18

Direct-Current Circuits

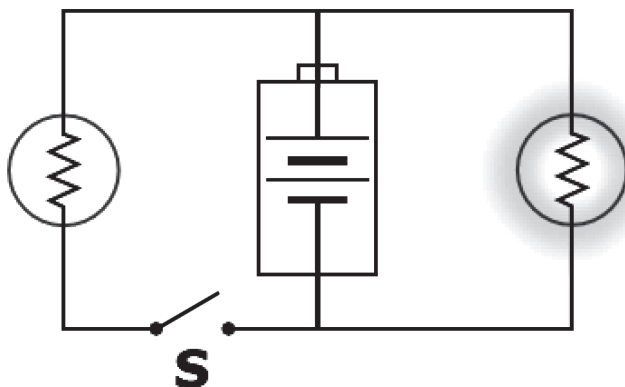### Clicker Questions

Question N2.01

Description: Understanding circuits with parallel resistances.

Question

A battery is used to light a bulb as shown. A second bulb is connected by closing switch S. What happens to the bulbs?

1. The right bulb goes out; the left bulb lights up.
2. The right bulb gets noticeably dimmer; both bulbs have the same brightness.
3. The right bulb gets very slightly dimmer; both bulbs have the same brightness.
4. The right bulb's brightness stays the same; both bulbs have the same brightness.
5. The right bulb gets slightly brighter; both bulbs have the same brightness.
6. The right bulb gets noticeably brighter; both bulbs have the same brightness.
7. The right bulb stays the same; the left bulb does not light.
8. Impossible to determine

Commentary

Purpose: To develop understanding of Kirchhoff's loop law and parallel resistors.

Discussion: Many people think that when the second light bulb is connected to the battery, the first bulb will become dimmer, since the battery must "work harder" to make two bulbs glow or the second light bulb will divert some of the current from the first bulb.

However, Kirchhoff's loop law must always be satisfied, and this can help sort out the ideas here. The sum of the voltage drops around any closed loop must be equal to the sum of the voltage rises. Therefore, the voltage drop across each bulb is the voltage of the battery, whether there is one bulb or two connected. This means that each bulb glows just as brightly after the switch is closed as the one bulb glowed beforehand.

This does not mean that nothing changes in the circuit. With two bulbs glowing, twice as much current is flowing through the battery as before, so it is also delivering twice as much power as before, and will run out of energy twice as quickly.

Note that ideal batteries are *not* sources of constant current. In other words, the same current does not flow through the battery at all times. Instead, an ideal battery maintains a certain voltage across its terminals, adjusting the current to make sure the voltage is constant.

If we include the effects of internal resistance in the battery, we expect the terminal voltage to be slightly smaller, which means the bulbs would glow a little less brightly after the switch is closed. However, unless the resistance of each bulb is very small, this effect is not observable.

Key Points:

• For any closed loop in a circuit, the sum of the voltage rises must equal the sum of the voltage drops.

• The current a battery must provide does not affect its voltage significantly; if the battery's internal resistance is ignored, the voltage is not affected at all.

• Batteries are not sources of constant current. They maintain a certain "terminal voltage" and the current changes to make sure this happens.

For Instructors Only

Many students will think that the glowing bulb will dim noticeably when the second bulb is connected.

Some students will use experience to answer the question. They might have noticed that sometimes the lights in their houses dim when something turns on, such as the compressor of an air conditioner. This experience is not particularly relevant, since internal resistance is a factor. In other words, when the compressor turns on, it draws a very large current, which causes the voltage to dip slightly, which in turn causes the lights in the house to dim. An appropriate experience is simply turning on a light when another light is already on. Houses are wired "in parallel" just as this circuit is wired. We do not generally experience noticeable changes in brightness when various appliances turn on and off in the home, so we should not expect any changes here either.
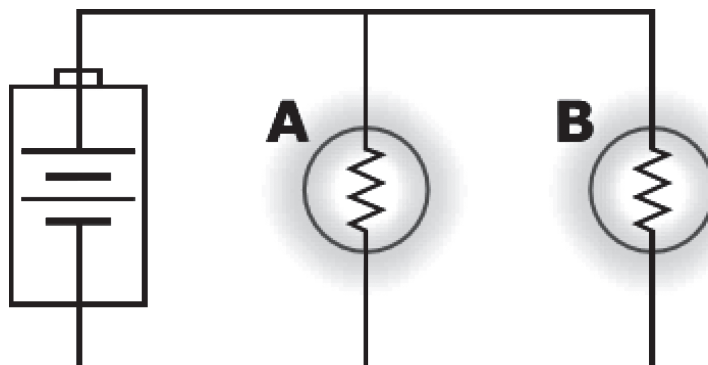
This question can be followed up with a live demonstration.

Question N2.02

Description: Understanding circuits with parallel resistances.

Question

A battery is used to light two bulbs as shown. What happens to bulb B when bulb A is unscrewed from its socket?

1. Bulb B goes out.
2. Bulb B gets noticeably dimmer, without going out.
3. Bulb B gets slightly dimmer.
4. Bulb B's brightness stays the same.
5. Bulb B gets slightly brighter.
6. Bulb B gets noticeably brighter.
7. Impossible to determine

Commentary

Purpose: To develop understanding of Kirchhoff's loop law and parallel resistors.

Discussion: When two circuit elements are arranged in parallel, they necessarily have the same voltage across them. When arranged in series, they have the same current through them. In this situation, the voltages must be the same. The currents will only be the same if the bulbs have the same resistance.

Also, when elements are arranged in parallel, disconnecting one does not disconnect the rest. When bulb A is removed, there is still a closed circuit with the battery and bulb B. No current flows through A, but this does not affect the flow of current through B. Further, the voltage across bulb B does not change when bulb A is removed; it is the voltage across the battery, according to Kirchhoff's loop law. Therefore, the brightness of bulb B does not change.

Your house is wired "in parallel." When you replace a bulb, the rest of the lights in the house do not go out, and they do not change in brightness. The voltage across each light and each appliance remains the same, no matter how many are on and how many are off.

Key Points:

• Circuit elements in parallel have the same voltage across them.

• Removing one path from a circuit does not break paths in parallel with it.

For Instructors Only

Some students will think that bulb B goes out when bulb A is removed, perhaps because they do not recognize that the flow of current is not broken.

They also might recognize that the current through the bulbs beforehand is the same, which suggests to them that they are in series, overgeneralizing a critical feature of the series arrangement. In other words, instructors often emphasize that the current through elements in series is the same. Students might misapply that "rule" here.

Other students might think that bulb B is brighter after A is removed, because then only one bulb is connected to the battery. They might even be surprised when there is no change. It is useful, therefore, to focus some attention on the battery, explaining that the current through it is half as much as before, so the power it delivers is also cut in half.

Internal resistance should not be a factor here. Including its effects would make bulb B glow slightly brighter after A is removed, since the terminal voltage across the battery would be slightly higher than before. If this is issue is raised, you must discuss it carefully, since they may take the message that "bulb B is brighter afterwards" and interpret it as a consequence of the circuit arrangement rather than the battery's internal resistance.
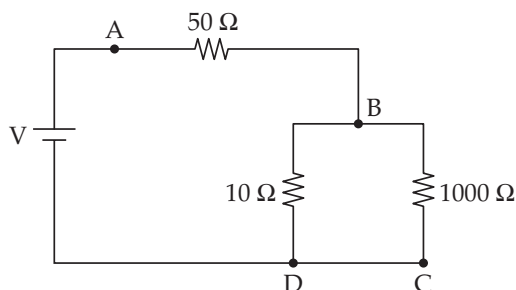
This question can be followed up with a live demonstration.

Question N2.03

Description: Developing problem solving skills with resistor circuits.

Question

Consider the circuit below. Which resistor has the largest power consumption?

1. The $50\,\Omega$ resistor
2. The $10\,\Omega$ resistor
3. The $1000\,\Omega$ resistor
4. The $50\,\Omega$ and $10\,\Omega$ resistors
5. The $50\,\Omega$ and $1000\,\Omega$ resistors
6. The $10\,\Omega$ and $1000\,\Omega$ resistors
7. All have the same power.

Commentary

Purpose: To develop your problem solving skills by considering both forms of the equation for a resistor's power dissipation.

Discussion: There are two common forms for the power dissipated by a resistor, $\mathcal{P}_R = I^2 R = V^2/R$. Which one is more useful depends on the situation, and sometimes using both on a problem is helpful.

First, let's compare the power dissipated by the $10\,\Omega$ and $1000\,\Omega$ resistors. They are arranged in parallel and the potential drop is the same across both ($V_{BC} = V_{BD}$). As a result, it is easy to use $\mathcal{P}_R = V^2/R$ to see that $10\,\Omega$ dissipates more power than $1000\,\Omega$. (Note that V in this equation is not the voltage of the power source, but the voltage drop across the two resistors.)

Now, let's compare the $50\,\Omega$ and $10\,\Omega$ resistors. They have different currents, but we know that the current through $10\,\Omega$ is smaller. So, using $\mathcal{P}_R = I^2 R$, we can say that the $50\,\Omega$ resistor must absorb more power because both I and R are larger. (Note that I is the current through one resistor or the other, not through the circuit as a whole.)

Since $50\,\Omega$ absorbs more than $10\,\Omega$ and $10\,\Omega$ absorbs more than $1000\,\Omega$ in this particular circuit, answer (1) is correct.

Key Points:

- You have two equations for the power dissipated by a resistor: one in terms of current and resistance, and one in terms of voltage drop and resistance.

- Both of these power dissipation equations are useful, sometimes on the same problem.

- Pay attention to what the variables *R*, *V*, and *I* mean in the context you are applying the equation — not all *V*s or *I*s mean the same thing!

For Instructors Only

Students selecting answer (2) may be reasoning from $\mathcal{P} = V^2/R$ that the lower the resistance, the higher the power dissipated. Students selecting answer (3) may be reasoning from $P = I^2 R$ that the higher the resistance, the higher the power dissipated. Either way, they should be engaged in a discussion about what the variables in the equation mean.

Common mistakes include using the voltage of the power supply ("*V*") in $\mathcal{P} = V^2/R$ and using one generic current value ("*I*") for all three resistors. Common sources of confusion include failing to appreciate that the 10 Ω and 1000 Ω resistors share the same voltage drop.

Question N2.04

Description: Developing reasoning ability with resistor circuits.

