to oppose the decreasing flux. The current has magnitude $I = |\mathcal{E}|/R = NBwv/R$. This current flowing upward, through conductors of total length Nw, in the left end of the loop, experiences a magnetic force given by

$$F = BI(Nw) = B\left(\frac{NBwv}{R}\right)(Nw) = \boxed{\frac{N^2B^2w^2v}{R}} \text{ directed } \boxed{\text{toward the left}}$$

20.32 (a) The motional emf induced in the bar must be $\mathcal{E} = IR$, where I is the current in this series circuit. Since $\mathcal{E} = B_\perp \ell v$, the speed of the moving bar must be

$$v = \frac{\mathcal{E}}{B_\perp \ell} = \frac{IR}{B_\perp \ell} = \frac{(8.5\times10^{-3}\text{ A})(9.0\ \Omega)}{(0.30\text{ T})(0.35\text{ m})} = \boxed{0.73\text{ m/s}}$$

The flux through the closed loop formed by the rails, the bar, and the resistor is directed into the page and is increasing in magnitude. To oppose this change in flux, the current must flow in a manner so as to produce flux out of the page through the area enclosed by the loop. This means the current will flow $\boxed{\text{counterclockwise}}$.

(b) The rate at which energy is delivered to the resistor is

$$\mathcal{P} = I^2R = (8.5\times10^{-3}\text{ A})^2(9.0\ \Omega) = 6.5\times10^{-4}\text{ W} = 0.65\text{ mW} = \boxed{0.65\text{ mJ/s}}$$

(c) An external force directed to the right acts on the bar to balance the magnetic force to the left. Hence, $\boxed{\text{work is being done by the external force}}$, which is transformed into the resistor's thermal energy.

20.33 The emf induced in a rotating coil is directly proportional to the angular frequency of the coil. Thus,

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{\omega_2}{\omega_1} \qquad \text{or} \qquad \mathcal{E}_2 = \left(\frac{\omega_2}{\omega_1}\right)\mathcal{E}_1 = \left(\frac{500\text{ rev/min}}{900\text{ rev/min}}\right)(24.0\text{ V}) = \boxed{13.3\text{ V}}$$

20.34 $$\mathcal{E}_{\max} = NB_{\text{horizontal}}A\omega = 100(2.0\times10^{-5}\text{ T})(0.20\text{ m})^2\left[\left(1500\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]$$

$$= 1.3\times10^{-2}\text{ V} = \boxed{13\text{ mV}}$$

20.35 Note the similarity between the situation in this problem and a generator. In a generator, one normally has a loop rotating in a constant magnetic field so the flux through the loop varies sinusoidally in time. In this problem, we have a stationary loop in an oscillating magnetic field, and the flux through the loop varies sinusoidally in time. In both cases, a sinusoidal emf $\mathcal{E} = \mathcal{E}_{\max}\sin\omega t$ where $\mathcal{E}_{\max} = NBA\omega$ is induced in the loop.

The loop in this case consists of a single band $(N = 1)$ around the perimeter of a red blood cell with diameter $d = 8.0\times10^{-6}$ m. The angular frequency of the oscillating flux through the area of this loop is $\omega = 2\pi f = 2\pi(60\text{ Hz}) = 120\pi$ rad/s. The maximum induced emf is then

$$\mathcal{E}_{\max} = NBA\omega = B\left(\frac{\pi d^2}{4}\right)\omega = \frac{(1.0\times10^{-3}\text{ T})\pi(8.0\times10^{-6}\text{ m})^2(120\pi\text{ s}^{-1})}{4} = \boxed{1.9\times10^{-11}\text{ V}}$$

20.36 (a) Using $\mathcal{E}_{\max} = NBA\omega$,

$$\mathcal{E}_{\max} = 1\,000(0.20\text{ T})(0.10\text{ m}^2)\left[\left(60\ \frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\right] = 7.5\times 10^3 = \boxed{7.5\text{ kV}}$$

(b) $\mathcal{E}_{\max}$ occurs when the flux through the loop is changing the most rapidly. This is when the plane of the loop is $\boxed{\text{parallel to the magnetic field}}$.

20.37 $\omega = \left(120\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 4\pi\ \frac{\text{rad}}{\text{s}}$

and the period is $T = \frac{2\pi}{\omega} = 0.50\text{ s}$

(a) $\mathcal{E}_{\max} = NBA\omega = 500(0.60\text{ T})[(0.080\text{ m})(0.20\text{ m})](4\pi\ \text{rad/s}) = \boxed{60\text{ V}}$

(b) $\mathcal{E} = \mathcal{E}_{\max}\sin(\omega t) = (60\text{ V})\sin\left[(4\pi\ \text{rad/s})\left(\frac{\pi}{32}\ \text{s}\right)\right] = \boxed{57\text{ V}}$

(c) The emf is first maximum at $t = \frac{T}{4} = \frac{0.50\text{ s}}{4} = \boxed{0.13\text{ s}}$.