Question

Consider the circuit below. Which statement(s) is correct?
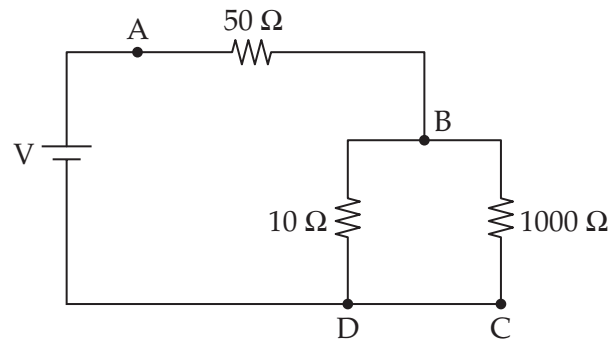
1. $I_{AB} = I_{BD} + I_{BC}$
2. $I_{BD} > I_{BC}$
3. $I_{BD} < I_{BC}$

1. 1 only
2. 2 only
3. 3 only
4. 1 and 2
5. 1 and 3

Commentary

Purpose: To develop your ability to reason qualitatively about currents and resistances.

Description: Current will flow out of the battery and through point A. It will split at point B, some flowing through each of the two parallel resistors, rejoining at point D and returning to the battery. Current is conserved, so the total current flowing into any junction equals the total current flowing out. This means that the currents through the 10 Ω and 1000 Ω resistors must add up to equal the current through the 50 Ω resistor (statement 1).

The potential difference across the 10 Ω and 1000 Ω resistors is the same — ideal wires have the same potential everywhere on them — so according to Ohm's law ($\Delta V = IR$), the smaller resistor must have the larger current through it (statement 2). Thus, the best answer is (4).

Key Points:

- At any wire junction, the total current entering equals the total current leaving.

- Two parallel circuit branches have the same potential difference across them.

- Ohm's law ($\Delta V = IR$) relates the potential difference across a resistor to its resistance and the current flowing through it.

For Instructors Only

This is a fairly straightforward question about currents, resistances, and potential differences, and serves well when students are first beginning to analyze nontrivial complete circuits after having learned about the elements (current conservation, Ohm's law, etc.).

Note that the question does not specify a numerical value for the battery voltage. If it did, students would be far more inclined to calculate the currents rather than reason about them.

Question N2.05

Description: Understanding Kirchhoff's loop law (batteries in parallel).

Question

A battery is used to light a bulb as shown. An identical battery is connected by closing switch S. What happens to the bulb?
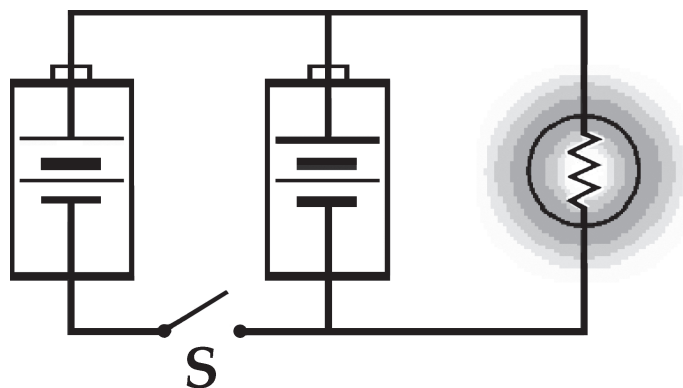
1. The bulb goes out.
2. The bulb gets noticeably dimmer.
3. The bulb gets very slightly dimmer.
4. The bulb's brightness stays the same.
5. The bulb gets slightly brighter.
6. The bulb gets noticeably brighter.
7. A combination of 2 of the answers above; first one happens, then the other.

Commentary

Purpose: To check your awareness and understanding of Kirchhoff's loop law.

Discussion: It is tempting to think that the bulb will glow brighter with 2 batteries connected than with only 1 connected, but this would not satisfy Kirchhoff's loop law.

The voltage rises in a closed loop must equal the voltage drops, and this is true even when there are multiple loops to consider.

Kirchhoff's loop law is satisfied easily for the left loop, since the batteries are "identical." Going around the right loop, the voltage drop across the bulb (a resistor) must be equal to the voltage rise across *one* battery. But that is exactly what was true before the switch was closed, so we should expect no change in the brightness of the bulb. The current through it is the same as before.

Even though there is no change in the brightness of the bulb, this does not mean there are no changes in the circuit. With 2 batteries connected "in parallel," each battery has only half as much current going through it as the one battery originally, which means each battery provides half of the power dissipated by the bulb. This means that, for instance, the 2-battery arrangement would be able to keep the bulb glowing about twice as long as the 1-battery arrangement.

If we include the effects of internal resistance, then the terminal voltage would be slightly larger than before since each battery has less current flowing through it, which means the bulb would glow slightly brighter than before. However, unless the resistance of the bulb is extremely small, we do not expect to be able to see any difference in brightness.

The orientation of the two batteries is critical. If one of them is reversed, then the result is very different.

Key Points:

•   For any closed loop in a circuit, the sum of the voltage rises must equal the sum of the voltage drops.

•   Two identical batteries in parallel provide the same voltage as one alone (though they will last longer).

For Instructors Only

Students are likely to say that the bulb will glow much more brightly with 2 batteries attached than with only 1 attached. They are not distinguishing between parallel and series arrangements, but relying on intuition.

This question can be followed up with a live demonstration. Students will be surprised by the result, since it appears that nothing changes. What they cannot see is that the current drawn from each battery is much reduced.

Question N2.06

Description: Understanding internal resistance of batteries and recognizing common but imperfect circuit assumptions.

Question

A battery is used to light a bulb as shown. An identical battery is connected by closing switch S. What happens to the bulb?
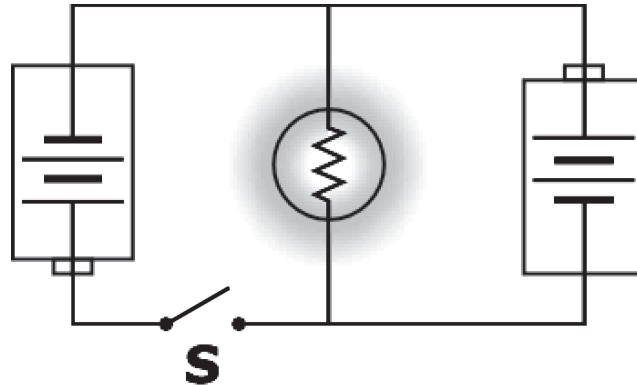
1. The bulb goes out.
2. The bulb gets noticeably dimmer.
3. The bulb gets very slightly dimmer.
4. The bulb's brightness stays the same.
5. The bulb gets slightly brighter.
6. The bulb gets noticeably brighter.
7. Impossible to determine

Commentary

Purpose: To help you to understand internal resistance and sensitize you to the kinds of situations where it cannot be ignored.

Discussion: Applying Kirchhoff's loop laws to this situation leads to some contradictions. Going around the left loop suggests that the current through the bulb should go from the bottom of the circuit to the top. Going around the right loop suggest the opposite: the current flows from top to bottom. Both statements cannot be true. Going around the outside is worst of all: it appears as though there are two voltage rises and no voltage drops. What is going on here?

The problem is that we are ignoring the internal resistance of each battery (as well as the very small resistance in the wires of the circuit). We can answer many questions neglecting batteries' internal resistance, but not this one.

If the batteries are identical, then there must be a voltage drop across the internal resistor exactly equal to the voltage of the battery, which means the voltage drop across each battery (i.e., the measured voltage from one terminal to the other) is zero. This is the only way Kirchhoff's loop law for the outermost loop can be satisfied. Thus, the voltages at both ends of the middle branch are the same, and no current flows through the bulb.

This does not mean that nothing is happening. Current is flowing around the outer loop, out of one battery and into the next. This current is fairly large, since the internal resistance of each battery is small. Therefore, this circuit should not remain connected for too long — it will heat up and drain the batteries! A more accurate circuit diagram would show a small resistance inside each battery, in series with its voltage source.

Key Points:

- The internal resistance of a battery is generally very small, but it cannot be ignored when the current flowing through the battery is very large.

- When analyzing a situation leads to a contradiction, check your assumptions and approximations.

For Instructors Only

Students might not notice that the polarity of one of the batteries is reversed.

Most circuit problems can be solved ignoring internal resistance, but this situation is impossible to understand without it. When we occasionally discuss situations in which our "common" approximations fail, it helps students to understand the approximations better: their implications, when they are or are not warranted, etc.

It is useful to label the nodes above and below the bulb and use them to discuss this situation. It is also useful to redraw the circuit with small resistors inside the outlines of the batteries to represent internal resistance.

This question can be followed up with a live demonstration, which is quite dramatic when the bulb goes out. Note that you should not leave the switch closed for any length of time, since a very large current is flowing elsewhere in the circuit!

Question N2.07

Description: Understanding internal resistance of batteries and recognizing common but imperfect circuit assumptions.

Question

Two batteries are used to light a bulb as shown. A third identical battery is connected by closing switch S. What happens to the bulb?
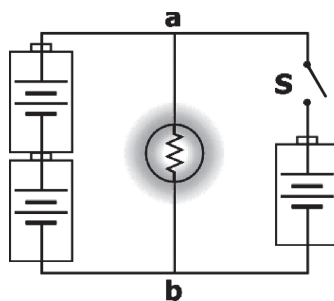

1. The bulb goes out.
2. The bulb gets noticeably dimmer.
3. The bulb gets very slightly dimmer.
4. The bulb's brightness stays the same.
5. The bulb gets slightly brighter.
6. The bulb gets noticeably brighter.
7. Impossible to determine

Commentary

Purpose: To better understand the internal resistance of batteries.

Discussion: Given the circuit as drawn, Kirchhoff's loop law cannot be satisfied around the outer loop. If the batteries are identical and ideal, each with a potential difference of *V* across it, then it is impossible for the voltage rise to equal the voltage drop.

This is a question for which we cannot ignore the internal resistance of real batteries. The only way for Kirchhoff's loop law to be satisfied is to recognize that there is a small resistance associated with each battery (see diagram). So, for instance, if each battery is 1.5 V, the voltage between points a and b is less than 3 V because there is a small voltage drop across each internal resistor on the left. Further, current is actually flowing into the battery on the right, so V_{ab} is larger than 1.5 V, because there is a voltage rise across the resistor there.
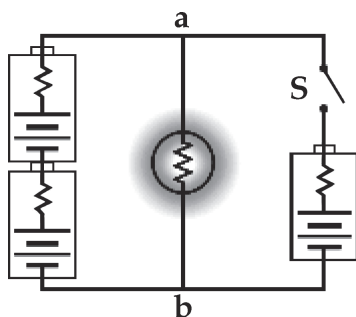
It turns out that the bulb will become slightly dimmer when the third battery is connected, since the voltage drop across it is smaller than before. Because more current flows through the pair of batteries in series than before, the voltage drop across their internal resistances will be greater.

Key Points:

- The internal resistance of a battery is generally very small, but it cannot be ignored when the current flowing through the battery is very large.

- It is possible for current to be forced through a battery the "wrong way."

- When analyzing a situation leads to a contradiction, check your assumptions and approximations.

For Instructors Only

Students are likely to think that the bulb will glow brighter than before, since there is a third source connected.

Other students will be confused by the apparent contradictions in the circuit. It will seem impossible to them (which it is if the batteries are treated as ideal).

This question can be followed up with a live demonstration.

Additional Questions:

- Let the internal resistance of each 1.5 V battery be equal to 0.1 Ω and the resistance of the bulb be equal to 2 Ω. What is the current through each branch of the circuit? What is V_{ab}?

Question N2.08a

Description: Problem solving: reasoning about complex resistor circuits.

Question

A battery and 5 identical bulbs are arranged as shown. What happens when switch S is closed?
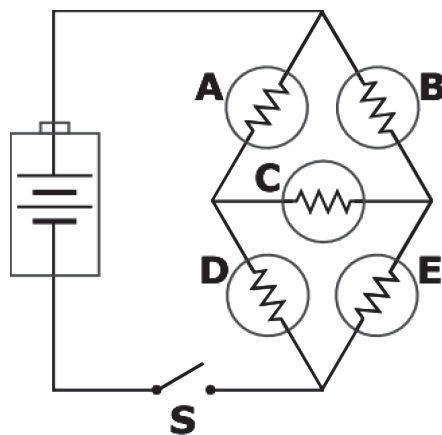
1. All 5 bulbs turn on with equal brightness.
2. All 5 bulbs turn on, but they do not all have equal brightness.
3. One of the bulbs does not turn on; the other 4 turn on with equal brightness.
4. One of the bulbs does not turn on; the other 4 turn on, but they do not all have equal brightness.
5. Two or more bulbs do not turn on.
6. I have no idea!

Commentary

Purpose: To develop your ability to reason about less conventional resistor arrangements.

Discussion: This arrangement of 5 bulbs is not easily decomposed into "series" and "parallel" chunks with an equivalent resistance. Bulb C makes it impossible to combine any of the other resistors. Therefore, we must use other techniques to analyze this situation.

Symmetry demands that the current through bulbs A and B is the same, since they are identical and the paths through them are equivalent. Likewise, the current through bulbs D and E is the same.

Now look at bulb C. Assume first that current flows left to right through it. Since the current through A and B is the same, this would make the current through E larger than the current through D, which is impossible. In the same way, current flowing right to left would make the current through D larger than that through E.

The only possibility is that no current flows through C. This makes sense because the potential at the bottom end of A must be the same as the potential at the bottom end of B, since their currents and resistances are the same as each other. Another way to think of this is to apply Kirchhoff's loop law to the triangle made by bulbs A, B, and C. If A and B have the same resistance and the same current, they must have the same voltage drop. So, the voltage rise of one (going up the circuit) will cancel the voltage drop of the other (going down the circuit), and according to Kirchhoff's loop law C must have zero voltage rise/drop.

Since no current is flowing through C, it can be removed without changing the current flow in the circuit. The result is that bulbs A and D are now in series, as are B and E, and each pair is in parallel with the other pair. This means that each bulb glows as brightly as when 2 bulbs are connected in series with the battery.

Note that if A and B are not identical, or if D and E are not identical, the symmetry is broken, and current flows through C.

Key Points:

• Sometimes clever reasoning can help you analyze a circuit and figure out how it behaves.

• Not all circuit arrangements can be thought of as sets of resistors in series and parallel.

• A branch of a circuit with zero current can be ignored, since its absence would not change the currents or voltages anywhere else in the circuit.

For Instructors Only

This is the first of two questions using this situation.

Arrangements like this are often avoided at the introductory level because they cannot be simplified using standard techniques of series and parallel combinations. But even though we cannot easily solve this problem in general, we can still reason to an answer using other simple arguments, as above. Engaging in this reasoning process is valuable for students.

Note that Kirchhoff's loop law and node law can be used to find the currents through each resistor, though the resulting system has six equations in six unknowns. This approach should be discouraged by giving students a relatively short amount of time to answer the question.

It is useful to label the nodes and use these labels when discussing this situation. There are 4 relevant nodes.

This question can be followed up with a live demonstration.

One drawback to be guarded against when using this question is inappropriate generalization: students might incorrectly conclude that any time a resistor is connected between two branches like C is here, the current through it is zero. It happens here only because identical resistors are paired up, A and B, D and E. In other words, all 5 resistors do not need to be the same, but certain pairs do: A and B must be identical, and D and E must be identical, though A and B do not need to be the same as D and E, and C can be different from all of the others.

Additional Questions

• Using the same network of 5 identical bulbs, connect the battery across bulb C. Which bulbs will light and which will not? Which will glow more brightly than the others? Which will glow with the same brightness as when only one bulb is connected to the battery? Which will glow with the same brightness as when two are connected in series with the battery?

Question N2.08b

Description: Problem solving: reasoning about complex resistor circuits; distinguishing zero current from an open circuit.

Question

A battery and 5 bulbs are arranged as shown, and when the switch is closed 4 of the bulbs are lit. Which bulbs **change** when bulb B is unscrewed from its socket?
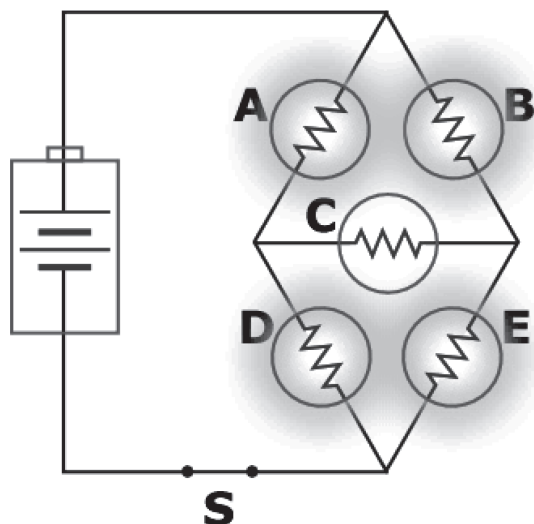
1. Bulb E goes out. (Bulbs A and D stay on; bulb C stays off.)
2. Bulb E goes out; bulb C goes on. (Bulbs A and D stay on.)
3. Bulb A goes out; bulb C goes on. (Bulbs D and E stay on.)
4. Bulb C goes on. (Bulbs A, D, and E stay on.)
5. Bulb C goes on; bulb D goes out. (Bulbs A and E stay on.)
6. I have no idea!

Commentary

Purpose: To develop intuition for current flow in circuits, and help you distinguish between an *open* circuit and one that is closed but has zero current.

Discussion: As discussed in the previous question, no current flows through C in the initial arrangement (i.e., while B is still part of the circuit). However, C is still a viable path for current. It is not "open," but merely does not have any current in that set of circumstances.

Once B is removed, the voltages in the circuit change. B is now "open," and all the current will flow through A. When it reaches the node after A, the current has two paths it can follow to return to the battery: through D, or through C and then E. C and E are in series, and the C+E combination is in parallel with D. The current will therefore divide, some going through each path, and then join up again at the bottom.

So, A, D, and E will stay lit, and C will light up when B is removed. (If you work out what the currents must be, you'll see that A will glow brightest, then D, and C and E will be the dimmest and equally dim. All will change in brightness when B is removed.)

Key Points:

- An open circuit is one in which current cannot flow because no complete conducting path exists. This is not the same as a closed circuit with zero current.

- When an element of a complex circuit is changed or removed, it can affect voltages and current flows elsewhere in the circuit.

For Instructors Only

Students are likely to have oversimplified ideas of open circuits and closed paths. Some will think that bulb C must be an open circuit because it is not glowing. They might wonder why it is not glowing and ask if there is something wrong with the bulb.

It is not necessary to discuss which bulb is brightest, dimmest, etc. The focus of this question should be to identify paths for which current can flow vs. paths for which no current flows.

It is useful to label the 4 nodes in the resistive network and use them to discuss this situation. It is useful also to redraw the situation with an open circuit at B.

This question can be followed up with a live demonstration.

Additional Questions:

- If each bulb has a constant resistance R, what is the equivalent resistance of the network after bulb B is removed, and what is the current through each bulb?

- What is the power dissipated by each resistor before and after B is removed? Which bulbs will become brighter and which will become dimmer after B is removed?

Question N2.09a

Description: Linking theory to real-world experience: bulbs and batteries.

Question

A light bulb is connected to a battery as shown in Figure A below. When a second bulb is connected as shown in Figure B, what happens to the brightness of the original bulb?

Fig. A   **Fig. B**
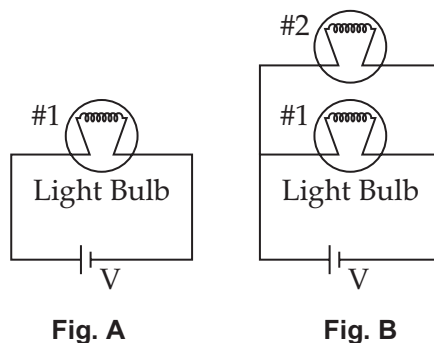
1.   Increases
2.   Decreases
3.   Stays the same
4.   Bulb goes out
5.   Can't determine

Commentary

Purpose: To connect your understanding of resistances and parallel circuits to the observable behavior of real-world light bulbs and batteries.

Discussion: An incandescent light bulb can be thought of as a resistor. The brightness of the bulb depends on the amount of power dissipated by it: $\mathcal{P} = I^2 R = IV = V^2/R$.

The potential difference across bulb #1 is equal to the potential difference of the battery, regardless of whether bulb #2 is placed in parallel with it. For an ideal circuit, the potential difference of the battery is always the same, so the power dissipated by bulb #1 won't change when bulb #2 is connected. (Bulb #2 will light up, too.) So, answer (3) is correct for an ideal circuit.

But what if the circuit isn't ideal? Real batteries have a maximum current output as well as a rated potential difference ("voltage"). As more and more current is drawn from a battery, its output voltage drops: slightly at first, but significantly as the current approaches the battery's maximum. When the second light bulb is attached, it draws additional current from the battery, so the first bulb's brightness will decrease. It will decrease very slightly — probably not enough to notice — unless the battery is already near its maximum current capacity, in which case bulb #1 may dim significantly. So, answer (2) is possible for a circuit with a real battery.

Since real light bulbs don't appear to light up at all for very low current flows and power dissipation (they get slightly warm, but not hot enough to glow visibly), bulb #1 may even appear to go out completely if attaching the second bulb and drawing more current causes the battery's voltage to drop, which causes the current through bulb #1 to decrease below the bulb's threshold for "lighting up." So, answer (4) is possible for a circuit with a real battery and real bulbs.

An additional real-world complication is that no wire really has zero resistance, so that when current flows through the circuit, some power is dissipated by the wires. That means there is some nonzero potential difference between the battery's terminal and the light bulbs' contacts, which means that the voltage across the bulbs won't be exactly equal to the voltage across the battery; the more current that flows, the bigger this difference will be. This is unlikely to be a significant effect for typical wires, batteries, and bulbs, however.