20.38 (a) Immediately after the switch is closed, the motor coils are still stationary and the back emf is zero. Thus, $I = \frac{\mathcal{E}}{R} = \frac{240\text{ V}}{30\ \Omega} = \boxed{8.0\text{ A}}$.

(b) At maximum speed, $\mathcal{E}_{back} = 145\text{ V}$ and

$$I = \frac{\mathcal{E} - \mathcal{E}_{back}}{R} = \frac{240\text{ V} - 145\text{ V}}{30\ \Omega} = \boxed{3.2\text{ A}}$$

(c) $\mathcal{E}_{back} = \mathcal{E} - IR = 240\text{ V} - (6.0\text{ A})(30\ \Omega) = \boxed{60\text{ V}}$

20.39 (a) When a coil having N turns and enclosing area A rotates at angular frequency ω in a constant magnetic field, the emf induced in the coil is

$$\mathcal{E} = \mathcal{E}_{\max}\sin\omega t \quad \text{where} \quad \mathcal{E}_{\max} = NB_{\perp}A\omega$$

Here, $B_{\perp}$ is the magnitude of the magnetic field perpendicular to the rotation axis of the coil. In the given case, $B_{\perp} = 55.0\ \mu\text{T}$; $A = \pi ab$ where $a = (10.0\text{ cm})/2$ and $b = (4.00\text{ cm})/2$; and

$$\omega = 2\pi f = 2\pi\left(100\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = 10.5\ \text{rad/s}$$

Thus, $\mathcal{E}_{\max} = (10.0)(55.0\times 10^{-6}\text{ T})\left[\frac{\pi}{4}(0.100\text{ m})(0.040\,0\text{ m})\right](10.5\ \text{rad/s})$

or $\mathcal{E}_{\max} = 1.81\times 10^{-5}\text{ V} = \boxed{18.1\ \mu\text{V}}$

(b) When the rotation axis is parallel to the field, then $B_{\perp} = 0$ giving $\mathcal{E}_{\max} = \boxed{0}$

It is easily understood that the induced SSM is always zero in this case if you recognize that the magnetic field lines are always parallel to the plane of the coil, and the flux through the coil has a constant value of zero.

20.40 (a) In terms of its cross-sectional area (A), length (ℓ), and number of turns (N), the self inductance of a solenoid is given as $L = \mu_0 N^2 A/\ell$. Thus, for the given solenoid,

$$L = \frac{\mu_0 N^2 \left(\pi d^2/4\right)}{\ell} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(580)^2 \pi \left(8.0\times 10^{-2}\ \text{m}\right)^2}{4(0.36\ \text{m})} = 5.9\times 10^{-3}\ \text{H} = \boxed{5.9\ \text{mH}}$$

(b) $$\mathcal{E} = -L\left(\frac{\Delta I}{\Delta t}\right) = -\left(5.9\times 10^{-3}\ \text{H}\right)(+4.0\ \text{A/s}) = -24\times 10^{-3}\ \text{V} = -24\ \text{mV}$$

20.41 From $|\mathcal{E}| = L|\Delta I/\Delta t|$, we have

$$L = \frac{|\mathcal{E}|}{|\Delta I/\Delta t|} = \frac{|\mathcal{E}|(\Delta t)}{|\Delta I|} = \frac{\left(12\times 10^{-3}\ \text{V}\right)(0.50\ \text{s})}{|2.0\ \text{A} - 3.5\ \text{A}|} = 4.0\times 10^{-3}\ \text{H} = \boxed{4.0\ \text{mH}}$$

20.42 The units of $\dfrac{N\Phi_B}{I}$ are $\dfrac{\text{T}\cdot\text{m}^2}{\text{A}}$

From the force on a moving charged particle, $F = qvB$, the magnetic field is $B = \dfrac{F}{qv}$ and we find that

$$1\ \text{T} = 1\ \frac{\text{N}}{\text{C}\cdot(\text{m/s})} = 1\ \frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}}$$

Thus, $\text{T}\cdot\text{m}^2 = \left(\dfrac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}}\right)\cdot\text{m}^2 = \dfrac{(\text{N}\cdot\text{m})\cdot\text{s}}{\text{C}} = \left(\dfrac{\text{J}}{\text{C}}\right)\cdot\text{s} = \text{V}\cdot\text{s}$

and $\dfrac{\text{T}\cdot\text{m}^2}{\text{A}} = \dfrac{\text{V}\cdot\text{s}}{\text{A}}$ which is the same as the units of $\dfrac{\mathcal{E}}{\Delta I/\Delta t}$