Key Points:

- Incandescent light bulbs can be modeled as resistors, and the brightness of the bulb depends on the power dissipated.

- Adding one light bulb in parallel with another doesn't affect the brightness of the first, **if** the potential difference across the first remains the same.

- The voltage output by a real, non-ideal battery decreases as more current is drawn from it, though the change is small unless the current approaches the battery's maximum current capacity.

- Light bulbs won't light up visibly if too little power is dissipated.

For Instructors Only

This is the first of two related questions.

Students often can't, or don't try to, associate what they learn about "ideal" circuits with their experience of real electrical devices like light bulbs, batteries, and the like. This inhibits both their understanding of the physics and their ability to use physics to understand their world.

One or more demonstrations can significantly enhance the learning from this question. Optimally, you would have one set of components that approximates an ideal circuit well, and another with a marginal enough battery that non-ideal effects can be observed.

Question N2.09b

Description: Linking theory to real-world experience: bulbs and batteries.

Question

A light bulb is connected to a battery as shown in Figure A below. When a second bulb is connected as shown in Figure B, what happens to the brightness of the original bulb?
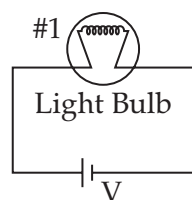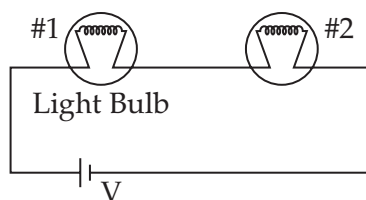
Fig. A   **Fig. B**

1. Increases
2. Decreases
3. Stays the same
4. Bulb goes out
5. Can't determine

Commentary

Purpose: To connect your understanding of resistances and series circuits to the observable behavior of real-world light bulbs and batteries.

Discussion: When the second light bulb is inserted into the circuit, the total resistance of the loop increases (series resistances add), so the current flowing decreases. If the second bulb is identical, the current drops to one-half its previous value. Since the current through bulb #1 is now significantly less than its previous value, the power dissipated by bulb #1 ($\mathcal{P} = I^2 R$) is now much less than it was — perhaps 75% less — so the bulb will be much dimmer. Thus, answer (2) is defensible.

A light bulb will not actually light up visibly for very small currents; the current must be above some threshold before the power dissipated heats up the filament enough to emit detectable visible light. So, depending on the details of the battery and bulbs, bulb #1 may appear to go completely out. Thus, answer (4) is also possible. And answer (5) is defensible, if you choose it because you don't know whether the bulb will go out or just get dimmer.

If we don't treat the circuit as ideal, it still behaves essentially the same way; the exact change in brightness might be slightly different than for an ideal circuit, but not significantly.

Key Points:

• Incandescent light bulbs can be modeled as resistors, and the brightness of the bulb depends on the power dissipated.

• The current through two resistors in series, and the power dissipated by each, is smaller than it would be for either resistor alone (with the same total voltage).

• Light bulbs won't light up visibly unless the current through them is large enough to dissipate enough power to make them glow.

For Instructors Only

This is the second of two related questions. It gives students a chance to apply some of the ideas they may have been exposed to in the first question, and introduces series (as opposed to parallel) resistances.

Again, one answer is "correct" with idealizing assumptions, but we want to dig deeper to identify these assumptions and explore how a real circuit might behave.

Question N2.10

Description: Reasoning with power and understanding the "wattage" rating of light bulbs.

Question

Two light bulbs are connected to a wall outlet as shown below. Bulb #1 is 100 W and Bulb #2 is 40 W. Which statement is true?
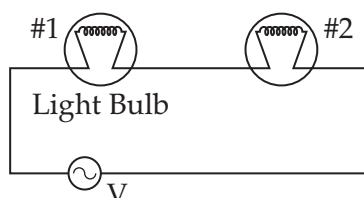
1. Both bulbs are at their normal brightness.
2. The 100 W bulb is brighter than the 40 W bulb.
3. The 40 W bulb is brighter than the 100 W bulb.
4. Both bulbs are at equal brightness.
5. Cannot determine their relative brightness.

Commentary

Purpose: To develop your ability to reason about resistors and power, and understand more about light bulbs.

Discussion: The "wattage" rating of a light bulb is the power it dissipates when connected to standard household current (120 V AC). Since the power dissipated by a resistor can be written $\mathcal{P} = V^2/R$, we know that the higher-wattage bulb (#1) must have the lower resistance.

In this situation, the potential difference across the two bulbs is not necessarily the same. However, since they are in series, the current through them is the same. Since the power dissipated by a resistor can also be written $\mathcal{P} = I^2 R$, we know that the lower-resistance bulb (#1) must dissipate less power. Since it dissipates less power, it appears less bright. So, in this situation, the 40 W bulb appears brighter than the 100 W bulb! Thus, statement (3) is true and the others must be false.

Key Points:

- The rating ("wattage") of an incandescent light bulb is the power it dissipates when connected directly to standard household current. It may dissipate a different amount of power in other situations.

- Higher-wattage bulbs have lower resistances (so more current flows).

- The power dissipated by a resistor (Watt's law) can be written in several forms: $\mathcal{P} = I^2 R = V^2/R = IV$. If you know one, you can derive the others using Ohm's law ($V = IR$).

- When comparing circuits or components within a circuit, make sure to identify what quantity is constant (in this case, current).

For Instructors Only

Simple intuitive reasoning can lead students to think that the higher-wattage bulb must be brighter. Implicitly, they are taking voltage to be the same rather than current.

This question can reveal confusion about what the "wattage" of an incandescent bulb really means, especially for those choosing answer (1). It can also reveal confusion about what the "brightness" of a bulb corresponds to.
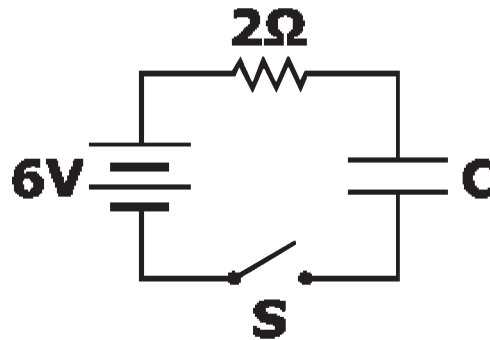
Errors of relational reasoning — using proportional rather than inverse relationships — are also likely.

Question N3.06a

Description: Introducing capacitor behavior in DC circuits: charge vs. voltage.

Question

Consider the circuit below. Switch S is closed at $t = 0$.

What is the voltage across the capacitor C just after the switch is closed?

1. 0
2. 1 V
3. 2 V
4. 3 V
5. 4 V
6. 5 V
7. 6 V
8. 9 V
9. None of the above
10. Impossible to determine

Commentary

Purpose: To introduce you to the behavior of capacitors, especially the relationship between a capacitor's charge and the voltage across it.

Discussion: The voltage on a capacitor is proportional to the amount of charge "separated" or "stored" on its plates: $Q = CV$. (Positive charge accumulates on one plate and an equal amount of negative charge accumulates on the other.) Assuming the capacitor is uncharged before the switch is closed, it must still be uncharged immediately afterwards, so the voltage across it is zero.

Since the charge on the plates of a capacitor cannot change instantaneously, the voltage across it also cannot change instantaneously. In other words, the charge and voltage are both continuous functions of time for a capacitor.

Key Points:

- The voltage difference between the plates of a capacitor is proportional to the charge stored on them.

- Neither charge nor voltage can change instantaneously for a capacitor; when current starts to flow through an uncharged capacitor, charge starts to build up and the voltage starts to change from zero.

For Instructors Only

This is first of three questions using this situation. It is a good question for developing a physical picture of capacitance and understanding why charge and voltage are intimately related.

In questions like this we typically assume that the capacitor is initially uncharged, but the question does not explicitly state that. Therefore, a valid (if not optimal) response is to say that the voltage is impossible to determine since the initial conditions are not given. If students pick up on this ambiguity, it can be used to motivate a discussion of how the initial conditions might affect the circuit's behavior.

Question N3.06b

Description: Introducing capacitor behavior in DC circuits: voltage vs. current.

Question

Consider the circuit below. Switch S is closed at $t = 0$.
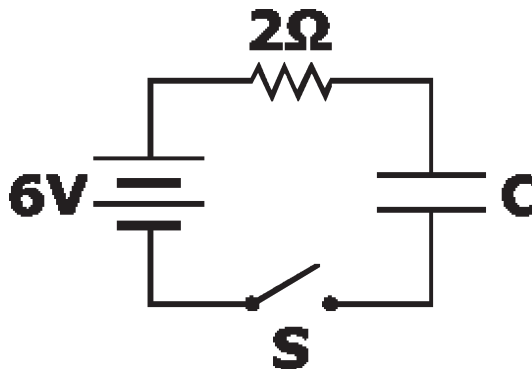
What is the current through the capacitor C just after the switch is closed?

1. 0
2. 1 A
3. 2 A
4. 3 A
5. 4 A
6. 5 A
7. 6 A
8. 9 A
9. None of the above
10. Impossible to determine

Commentary

Purpose: To introduce you to the behavior of capacitors, especially the relationship between the voltage across a capacitor and the current "through" it.

Discussion: Assuming there is no charge on the capacitor when the switch is closed, there is no voltage across it, and it behaves like a conductor. In other words, at the instant $t = 0$ we can imagine replacing the capacitor with a wire. With no voltage across C, Kirchhoff's loop law requires that the voltage across the resistor be 6 V, so the current everywhere must be 3 A, including through the capacitor.

(Current flow "through" a capacitor does not mean that charge actually travels across the gap from one plate to the other. Rather, current flows into one terminal and accumulates as charge on the connected plate; meanwhile, current flows out of the other terminal as the opposite charge accumulates on that plate. As long as one does not look inside the capacitor itself, however, the capacitor behaves as if current is flowing "through" it.)

The behavior of a capacitor is different from that of a resistor. If there is zero voltage across a resistor, there must be zero current; if there is a current through a resistor, there must a voltage across it. For capacitors, however, zero voltage means current can flow unimpeded, while a voltage difference (due to charge build-up) hinders current flow.

Note that the current through a capacitor can change instantaneously. The current was zero just before the switch is closed, and is 3 A just afterwards. This situation is analogous to dropping an object at $t = 0$. The velocity is zero but changing, so the net force on it is nonzero. Likewise, the voltage across the capacitor is zero but changing, so there is a current through it. (The current through a capacitor is proportional to the rate of change of its voltage, just as the net force on an object is proportional to the rate of change of its velocity.)

Key Points:

- A capacitor with zero voltage across it allows current to flow unimpeded, like a zero-resistance wire.

- The charge on or voltage across a capacitor cannot change instantaneously, but the current through it can.

- The relationship between voltage and current differs for capacitors and resistors.

For Instructors Only

This is second of three questions using this situation.