20.43 (a) $$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(400)^2\left[\pi\left(2.5\times 10^{-2}\ \text{m}\right)^2\right]}{0.20\ \text{m}}$$

$$= 2.0\times 10^{-3}\ \text{H} = \boxed{2.0\ \text{mH}}$$

(b) From $|\mathcal{E}| = L(\Delta I/\Delta t)$, $\dfrac{\Delta I}{\Delta t} = \dfrac{|\mathcal{E}|}{L} = \dfrac{75\times 10^{-3}\ \text{V}}{2.0\times 10^{-3}\ \text{H}} = \boxed{38\ \text{A/s}}$

20.44 From $|\mathcal{E}| = L(\Delta I/\Delta t)$, the self-inductance is

$$L = \frac{|\mathcal{E}|}{\Delta I/\Delta t} = \frac{24.0\times 10^{-3}\ \text{V}}{10.0\ \text{A/s}} = 2.40\times 10^{-3}\ \text{H}$$

Then, from $L = N\Phi_B/I$, the magnetic flux through each turn is

$$\Phi_B = \frac{L\cdot I}{N} = \frac{\left(2.40\times 10^{-3}\ \text{H}\right)(4.00\ \text{A})}{500} = \boxed{1.92\times 10^{-5}\ \text{T}\cdot\text{m}^2}$$

20.45 (a) $$\tau = \frac{L}{R} = \frac{12\times 10^{-3}\ \text{H}}{3.0\ \Omega} = 4.0\times 10^{-3}\ \text{s} = \boxed{4.0\ \text{ms}}$$

(b) $I_{max} = \dfrac{\mathcal{E}}{R}$, so $\mathcal{E} = I_{max}R = \left(150\times 10^{-3}\ \text{A}\right)(3.0\ \Omega) = 4.5\times 10^{-1}\ \text{V} = \boxed{0.45\ \text{V}}$

20.46 (a) The time constant of the *RL* circuit is $\tau = L/R$, and that of the *RC* circuit is $\tau = RC$. If the two time constants have the same value, then

$$RC = \frac{L}{R}, \quad \text{or} \quad R = \sqrt{\frac{L}{C}} = \sqrt{\frac{3.00\text{ H}}{3.00\times10^{-6}\text{ F}}} = 1.00\times10^{3}\ \Omega = \boxed{1.00\text{ k}\Omega}$$

(b) The common value of the two time constants is

$$\tau = \frac{L}{R} = \frac{3.00\text{ H}}{1.00\times10^{3}\ \Omega} = 3.00\times10^{-3}\text{ s} = \boxed{3.00\text{ ms}}$$

20.47 (a) $I_{\max} = \dfrac{\mathcal{E}}{R}$, so $\mathcal{E} = I_{\max}R = (8.0\text{ A})(0.30\ \Omega) = \boxed{2.4\text{ V}}$

(b) The time constant is $\tau = \dfrac{L}{R}$, giving

$$L = \tau R = (0.25\text{ s})(0.30\ \Omega) = 7.5\times10^{-2}\text{ H} = \boxed{75\text{ mH}}$$

(c) The current as a function of time is $I = I_{\max}\left(1-e^{-t/\tau}\right)$, so at $t=\tau$,

$$I = I_{\max}\left(1-e^{-1}\right) = 0.632 I_{\max} = 0.632(8.0\text{ A}) = \boxed{5.1\text{ A}}$$

(d) At $t=\tau$, $I = 5.1\text{ A}$ and the voltage drop across the resistor is

$$\Delta V_R = -IR = -(5.1\text{ A})(0.30\ \Omega) = \boxed{-1.5\text{ V}}$$

Applying Kirchhoff's loop rule to the circuit shown in Figure 20.27 gives $\mathcal{E} + \Delta V_R + \Delta V_L = 0$. Thus, at $t=\tau$, we have

$$\Delta V_L = -\left(\mathcal{E} + \Delta V_R\right) = -(2.4\text{ V} - 1.5\text{ V}) = \boxed{-0.90\text{ V}}$$

20.48 The current in the *RL* circuit at time *t* is $I = \dfrac{\mathcal{E}}{R}\left(1-e^{-t/\tau}\right)$. The potential difference across the resistor is $\Delta V_R = RI = \mathcal{E}\left(1-e^{-t/\tau}\right)$, and from Kirchhoff's loop rule, the potential difference across the inductor is

$$\Delta V_L = \mathcal{E} - \Delta V_R = \mathcal{E}\left[1-\left(1-e^{-t/\tau}\right)\right] = \mathcal{E}e^{-t/\tau}$$

(a) At $t=0$, $\Delta V_R = \mathcal{E}\left(1-e^{-0}\right) = \mathcal{E}(1-1) = \boxed{0}$

(b) At $t=\tau$, $\Delta V_R = \mathcal{E}\left(1-e^{-1}\right) = (6.0\text{ V})(1-0.368) = \boxed{3.8\text{ V}}$

(c) At $t=0$, $\Delta V_L = \mathcal{E}e^{-0} = \mathcal{E} = \boxed{6.0\text{ V}}$

(d) At $t=\tau$, $\Delta V_L = \mathcal{E}e^{-1} = (6.0\text{ V})(0.368) = \boxed{2.2\text{ V}}$

20.49 From $I = I_{\max}\left(1-e^{-t/\tau}\right)$, $e^{-t/\tau} = 1 - \dfrac{I}{I_{\max}}$

If $\dfrac{I}{I_{\max}} = 0.900$ at $t = 3.00$ s, then

$$e^{-3.00\text{ s}/\tau} = 0.100 \quad \text{or} \quad \tau = \frac{-3.00\text{ s}}{\ln(0.100)} = 1.30\text{ s}$$

Since the time constant of an *RL* circuit is $\tau = L/R$, the resistance is

$$R = \frac{L}{\tau} = \frac{2.50\text{ H}}{1.30\text{ s}} = \boxed{1.92\ \Omega}$$

20.50 (a) $\tau = \dfrac{L}{R} = \dfrac{8.00 \text{ mH}}{4.00\ \Omega} = \boxed{2.00 \text{ ms}}$

(b) $I = \dfrac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) = \left(\dfrac{6.00 \text{ V}}{4.00\ \Omega}\right)\left(1 - e^{-250\times 10^{-6} \text{ s}/2.00\times 10^{-3} \text{ s}}\right) = \boxed{0.176 \text{ A}}$

(c) $I_{\max} = \dfrac{\mathcal{E}}{R} = \dfrac{6.00 \text{ V}}{4.00\ \Omega} = \boxed{1.50 \text{ A}}$

(d) $I = I_{\max}\left(1 - e^{-t/\tau}\right)$ yields $e^{-t/\tau} = 1 - I/I_{\max}$,

and $t = -\tau \ln\left(1 - I/I_{\max}\right) = -\left(2.00 \text{ ms}\right)\ln\left(1 - 0.800\right) = \boxed{3.22 \text{ ms}}$

20.51 (a) The energy stored by an inductor is $PE_L = \frac{1}{2}LI^2$, so the self inductance is

$$L = \frac{2\left(PE_L\right)}{I^2} = \frac{2\left(0.300\times 10^{-3} \text{ J}\right)}{\left(1.70 \text{ A}\right)^2} = 2.08\times 10^{-4} \text{ H} = \boxed{0.208 \text{ mH}}$$

(b) If $I = 3.0$ A, the stored energy will be

$$PE_L = \frac{1}{2}LI^2 = \frac{1}{2}\left(2.08\times 10^{-4} \text{ H}\right)\left(3.0 \text{ A}\right)^2 = 9.36\times 10^{-4} \text{ J} = \boxed{0.936 \text{ mJ}}$$

20.52 (a) The inductance of a solenoid is given by $L = \mu_0 N^2 A/\ell$, where N is the number of turns on the solenoid, A is its cross-sectional area, and ℓ is its length. For the given solenoid,

$$L = \frac{\mu_0 N^2\left(\pi r^2\right)}{\ell} = \frac{\left(4\pi\times 10^{-7} \text{ T}\cdot\text{m/A}\right)\left(300\right)^2 \pi\left(5.00\times 10^{-2} \text{ m}\right)^2}{0.200 \text{ m}} = \boxed{4.44\times 10^{-3} \text{ H}}$$

(b) When the solenoid described above carries a current of $I = 0.500$ A, the stored energy is

$$PE_L = \frac{1}{2}LI^2 = \frac{1}{2}\left(4.44\times 10^{-3} \text{ H}\right)\left(0.500 \text{ A}\right)^2 = \boxed{5.55\times 10^{-4} \text{ J}}$$

20.53 The current in the circuit at time t is $I = \dfrac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$, and the energy stored in the inductor is $PE_L = \dfrac{1}{2}LI^2$

(a) As $t \to \infty$, $I \to I_{\max} = \dfrac{\mathcal{E}}{R} = \dfrac{24 \text{ V}}{8.0\ \Omega} = 3.0$ A, and

$$PE_L \to \frac{1}{2}LI_{\max}^2 = \frac{1}{2}\left(4.0 \text{ H}\right)\left(3.0 \text{ A}\right)^2 = \boxed{18 \text{ J}}$$