This discussion neglects the issue of charge building up and gradually decreasing the current. That will be visited in the final question in this set.

Be on the lookout for signs of confusion between the voltage-current relationships for resistors and capacitors; it is common.
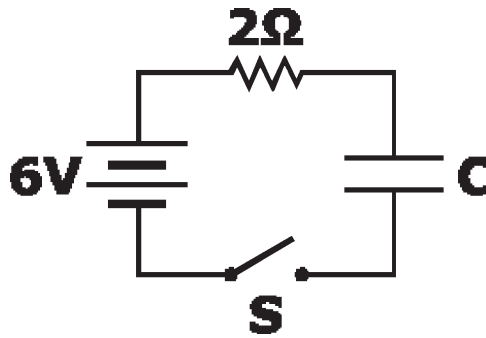
As with the previous question, this one does not identify the initial state of the capacitor. If the ambiguity and the significance of different initial conditions was not discussed then, it may be now.

Question N3.06c

Description: Introducing capacitor behavior in DC circuits: saturation.

Question

Consider the circuit below. Switch S is closed at $t = 0$.

What is the voltage across the capacitor C long after the switch is closed?

1. 0
2. 1 V
3. 2 V
4. 3 V
5. 4 V
6. 5 V
7. 6 V
8. 9 V
9. None of the above
10. Impossible to determine

Commentary

Purpose: To introduce you to the behavior of capacitors, especially their behavior when fully charged.

Discussion: As current flows through the circuit, charge accumulates on the capacitor. As charge accumulates, the voltage across the capacitor rises. As the voltage rises, the voltage difference across the resistor drops (according to Kirchhoff's loop law). As the voltage across the resistor drops, the current through it (and the rest of the circuit) drops.

Eventually, current stops flowing through the capacitor and it is at maximum voltage. We say the capacitor is "fully charged" or "saturated." In this situation, the capacitor behaves like an "open circuit," i.e., a break in the circuit, because no current is flowing through it.

Since no current is flowing through the capacitor, no current is flowing anywhere in the circuit, and the voltage across the resistor is zero. Therefore, the voltage across C must be the same as the battery: 6 V.

For a resistor, nonzero voltage means nonzero current. For a capacitor, nonzero voltage can mean zero current if the voltage is enough to prevent current flow in the rest of the circuit.

Key Points:

- As current flows through a capacitor, charge builds up on its plates, causing a voltage across the capacitor. This voltage will reduce the current flow.

- In a DC circuit, a capacitor will charge until it prevents any current from flowing through the circuit branch it belongs to. A fully charged capacitor acts like an "open circuit" because no current is flowing through it.

- The current through a resistor can be determined by knowing *only* the voltage across it. The current through a capacitor cannot be determined from the voltage across it without understanding the rest of the circuit.

For Instructors Only

This is last of three questions using this situation. It provides an opportunity to discuss the charging behavior of capacitors and further make sense of the relationship between voltage, charge, and capacitance.

Some students will say that the voltage on the capacitor is 6 V without fully appreciating the reasoning needed to determine this. A slightly more complicated circuit can help tease this out.
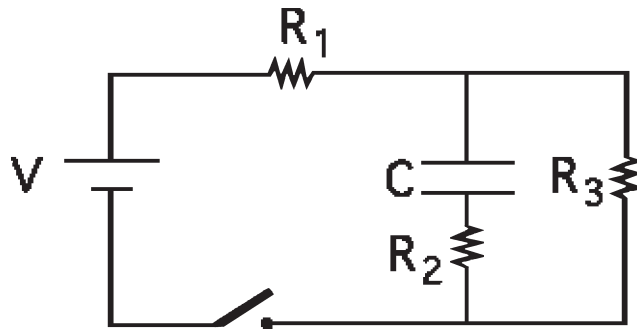
Additional Questions:

*   How would your answer change if (a) 2 Ω was changed to 3 Ω, (b) *C* was changed to *C*/2, and (c) 6 V was changed to 8 V?

*   What would happen if the switch is opened after a long time?

Question N3.07

Description: Developing an understanding of how an uncharged capacitor behaves in a circuit.

Question

Consider the following circuit.

The capacitor is uncharged when the switch is closed at $t = 0$. Which circuit is equivalent to this circuit for the instant immediately after the switch is closed?
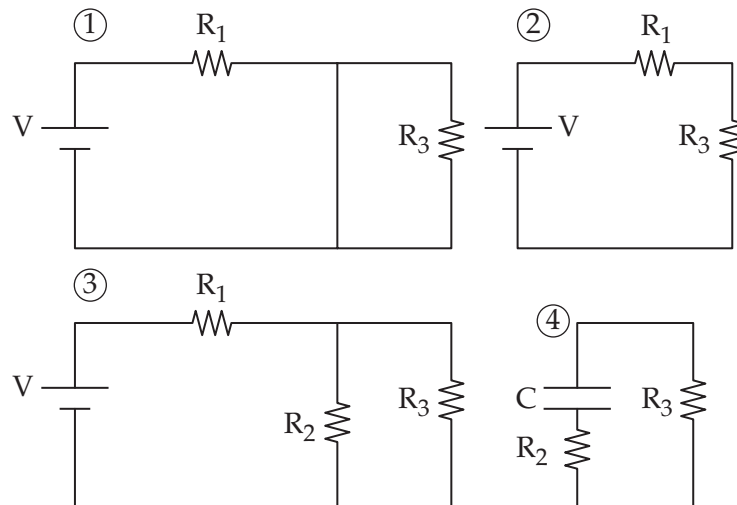
Commentary

Purpose: To develop your understanding of how an uncharged capacitor behaves in a circuit.

Discussion: When there is no charge on a capacitor, it behaves like a short circuit, which means it does not impede the free flow of current. We can imagine replacing it with a wire for that instant. However, R_2 can **not** be replaced by a wire as well. Thus, answer (3) is best.

Answer (2) is a correct representation of the long-term or steady-state behavior of the circuit: when the capacitor is fully charged, the current through it and R_2 are zero, and we can replace the entire branch with an open circuit.

Key Points:

- An uncharged capacitor acts like a short circuit (a resistanceless wire) until it accumulates some charge.

- Any resistors in series with the capacitor still impede current flow; they do not act like short circuits.

For Instructors Only

This is a good question to present just after a question set like Questions 39a–39c: it asks students to apply knowledge they've gained there in a slightly more complicated situation. This is an example of the "extend the context" tactic: present a concept, and then challenge students to apply it in a different context to push them to generalize the concept and dissociate it from the details of the original context in which they learned it.

Circuit (1) is never equivalent to the original. Circuit (2) is the equivalent circuit at steady state. Circuit (4) is the equivalent at the instant the switch is opened after being closed for a long time.
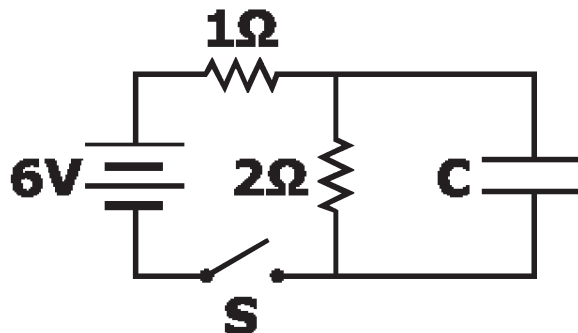
Additional Questions:

1. Just after the switch is closed, determine the current through each resistor.
2. Long after the switch is closed, (a) draw the equivalent circuit, then determine (b) the currents through the resistors and (c) the voltage across the capacitor.
3. The switch is closed for a long time, then opened again. (a) Which diagram would be equivalent just after the switch is open again? (b) Determine the current through each circuit element.

Question N3.08a

Description: Extending understanding of capacitor behavior in DC circuits.

Question

Consider the circuit below. Switch S is closed at $t = 0$.

What is the voltage across the capacitor C just after the switch is closed?

1.   0
2.   1 V
3.   2 V
4.   3 V
5.   4 V
6.   5 V
7.   6 V
8.   9 V
9.   None of the above
10.   Impossible to determine

Commentary

Purpose: To develop your understanding of how capacitors behave in circuits with other elements.

Discussion: The voltage on a capacitor is proportional to the amount of charge "separated" or "stored" on its plates. Assuming the capacitor is uncharged before the switch is closed, it must still be uncharged immediately afterwards, so the voltage across it is zero.

Since the charge on the plates of a capacitor cannot change instantaneously, the voltage across it also cannot change instantaneously. In other words, the charge and voltage are both continuous functions of time for a capacitor. (For resistors, however, voltage is *not* a continuous function of time.)

The answer "impossible to determine" is not particularly defensible here, even though the initial charge on the capacitor at $t = 0$ is not given. While the switch is open, the capacitor discharges through the 2 Ω resistor. Thus, for most typical capacitors, after just a few seconds of the switch being open, the charge on the capacitor is zero. We generally assume that these kinds of situations have no relevant history, that is, that the initial condition is steady-state and has existed for a long time.

Key Points:

•   The voltage across a capacitor is proportional to the charge stored on its plates.

•   Neither the charge on nor the voltage across a capacitor can change instantaneously.

•   Sometimes we must make "reasonable" assumptions to answer a question, taking into account the question's intent.

For Instructors Only

This is first of three questions using this situation.

Students might think that the answer is impossible to determine, perhaps because they do not know how much charge is stored on the capacitor when the switch is closed. In this case, however, the capacitor discharges when the switch is open, so it is not really a defensible position.

Students might recognize that current is flowing to the capacitor, but they might not realize that its voltage is zero. Having current flowing to a circuit element with no voltage across it might be hard for them to reconcile.

This is the simplest of the three questions in this set. Instructors do not need to spend much time on it. The main idea (that charge and voltage are intimately related for a capacitor) should have been settled for most students before asking this question. However, it is worth checking this quickly before asking the other two questions in this set.
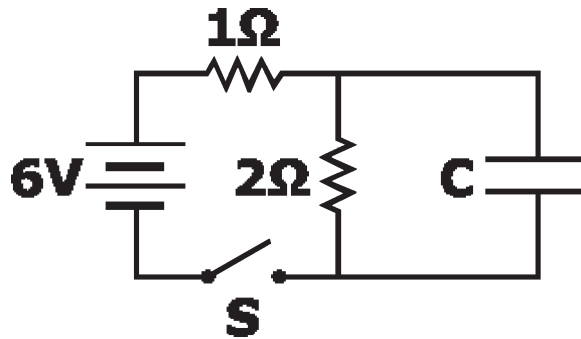
Question N3.08b

Description: Extending understanding of capacitor behavior in DC circuits.

Source: Leonard

Question

Consider the circuit below. Switch S is closed at $t = 0$.

What is the current through capacitor C just after the switch is closed?

1. 0
2. 1 A
3. 2 A
4. 3 A
5. 4 A
6. 5 A
7. 6 A
8. 9 A
9. None of the above
10. Impossible to determine

Commentary

Purpose: To develop your understanding of how capacitors behave in circuits with other elements.