(b) At $t = \tau$, $I = I_{\max}\left(1 - e^{-1}\right) = \left(3.0 \text{ A}\right)\left(1 - 0.368\right) = 1.9$ A

and $PE_L = \dfrac{1}{2}\left(4.0 \text{ H}\right)\left(1.9 \text{ A}\right)^2 = \boxed{7.2 \text{ J}}$

20.54 (a) Use Table 17.1 to obtain the resistivity of the copper wire and find

$$R_{\text{wire}} = \frac{\rho_{\text{Cu}} L}{A_{\text{wire}}} = \frac{\rho_{\text{Cu}} L}{\pi r_{\text{wire}}^2} = \frac{\left(1.7\times 10^{-8}\ \Omega\cdot\text{m}\right)\left(60.0 \text{ m}\right)}{\pi\left(0.50\times 10^{-3} \text{ m}\right)^2} = \boxed{1.3\ \Omega}$$

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(b) $$N = \frac{L}{\text{Circumference of a loop}} = \frac{L}{2\pi r_{\text{solenoid}}} = \frac{60.0\ \text{m}}{2\pi\left(2.0\times10^{-2}\ \text{m}\right)} = \boxed{4.8\times10^{2}\ \text{turns}}$$

(c) The length of the solenoid is

$$\ell = N(\text{diameter of wire}) = N\left(2r_{\text{wire}}\right) = (480)2\left(0.50\times10^{-3}\ \text{m}\right) = \boxed{0.48\ \text{m}}$$

(d) $$L = \frac{\mu_0 N^2 A_{\text{solenoid}}}{\ell} = \frac{\mu_0 N^2 \pi r_{\text{solenoid}}^2}{\ell} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(480)^2\,\pi\left(2.0\times10^{-2}\ \text{m}\right)^2}{0.48\ \text{m}}$$

giving $L = 7.6\times10^{-4}\ \text{H} = \boxed{0.76\ \text{mH}}$

(e) $$\tau = \frac{L}{R_{\text{total}}} = \frac{L}{R_{\text{wire}} + r_{\text{internal}}} = \frac{7.6\times10^{-4}\ \text{H}}{1.3\ \Omega + 0.350\ \Omega} = 4.6\times10^{-4}\ \text{s} = \boxed{0.46\ \text{ms}}$$

(f) $$I_{\max} = \frac{\mathcal{E}}{R_{\text{total}}} = \frac{6.0\ \text{V}}{1.3\ \Omega + 0.350\ \Omega} = \boxed{3.6\ \text{A}}$$

(g) $I = I_{\max}\left(1 - e^{-t/\tau}\right)$, so when $I = 0.999 I_{\max}$, we have $1 - e^{-t/\tau} = 0.999$ and $e^{-t/\tau} = 1 - 0.999 = 0.001$. Thus, $-\dfrac{t}{\tau} = \ln(0.001)$ or $t = -\tau\cdot\ln(0.001)$

giving $t = -(0.46\ \text{ms})\cdot\ln(0.001) = \boxed{3.2\ \text{ms}}$

(h) $$\left(PE_L\right)_{\max} = \frac{1}{2}LI_{\max}^2 = \frac{1}{2}\left(7.6\times10^{-4}\ \text{H}\right)(3.6\ \text{A})^2 = 4.9\times10^{-3}\ \text{J} = \boxed{4.9\ \text{mJ}}$$

20.55 According to Lenz's law, a current will be induced in the coil to oppose the change in magnetic flux due to the magnet. Therefore, current must be directed from *b* to *a* through the resistor, and $V_a - V_b$ will be $\boxed{\text{negative}}$.

20.56 $$|\mathcal{E}| = \frac{\Delta\Phi_B}{\Delta t} = \frac{NBA\left[\Delta(\cos\theta)\right]}{\Delta t},\ \text{so}\ B = \frac{|\mathcal{E}|\cdot\Delta t}{NA\left[\Delta(\cos\theta)\right]}$$

or $$B = \frac{(0.166\ \text{V})\left(2.77\times10^{-3}\ \text{s}\right)}{500\left[\pi(0.150\ \text{m})^2/4\right]\left[\cos 0^\circ - \cos 90^\circ\right]} = 5.20\times10^{-5}\ \text{T} = \boxed{52.0\ \mu\text{T}}$$

20.57 (a) The current in the solenoid reaches $I = 0.632\,I_{\max}$ in a time of $t = \tau = L/R$, where

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(12\ 500)^2\left(1.00\times10^{-4}\ \text{m}^2\right)}{7.00\times10^{-2}\ \text{m}} = 0.280\ \text{H}$$

Thus, $$t = \frac{0.280\ \text{H}}{14.0\ \Omega} = 2.00\times10^{-2}\ \text{s} = \boxed{20.0\ \text{ms}}$$