Discussion: Assuming there is no charge on the capacitor immediately after the switch is closed, there is zero voltage across it, and it behaves like a "short circuit" — a zero-resistance wire. According to Kirchhoff's loop law for a loop through the battery, capacitor, and 1 Ω resistor, the voltage drop across the 1 Ω resistor must be 6 V. Therefore, by Ohm's law, the current through it must be 6 A.

If the voltage across the capacitor is zero, the voltage across the 2 Ω resistor must also be zero, which means no current flows through that resistor. So, all 6 A flows through the capacitor.

Note that the current through a capacitor can change instantaneously. The current was zero just before the switch is closed, and is 6 A just afterwards. This situation is analogous to dropping an object: immediately after releasing it, the velocity is zero but changing, so the net force on it must be nonzero. Likewise, the voltage across the capacitor is zero but changing, so there is a current through it. The current through a capacitor is proportional to the rate of change of its voltage, just as the net force on an object is proportional to the rate of change of its velocity.

Key Points:

- For a capacitor, zero voltage does not mean zero current. (For a resistor, it does.)

- A capacitor with zero voltage acts like a zero-resistance wire (until some charge builds up), and can "short out" other branches of a circuit in parallel with it.

- Kirchhoff's loop law and node law remain useful analysis tools, even though we are now dealing with capacitors.

- To figure out the current through a capacitor, you need to know about the rest of the circuit.

For Instructors Only

This is second of three questions using this situation.

As with the previous question, we assume but are not told explicitly that at $t = 0$ the switch has been closed long enough so that the capacitor is fully discharged. The answer "impossible to determine," because the initial charge is not given, is somewhat defensible but not preferred.

Some students will not realize that they need to use Kirchhoff's laws to answer this question. In other words, once they recognize that the voltage across the capacitor is zero, they need Kirchhoff's laws to deduce what the current through it will be at that instant.

A diagram showing the equivalent circuit (i.e., with the capacitor replaced by a wire) might help some students.

Additional Questions:

1. How would the answer change if (a) the 1 Ω resistor is replaced by a 2 Ω one, (b) the 2 Ω resistor is replaced by a 1 Ω one, (c) the 6 V battery is changed to a 12 V one, and (d) capacitor C is replaced by a capacitor $2C$?
2. When the voltage across the capacitor is 3 V, what are the currents through (a) 1 Ω, (b) 2 Ω, and (c) C?

Question N3.08c

Description: Extending understanding of capacitor behavior in DC circuits.

Question

Consider the circuit below. Switch S is closed at $t = 0$.

What is the voltage across the capacitor C long after the switch is closed?

1. 0
2. 1 V
3. 2 V
4. 3 V
5. 4 V
6. 5 V
7. 6 V
8. 9 V
9. None of the above
10. Impossible to determine
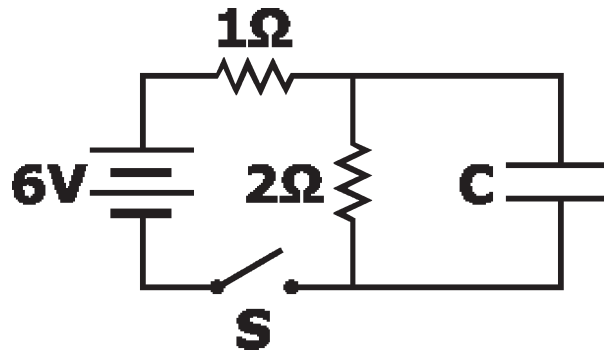
Commentary

Purpose: To develop your understanding of how capacitors behave in circuits with other elements.

Discussion: As discussed in the previous questions, current will start flowing through the capacitor when the switch is closed. As it flows, charge accumulates on the capacitor's plates, increasing the voltage across them. This voltage opposes current flow through the capacitor, causing some of the current to flow through the 2 Ω resistor. Eventually, enough charge will have accumulated on the capacitor to prevent any more current flow there; that is the capacitor's "steady-state" charge.

When all the current in the circuit flows through the 2 Ω resistor and none through the capacitor, the capacitor's branch of the circuit no longer impacts the rest of the circuit; the capacitor is effectively an "open circuit." In the simple two-resistor series circuit that remains, we can determine that the voltage across the 2 Ω resistor must be 4 V. (The current through the resistors is 2 A.) The voltage drop across the capacitor must be the same as that across the 2 Ω resistor, since they are in parallel.

Key Points:

- In a steady-state situation (nothing changing, i.e., "after a long time"), the current through a capacitor will be zero, and the voltage across it will be whatever value is consistent with that. The actual value depends on the rest of the circuit.

- A fully-charged capacitor (i.e., steady-state) acts like an open circuit and has no effect on other circuit branches in parallel with it.

- Kirchhoff's laws are useful for analyzing all circuits.

For Instructors Only

This is last of three questions using this situation.

Students who choose 6 V (answer 7) may be thinking that the voltage across the capacitor limits to the battery voltage, because that's what happens in simpler capacitor situations they have seen before (for example, if either of the resistors were removed). With this question, we are "extending the context" in which they consider capacitance. If students see only simpler cases, they may form inappropriate generalizations.

Some students will find it counterintuitive to focus on the 2 Ω resistor when the question asks specifically about the capacitor. Often, good problem solving is the ability to transform a question from an unanswerable to an answerable one. In this case, recognizing that the voltages are the same across C and 2 Ω is a key step. (Students must also recognize that no current flows through C.)

Students can also find the voltage across the 2 Ω resistor using a "voltage divider," since the two resistors are in series when the capacitor behaves like an open circuit.

A new diagram in which the capacitor is replaced by an open circuit might help some students sort out these ideas.
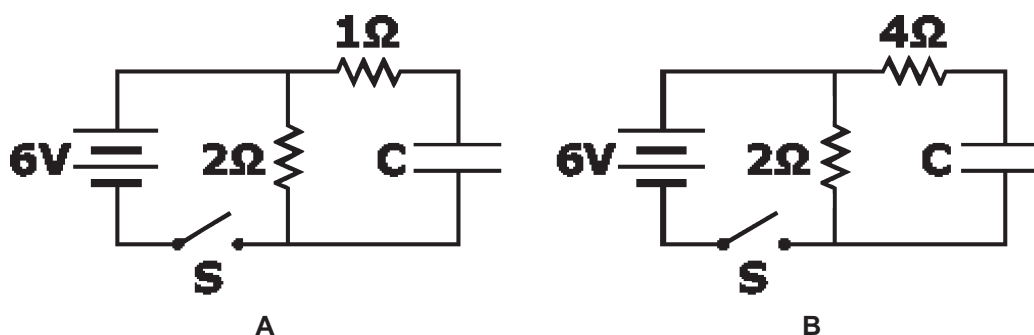
Students might not realize just how useful Kirchhoff's laws remain for analyzing these other circuits.

Question N3.09

Description: Linking energy conservation ideas to capacitor circuits.

Question

Consider the following circuits. The batteries and capacitors are identical, and the switches have been closed for a very long time.

At $t = 0$, both switches are opened. Which pair of resistors dissipates more total energy?

1.   The resistors in A dissipate more energy.
2.   The resistors in B dissipate more energy.
3.   Both pairs of resistors dissipate the same amount of energy.
4.   The amounts of energy dissipated cannot be compared.

Commentary

Purpose: To link *conservation of energy* ideas to a capacitive circuit.

Discussion: At steady-state (after a long time), the capacitor are fully charged, with no current flowing through them. When the switch is first opened, the voltage across each capacitor is 6 V. Since the voltages are the same, each capacitor stores the same energy: $U = \frac{1}{2}CV^2$.

When the switch is opened, current flows through the resistors, discharging each capacitor. A long time later, the capacitors have fully discharged, all of the energy stored in them has been dissipated. Energy must be conserved, so this energy went somewhere. The only mechanism to transfer energy in this situation is conversion to thermal energy via power dissipation in the resistors. So, each pair of resistors must dissipate exactly $\frac{1}{2}CV^2$ of energy. Therefore, the amounts of energy dissipated by the pair in each circuit must be the same.

Note that the *power* dissipated by the resistors in circuit A is **not** the same as the power dissipated by those in circuit B. Power is the *rate* at which energy is transferred. The circuit with the higher resistance (B) takes longer to dissipate the energy, but eventually dissipates the same total amount.

Key Points:

- The energy stored in a capacitor with capacitance C and voltage V across it is $U = \frac{1}{2}CV^2$.

- Although you learned the principle of *conservation of energy* in mechanics, it applies to other areas of physics as well.

- Do not confuse energy with power. Power is the rate at which energy is transferred, and has units of energy per unit time.

For Instructors Only

Fruitful use of this question requires that students are comfortable enough with capacitance to know that both capacitors will have the same initial voltage.

Students choosing answer (1) may be reasoning that circuit A dissipates more energy because it has a lower resistance and thus higher initial current. Students choosing answer (2) may be reasoning that circuit B dissipates more energy because current flows in it for longer (i.e., has a large time constant RC).

If any students choose answer (4), you should ask them what information they need to be able to answer the question. (They are likely to say they need a value for C in order to calculate the energy stored. In this case, stress that the question is qualitative and comparative, and does not require calculations.)

QUICK QUIZZES

1.   True. When a battery is delivering a current, there is a voltage drop within the battery due to internal resistance, making the terminal voltage less than the emf.

2.   Because of internal resistance, power is dissipated into the battery material, raising its temperature.

3.   (b). When the switch is opened, resistors R_1 and R_2 are in series, so that the total circuit resistance is larger than when the switch was closed. As a result, the current decreases.

4.   (a). When the switch is opened, resistors R_1 and R_2 are in series, so the total circuit resistance is larger, and the current through R_1 is less, with the switch open than when it is closed. Since the power delivered to R_1 is $\mathcal{P} = I^2 R_1$, $\mathcal{P}_o < \mathcal{P}_c$.

5.   (a). When the switch is closed, resistors R_1 and R_2 are in parallel, so that the total circuit resistance is smaller than when the switch was open. As a result, the current increases.

6.   (b). Observe that the potential difference across R_1 equals the terminal potential difference of the battery. If the battery has negligible internal resistance, the terminal potential difference is the same with the switch open or closed. Under these conditions, the power delivered to R_1, equal to $\mathcal{P} = (\Delta V)^2 / R_1$, is unchanged when the switch is closed.

7.   The voltage drop across each bulb connected in parallel with each other and across the battery equals the terminal potential difference of the battery. As more bulbs are added, the current supplied by the battery increases. However, if the internal resistance is negligible, the terminal potential difference is constant and the current through each bulb is the same regardless of the number of bulbs connected. Under these conditions: **(a)** The brightness of a bulb, determined by the current flowing in the bulb, is unchanged as bulbs are added. **(b)** The individual currents in the bulbs, $I = \Delta V / R$, are constant as bulbs are added since ΔV does not change. **(c)** The total power delivered by the battery increases by an amount $(\Delta V)^2 / R$ each time a bulb is added. **(d)** With the total delivered power increasing, energy is drawn from the battery at an increasing rate and the battery's lifetime decreases.

8.   Adding bulbs in series with each other and the battery increases the total load resistance seen by the battery. This means that the current supplied by the battery decreases with each new bulb that is added. **(a)** The brightness of a bulb is determined by the power delivered to that bulb, $\mathcal{P}_{bulb} = I^2 R$, decreases as bulbs are added and the current decreases. **(b)** For a series connection, the individual currents in the bulbs are the same and equal to the total current supplied by the battery. This decreases as bulbs are added. **(c)** The total power delivered by the battery is given by $\mathcal{P}_{total} = (\Delta V) I$, where ΔV is the terminal potential difference of the battery and I is the total current supplied by the battery. With negligible internal resistance, ΔV is constant. Thus, with I decreasing as bulbs are added, the total delivered power decreases. **(d)** With the delivered power decreasing, energy is drawn from the battery at a decreasing rate, which increases the lifetime of the battery.

9.   (c). After the capacitor is fully charged, current flows only around the outer loop of the circuit. This path has a total resistance of $3\ \Omega$, so the 6-V battery will supply a current of 2 amperes.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1.   In a series connection, the same current exists in each element. The potential difference across a resistor in this series connection is directly proportional to the resistance of that resistor, $\Delta V = IR$, and independent of its location within the series connection. The only true statement among the listed choices is (c).

2.   The same potential difference exists across all elements connected in parallel with each other, while the current through each element is inversely proportional to the resistance of that element ($I = \Delta V / R$). Thus, both (b) and (c) are true statements while the other choices are false.

3.   For these parallel resistors, $\dfrac{1}{R_{eq}} = \dfrac{1}{1.00\ \Omega} + \dfrac{1}{2.00\ \Omega} = \dfrac{2+1}{2.00\ \Omega}$ and $R_{eq} = \dfrac{2.00\ \Omega}{3} = 0.667\ \Omega$, and (c) is the correct answer.

4.   The total power dissipated is $\mathcal{P}_{total} = \mathcal{P}_1 + \mathcal{P}_2 = 120\ \text{W} + 60\ \text{W} = 180\ \text{W}$ while the potential difference across this series combination is $\Delta V = 120\ \text{V}$. The current drawn through the series combination is then $I = \dfrac{\mathcal{P}_{total}}{\Delta V} = \dfrac{180\ \text{W}}{120\ \text{V}} = 1.5\ \text{A}$, and (b) is the correct choice.

5.   The potential difference across each element of this parallel combination is $\Delta V = 120\ \text{V}$ and the total power dissipated is $\mathcal{P}_{total} = 1\,200\ \text{W} + 600\ \text{W} = 1\,800\ \text{W}$. The total current through the parallel combination, and hence, the current drawn from the external source is then
$$I = \dfrac{P_{total}}{\Delta V} = \dfrac{1\,800\ \text{W}}{120\ \text{V}} = 15\ \text{A}.$$ The correct choice is (e).

6.   The equivalent resistance of the parallel combination consisting of the 4.0 Ω, 6.0 Ω, and 10 Ω resistors is $R_p = 1.9$ Ω. This resistance is in series with a 2.0 Ω resistor, making the total resistance of the circuit $R_{total} = 3.9$ Ω. The total current supplied by the battery is $I_{total} = \Delta V / R_{total} = 12$ V/3.9 Ω = 3.1 A. Thus, the potential difference across each resistor in the parallel combination is $\Delta V_p = R_p I_{total} = (1.9\ \Omega)(3.1\ \text{A}) = 5.9$ V and the current through the 10 Ω resistor is $I_{10} = \Delta V_p / 10\ \Omega = 5.9$ V/10 Ω = 0.59 A. Choice (a) is the correct answer.

7.   The equation of choice (b) is the result of a correct application of Kirchhoff's junction rule at either of the two junctions in the circuit. The equation of choice (c) results from a correct application of Kirchhoff's loop rule to the lower loop in the circuit, while the equation of choice (d) is obtained by correctly applying the loop rule to the loop forming the outer perimeter of the circuit. The equation of choice (a) is the result of an incorrect application (involving two sign errors) of the loop rule to the upper loop in the circuit. The correct answer is choice (a).

8.   When the two identical resistors are in series, the current supplied by the battery is $I = \Delta V / 2R$, and the total power delivered is $\mathcal{P}_s = (\Delta V) I = (\Delta V)^2 / 2R$. With the resistors connected in parallel, the potential difference across each resistor is ΔV and the power delivered to each resistor is $\mathcal{P}_1 = (\Delta V)^2 / R$. Thus, the total power delivered in this case is

$$\mathcal{P}_p = 2\mathcal{P}_1 = 2\frac{(\Delta V)^2}{R} = 4\left[\frac{(\Delta V)^2}{2R}\right] = 4\mathcal{P}_s = 4(8.0\ \text{W}) = 32\ \text{W}$$

and (b) is the correct choice.

9.   The equivalent resistance for the series combination of five identical resistors is $R_{eq} = 5R$, and the equivalent capacitance of five identical capacitors in parallel is $C_{eq} = 5C$. The time constant for the circuit is therefore $\tau = R_{eq} C_{eq} = (5R)(5C) = 25RC$, and (d) is the correct choice.

10.   When the switch is closed, the current has a large initial value but decreases exponentially in time. The bulb will glow brightly at first, but fade rapidly as the capacitor charges. After a time equal to many time constants of the circuit, the current is essentially zero and the bulb does not glow. The correct answer is choice (c).

11.   The equivalent resistance of a group of resistors connected in parallel is the reciprocal of the inverses of the individual resistances and is always less than the smallest resistance in the group. Therefore, both (b) and (e) are true statements while all other choices are false.

12.   The equivalent resistance of a series combination of resistors is the algebraic sum of the individual resistances and is always greater than any individual resistance. Therefore, choices (a) and (d) are true statements and all others are false.

13.   With the capacitor initially uncharged, the potential difference across the capacitor, $\Delta V = q/C$, starts at zero when the switch is first closed and rises exponentially toward the equilibrium value of $(\Delta V)_{max} = \Delta V_{battery} = 6.00$ V. The time constant of the circuit is the time required for the charge (and hence the potential difference) to increase from 0 to 63.2% of the maximum equilibrium value. Thus, after one time constant, the potential difference across the capacitor will be $\Delta V = 0.632(6.00\ \text{V}) = 3.79$ V. The correct answer is choice (d).

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2.
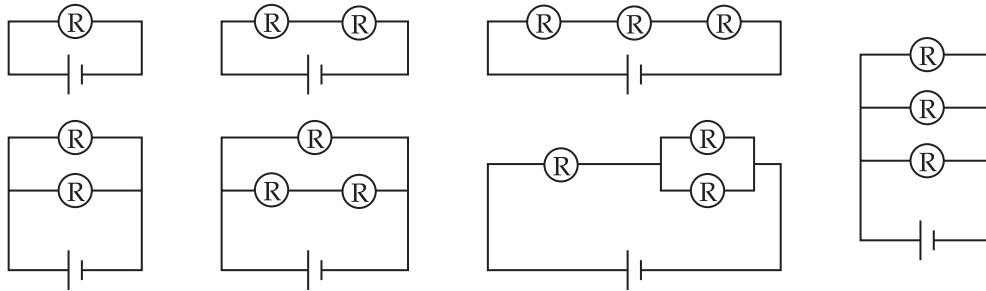
4. A short circuit can develop when the last bit of insulation frays away between the two conductors in a lamp cord. Then the two conductors touch each other, opening a low resistance branch in parallel with the lamp. The lamp will immediately go out, carrying no current and presenting no danger. A very large current will be produced in the power source, the house wiring, and the wire in the lamp cord up to and through the short. The circuit breaker will interrupt the circuit quickly but not before considerable heating and sparking is produced in the short-circuit path.

6. A wire or cable in a transmission line is thick and made of material with very low resistivity. Only when its length is very large does its resistance become significant. To transmit power over a long distance it is most efficient to use low current at high voltage. The power loss per unit length of the transmission line is $\mathcal{P}_{\text{loss}}/L = I^2 (R/L)$ where R/L is the resistance per unit length of the line. Thus, a low current is clearly desirable, but to transmit a significant amount of power, $\mathcal{P} = (\Delta V) I$, at low current a high voltage must be used.

8. The bulbs of set A are wired in parallel. The bulbs of set B are wired in series, so removing one bulb produces an open circuit with infinite resistance and zero current.

10. **(a)** ii. The power delivered may be expressed as $\mathcal{P} = I^2 R$, and while resistors connected in series have the same current in each, they may have different values of resistance.

(b) ii. The power delivered may also be expressed as $\mathcal{P} = (\Delta V)^2/R$, and while resistors connected in parallel have the same potential difference across them, they may have different values of resistance.

12. Compare two runs in series to two resistors connected in series. Compare three runs in parallel to three resistors connected in parallel. Compare one chairlift followed by two runs in parallel to a battery followed immediately by two resistors in parallel.

The junction rule for ski resorts says that the number of skiers coming into a junction must be equal to the number of skiers leaving. The loop rule would be stated as the total change in altitude must be zero for any skier completing a closed path.

14. Because water is a good conductor, if you should become part of a short circuit when fumbling with any electrical circuit while in a bathtub, the current would follow a pathway through you, the water, and to ground. Electrocution would be the obvious result.