(b) The change in the solenoid current during this time is

$$\Delta I = 0.632\,I_{\max} - 0 = 0.632\left(\frac{\Delta V}{R}\right) = 0.632\left(\frac{60.0\ \text{V}}{14.0\ \Omega}\right) = 2.71\ \text{A}$$

so the average back emf is

$$\mathcal{E}_{\text{back}} = L\left(\frac{\Delta I}{\Delta t}\right) = (0.280\ \text{H})\left(\frac{2.71\ \text{A}}{2.00\times10^{-2}\ \text{s}}\right) = \boxed{37.9\ \text{V}}$$

continued on next page

(c) $$\frac{\Delta\Phi_B}{\Delta t} = \frac{(\Delta B)A}{\Delta t} = \frac{\frac{1}{2}\left[\mu_0 n(\Delta I)\right]A}{\Delta t} = \frac{\mu_0 N(\Delta I)A}{2\,\ell\cdot(\Delta t)}$$

$$= \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(12\ 500)(2.71\ \mathrm{A})\left(1.00\times10^{-4}\ \mathrm{m}^2\right)}{2\left(7.00\times10^{-2}\ \mathrm{m}\right)\left(2.00\times10^{-2}\ \mathrm{s}\right)} = \boxed{1.52\times10^{-3}\ \mathrm{V}}$$

(d) $$I = \frac{|\mathcal{E}_{\text{coil}}|}{R_{\text{coil}}} = \frac{N_{\text{coil}}(\Delta\Phi_B/\Delta t)}{R_{\text{coil}}} = \frac{(820)\left(1.52\times10^{-3}\ \mathrm{V}\right)}{24.0\ \Omega} = 0.051\,9\ \mathrm{A} = \boxed{51.9\ \mathrm{mA}}$$

20.58 (a) The gravitational force exerted on the ship by the pulsar supplies the centripetal acceleration needed to hold the ship in orbit. Thus, $F_g = \dfrac{GM_{\text{pulsar}}m_{\text{ship}}}{r_{\text{orbit}}^2} = \dfrac{m_{\text{ship}}v^2}{r_{\text{orbit}}}$, giving

$$v = \sqrt{\frac{GM_{\text{pulsar}}}{r_{\text{orbit}}}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m/kg^2}\right)\left(2.0\times10^{30}\ \mathrm{kg}\right)}{3.0\times10^{7}\ \mathrm{m}}} = \boxed{2.1\times10^{6}\ \mathrm{m/s}}$$

(b) The magnetic force acting on charged particles moving through a magnetic field is perpendicular to both the magnetic field and the velocity of the particles (and therefore perpendicular to the ship's length). Thus, the charged particles in the materials making up the spacecraft experience magnetic forces directed from one side of the ship to the other, meaning that the induced emf is directed $\boxed{\text{from side to side}}$ within the ship.

(c) $\mathcal{E} = B_\perp \ell v$, where $\ell = 2r_{\text{ship}} = 0.080\ \mathrm{km} = 80\ \mathrm{m}$ is the side to side dimension of the ship. This yields

$$\mathcal{E} = \left(1.0\times10^{2}\ \mathrm{T}\right)(80\ \mathrm{m})\left(2.1\times10^{6}\ \mathrm{m/s}\right) = \boxed{1.7\times10^{10}\ \mathrm{V}}$$

(d) The very large induced emf would lead to powerful spontaneous electric discharges. The strong electric and magnetic fields would disrupt the flow of ions in their bodies.

20.59 (a) To move the bar at uniform speed, the magnitude of the applied force must equal that of the magnetic force retarding the motion of the bar. Therefore, $F_{\text{app}} = BI\ell$. The magnitude of the induced current is

$$I = \frac{|\mathcal{E}|}{R} = \frac{(\Delta\Phi_B/\Delta t)}{R} = \frac{B(\Delta A/\Delta t)}{R} = \frac{B\ell v}{R}$$

so the field strength is $B = \dfrac{IR}{\ell v}$, giving $F_{\text{app}} = I^2R/v$

Thus, the current is

$$I = \sqrt{\frac{F_{\text{app}}\cdot v}{R}} = \sqrt{\frac{(1.00\ \mathrm{N})(2.00\ \mathrm{m/s})}{8.00\ \Omega}} = \boxed{0.500\ \mathrm{A}}$$

(b) $$\mathcal{P} = I^2R = (0.500\ \mathrm{A})^2(8.00\ \Omega) = \boxed{2.00\ \mathrm{W}}$$

(c) $$\mathcal{P}_{\text{input}} = F_{\text{app}}\cdot v = (1.00\ \mathrm{N})(2.00\ \mathrm{m/s}) = \boxed{2.00\ \mathrm{W}}$$

20.60 (a) When the motor is first turned on, the coil is not rotating so the back emf is zero. Thus, the current is a maximum with only the resistance of the windings limiting its value. This gives $I_{\text{max}} = \mathcal{E}/R$, or

$$R = \frac{\mathcal{E}}{I_{\text{max}}} = \frac{120\ \text{V}}{11\ \text{A}} = \boxed{11\ \Omega}$$

(b) When the motor has reached maximum speed, the steady state value of the current is $I = (\mathcal{E} - \mathcal{E}_{\text{back}})/R$, giving the back emf as

$$\mathcal{E}_{\text{back}} = \mathcal{E} - IR = 120\ \text{V} - (4.0\ \text{A})(11\ \Omega) = \boxed{76\ \text{V}}$$

20.61 If d is the distance from the lightning bolt to the center of the coil, then

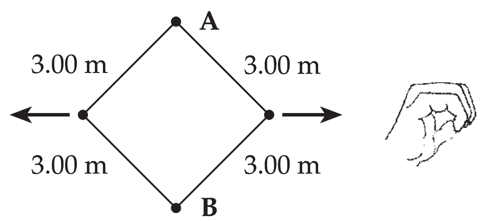
$$|\mathcal{E}_{\text{av}}| = \frac{N(\Delta\Phi_B)}{\Delta t} = \frac{N(\Delta B)A}{\Delta t} = \frac{N[\mu_0(\Delta I)/2\pi d]A}{\Delta t} = \frac{N\mu_0(\Delta I)A}{2\pi d(\Delta t)}$$

$$= \frac{100(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(6.02\times10^{6}\ \text{A}-0)[\pi(0.800\ \text{m})^2]}{2\pi(200\ \text{m})(10.5\times10^{-6}\ \text{s})}$$

$$= 1.15\times10^{5}\ \text{V} = \boxed{115\ \text{kV}}$$

20.62 When A and B are 3.00 m apart, the area enclosed by the loop consists of four triangular sections, each having hypotenuse of 3.00 m, altitude of 1.50 m, and base of $\sqrt{(3.00\ \text{m})^2 - (1.50\ \text{m})^2} = 2.60\ \text{m}$

The decrease in the enclosed area has been



$$\Delta A = A_i - A_f = (3.00\ \text{m})^2 - 4\left[\frac{1}{2}(1.50\ \text{m})(2.60\ \text{m})\right] = 1.20\ \text{m}^2$$

The average induced current has been

$$I_{\text{av}} = \frac{|\mathcal{E}_{\text{av}}|}{R} = \frac{(\Delta\Phi_B/\Delta t)}{R} = \frac{B(\Delta A/\Delta t)}{R} = \frac{(0.100\ \text{T})(1.20\ \text{m}^2/0.100\ \text{s})}{10.0\ \Omega} = \boxed{0.120\ \text{A}}$$

As the enclosed area decreases, the flux (directed into the page) through this area also decreases. Thus, the induced current will be directed $\boxed{\text{clockwise}}$ around the loop to create additional flux directed into the page through the enclosed area.

20.63 (a) $$|\mathcal{E}_{\text{av}}| = \frac{\Delta\Phi_B}{\Delta t} = \frac{B(\Delta A)}{\Delta t} = \frac{B[(\pi d^2/4) - 0]}{\Delta t}$$

$$= \frac{(25.0\ \text{mT})\pi(2.00\times10^{-2}\ \text{m})^2}{4(50.0\times10^{-3}\ \text{s})} = \boxed{0.157\ \text{mV}}$$

As the inward directed flux through the loop decreases, the induced current goes clockwise around the loop in an attempt to create additional inward flux through the enclosed area. With positive charges accumulating at B, $\boxed{\text{point } B \text{ is at a higher potential than } A}$.

continued on next page

(b) $$\left|\mathcal{E}_{\text{av}}\right| = \frac{\Delta\Phi_B}{\Delta t} = \frac{(\Delta B)A}{\Delta t} = \frac{\left[(100-25.0)\ \text{mT}\right]\pi\left(2.00\times10^{-2}\ \text{m}\right)^2}{4\left(4.00\times10^{-3}\ \text{s}\right)} = \boxed{5.89\ \text{mV}}$$

As the inward directed flux through the enclosed area increases, the induced current goes counterclockwise around the loop in an attempt to create flux directed outward through the enclosed area.

With positive charges now accumulating at A, $\boxed{\text{point } A \text{ is at a higher potential than } B}$.

20.64 The induced emf in the ring is

$$\left|\mathcal{E}_{\text{av}}\right| = \frac{\Delta\Phi_B}{\Delta t} = \frac{(\Delta B)A_{\text{solenoid}}}{\Delta t} = \frac{\left(\Delta B_{\text{solenoid}}/2\right)A_{\text{solenoid}}}{\Delta t} = \frac{1}{2}\left[\mu_0 n\left(\frac{\Delta I_{\text{solenoid}}}{\Delta t}\right)\right]A_{\text{solenoid}}$$

$$= \frac{1}{2}\left[\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(1\,000)(270\ \text{A/s})\left(\pi\left[3.00\times10^{-2}\ \text{m}\right]^2\right)\right] = 4.80\times10^{-4}\ \text{V}$$

Thus, the induced current in the ring is

$$I_{\text{ring}} = \frac{\left|\mathcal{E}_{\text{av}}\right|}{R} = \frac{4.80\times10^{-4}\ \text{V}}{3.00\times10^{-4}\ \Omega} = \boxed{1.60\ \text{A}}$$

20.65 (a) As the rolling axle (of length $\ell = 1.50$ m) moves perpendicularly to the uniform magnetic field, an induced emf of magnitude $|\mathcal{E}| = B\ell v$ will exist between its ends. The current produced in the closed-loop circuit by this induced emf has magnitude

$$I = \frac{\left|\mathcal{E}_{\text{av}}\right|}{R} = \frac{\left(\Delta\Phi_B/\Delta t\right)}{R} = \frac{B\left(\Delta A/\Delta t\right)}{R} = \frac{B\,\ell v}{R} = \frac{(0.800\ \text{T})(1.50\ \text{m})(3.00\ \text{m/s})}{0.400\ \Omega} = \boxed{9.00\ \text{A}}$$

(b) The induced current through the axle will cause the magnetic field to exert a retarding force of magnitude $F_r = BI\ell$ on the axle. The direction of this force will be opposite to that of the velocity $\vec{\mathbf{v}}$ so as to oppose the motion of the axle. If the axle is to continue moving at constant speed, an applied force in the direction of $\vec{\mathbf{v}}$ and having magnitude $F_{\text{app}} = F_r$ must be exerted on the axle.

$$F_{\text{app}} = BI\ell = (0.800\ \text{T})(9.00\ \text{A})(1.50\ \text{m}) = \boxed{10.8\ \text{N}}$$

(c) Using the right-hand rule, observe that positive charges within the moving axle experience a magnetic force toward the rail containing point b, and negative charges experience a force directed toward the rail containing point a. Thus, the rail containing b will be positive relative to the other rail. Point $\boxed{b \text{ is then at a higher potential than } a}$, and the current goes from b to a through the resistor R.

(d) $\boxed{\text{No}}$. Both the velocity $\vec{\mathbf{v}}$ of the rolling axle and the magnetic field $\vec{\mathbf{B}}$ are unchanged. Thus, the polarity of the induced emf in the moving axle is unchanged, and the current continues to be directed from b to a through the resistor R.

20.66 (a) The time required for the coil to move distance ℓ and exit the field is $t = \ell/v$, where v is the constant speed of the coil. Since the speed of the coil is constant, the flux through the area enclosed by the coil decreases at a constant rate. Thus, the instantaneous induced emf is the same as the average emf over the interval t seconds in duration, or

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{(0 - BA)}{t - 0} = N\frac{B\ell^2}{t} = \frac{NB\ell^2}{\ell/v} = \boxed{NB\ell v}$$

(b) The current induced in the coil is $I = \dfrac{\mathcal{E}}{R} = \boxed{\dfrac{NB\ell v}{R}}$

(c) The power delivered to the coil is given by $\mathcal{P} = I^2 R$, or

$$\mathcal{P} = \left(\frac{N^2B^2\ell^2v^2}{R^2}\right)R = \boxed{\frac{N^2B^2\ell^2v^2}{R}}$$

(d) The rate that the applied force does work must equal the power delivered to the coil, so $F_{\text{app}} \cdot v = \mathcal{P}$ or

$$F_{\text{app}} = \frac{\mathcal{P}}{v} = \frac{N^2B^2\ell^2v^2/R}{v} = \boxed{\frac{N^2B^2\ell^2v}{R}}$$

(e) As the coil is emerging from the field, the flux through the area it encloses is directed into the page and decreasing in magnitude. Thus, the *change* in the flux through the coil is directed out of the page. The induced current must then flow around the coil in such a direction as to produce flux into the page through the enclosed area, opposing the change that is occurring. This means that the current must flow $\boxed{\text{clockwise}}$ around the coil.

As the coil is emerging from the field, the left side of the coil is carrying an induced current directed toward the top of the page through a magnetic field that is directed into the page. Right-hand rule 1, then shows that this side of the coil will experience $\boxed{\text{a magnetic force directed to the left}}$, opposing the motion of the coil.