PROBLEM SOLUTIONS

18.1 From $\Delta V = I(R+r)$, the internal resistance is

$$r = \frac{\Delta V}{I} - R = \frac{9.00 \text{ V}}{0.117 \text{ A}} - 72.0 \text{ }\Omega = \boxed{4.92 \text{ }\Omega}$$

18.2 (a) When the three resistors are in series, the equivalent resistance of the circuit is

$$R_{eq} = R_1 + R_2 + R_3 = 3(9.0 \text{ }\Omega) = \boxed{27 \text{ }\Omega}$$

(b) The terminal potential difference of the battery is applied across the series combination of the three 9.0 Ω resistors, so the current supplied by the battery and the current through each resistor in the series combination is

$$I = \frac{\Delta V}{R_{eq}} = \frac{12 \text{ V}}{27 \text{ }\Omega} = \boxed{0.44 \text{ A}}$$

(c) If the three 9.0 Ω are now connected in parallel with each other, the equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{9.0 \text{ }\Omega} + \frac{1}{9.0 \text{ }\Omega} + \frac{1}{9.0 \text{ }\Omega} = \frac{3}{9.0 \text{ }\Omega} \quad \text{or} \quad R_{eq} = \frac{9.0 \text{ }\Omega}{3} = \boxed{3.0 \text{ }\Omega}$$

When this parallel combination is connected to the battery, the potential difference across each resistor in the combination is $\Delta V = 12$ V, so the current through each of the resistors is

$$I = \frac{\Delta V}{R} = \frac{12 \text{ V}}{9.0 \text{ }\Omega} = \boxed{1.3 \text{ A}}$$

18.3 For the bulb in use as intended,

$$R_{bulb} = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120 \text{ V})^2}{75.0 \text{ W}} = 192 \text{ }\Omega$$

Now, assuming the bulb resistance is unchanged, the current in the circuit shown is

$$I = \frac{\Delta V}{R_{eq}} = \frac{120 \text{ V}}{0.800 \text{ }\Omega + 192 \text{ }\Omega + 0.800 \text{ }\Omega} = 0.620 \text{ A}$$
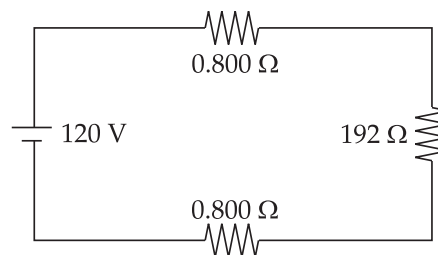
and the actual power dissipated in the bulb is

$$\mathcal{P} = I^2 R_{bulb} = (0.620 \text{ A})^2 (192 \text{ }\Omega) = \boxed{73.8 \text{ W}}$$

18.4 (a) When the 8.00-Ω resistor is connected across the 9.00-V terminal potential difference of the battery, the current through both the resistor and the battery is

$$I = \frac{\Delta V}{R} = \frac{9.00 \text{ V}}{8.00 \text{ }\Omega} = \boxed{1.13 \text{ A}}$$

(b) The relation between the emf and the terminal potential difference of a battery supplying current I is $\Delta V = \mathcal{E} - Ir$, where r is the internal resistance of the battery. Thus, if the battery has $r = 0.15 \text{ }\Omega$ and maintains a terminal potential difference of $\Delta V = 9.00$ V while supplying the current found above, the emf of this battery must be
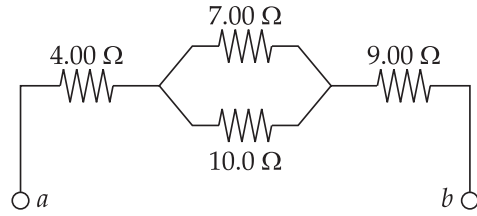
$$\mathcal{E} = \Delta V + Ir = 9.00 \text{ V} + (1.13 \text{ A})(0.15 \text{ }\Omega) = (9.00 + 0.17) \text{ }\Omega = \boxed{9.17 \text{ }\Omega}$$

18.5 (a) The equivalent resistance of the two parallel resistors is

$$R_p = \left(\frac{1}{7.00 \ \Omega} + \frac{1}{10.0 \ \Omega} \right)^{-1} = 4.12 \ \Omega$$

Thus,

$$R_{ab} = R_4 + R_p + R_9 = (4.00 + 4.12 + 9.00) \ \Omega = \boxed{17.1 \ \Omega}$$

(b) $I_{ab} = \dfrac{(\Delta V)_{ab}}{R_{ab}} = \dfrac{34.0 \ \text{V}}{17.1 \ \Omega} = 1.99 \ \text{A}$, so $\boxed{I_4 = I_9 = 1.99 \ \text{A}}$

Also, $(\Delta V)_p = I_{ab} R_p = (1.99 \ \text{A})(4.12 \ \Omega) = 8.18 \ \text{V}$

Then, $I_7 = \dfrac{(\Delta V)_p}{R_7} = \dfrac{8.18 \ \text{V}}{7.00 \ \Omega} = \boxed{1.17 \ \text{A}}$

and $I_{10} = \dfrac{(\Delta V)_p}{R_{10}} = \dfrac{8.18 \ \text{V}}{10.0 \ \Omega} = \boxed{0.818 \ \text{A}}$

18.6 (a) The parallel combination of the 6.0 Ω and 12 Ω resistors has an equivalent resistance of

$$\frac{1}{R_{p1}} = \frac{1}{6.0 \ \Omega} + \frac{1}{12 \ \Omega} = \frac{2+1}{12 \ \Omega} \quad \text{or} \quad R_{p1} = \frac{12 \ \Omega}{3} = 4.0 \ \Omega$$

Similarly, the equivalent resistance of the 4.0 Ω and 8.0 Ω parallel combination is

$$\frac{1}{R_{p2}} = \frac{1}{4.0 \ \Omega} + \frac{1}{8.0 \ \Omega} = \frac{2+1}{8.0 \ \Omega} \quad \text{or} \quad R_{p2} = \frac{8 \ \Omega}{3}$$

The total resistance of the series combination between points a and b is then

$$R_{ab} = R_{p1} + 5.0 \ \Omega + R_{p2} = 4.0 \ \Omega + 5.0 \ \Omega + \frac{8.0}{3} \ \Omega = \boxed{\frac{35}{3} \ \Omega}$$

(b) If $\Delta V_{ab} = 35$ V, the total current from a to b is $I_{ab} = \Delta V_{ab} / R_{ab} = 35 \ \text{V}/(35 \ \Omega/3) = 3.0$ A and the potential differences across the two parallel combinations are

$$\Delta V_{p1} = I_{ab} R_{p1} = (3.0 \ \text{A})(4.0 \ \Omega) = 12 \ \text{V} \quad \text{and} \quad \Delta V_{p2} = I_{ab} R_{p2} = (3.0 \ \text{A})\left(\frac{8.0}{3} \ \Omega \right) = 8.0 \ \text{V}$$

so the individual currents through the various resistors are:

$$I_{12} = \Delta V_{p1}/12 \ \Omega = \boxed{1.0 \ \text{A}}; \quad I_6 = \Delta V_{p1}/6.0 \ \Omega = \boxed{2.0 \ \text{A}}; \quad I_5 = I_{ab} = \boxed{3.0 \ \text{A}};$$

$$I_8 = \Delta V_{p2}/8.0 \ \Omega = \boxed{1.0 \ \text{A}}; \quad \text{and} \quad I_4 = \Delta V_{p2}/4.0 \ \Omega = \boxed{2.0 \ \text{A}}$$

18.7 The equivalent resistance of the parallel combination of three identical resistors is

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{3}{R} \quad \text{or} \quad R_p = \frac{R}{3}$$

The total resistance of the series combination between points a and b is then

$$R_{ab} = R + R_p + R = 2R + \frac{R}{3} = \boxed{\frac{7}{3}R}$$

18.8 (a) The equivalent resistance of this first parallel combination is

$$\frac{1}{R_{p1}} = \frac{1}{10.0\ \Omega} + \frac{1}{5.00\ \Omega}$$

or

$$R_{p1} = \boxed{3.33\ \Omega}$$

(b) For this series combination,

$$R_{upper} = R_{p1} + 4.00\ \Omega = \boxed{7.33\ \Omega}$$

(c) For the second parallel combination,

$$\frac{1}{R_{p2}} = \frac{1}{R_{upper}} + \frac{1}{3.00\ \Omega} = \frac{1}{7.33\ \Omega} + \frac{1}{3.00\ \Omega} \quad \text{or} \quad R_{p2} = \boxed{2.13\ \Omega}$$

(d) For the second series combination (and hence the entire resistor network),

$$R_{total} = 2.00\ \Omega + R_{p2} = 2.00\ \Omega + 2.13\ \Omega = \boxed{4.13\ \Omega}$$

(e) The total current supplied by the battery is

$$I_{total} = \frac{\Delta V}{R_{total}} = \frac{8.00\ V}{4.13\ \Omega} = \boxed{1.94\ A}$$

(f) The potential drop across the 2.00 Ω resistor is

$$\Delta V_2 = R_2 I_{total} = (2.00\ \Omega)(1.94\ A) = \boxed{3.88\ V}$$

(g) The potential drop across the second parallel combination must be

$$\Delta V_{p2} = \Delta V - \Delta V_2 = 8.00\ V - 3.88\ V = \boxed{4.12\ V}$$

(h) So the current through the 3.00 Ω resistor is $I_{total} = \frac{\Delta V_{p2}}{R_3} = \frac{4.12\ V}{3.00\ \Omega} = \boxed{1.37\ A}$.
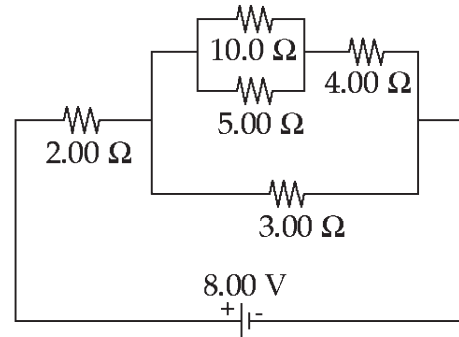
18.9   (a)   Turn the circuit given in Figure P18.9 90° counterclockwise to observe that it is equivalent to that shown in Figure 1 below. This reduces, in stages, as shown in the following figures.
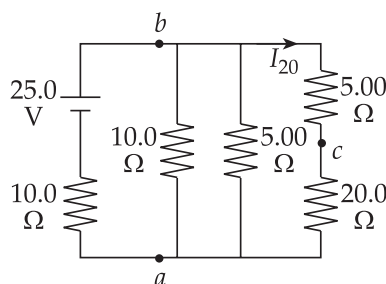
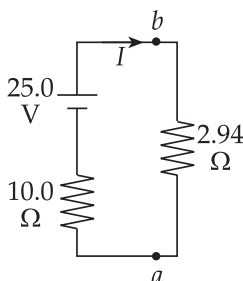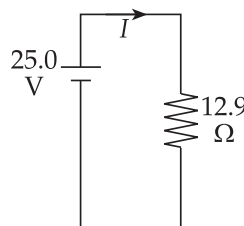**Figure 1**   **Figure 2**
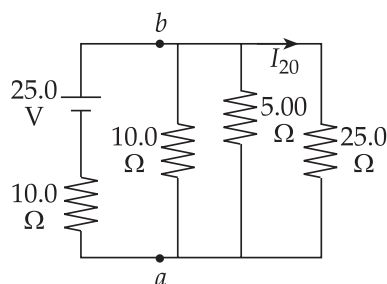
Figure 3   **Figure 4**

From Figure 4,

$$I = \frac{\Delta V}{R} = \frac{25.0\ \text{V}}{12.9\ \Omega} = 1.93\ \text{A}$$

(b)   From Figure 3,

$$\left(\Delta V\right)_{ba} = IR_{ba} = \left(1.93\ \text{A}\right)\left(2.94\ \Omega\right) = \boxed{5.68\ \text{V}}$$

(a)   From Figures 1 and 2, the current through the 20.0 Ω resistor is

$$I_{20} = \frac{\left(\Delta V\right)_{ba}}{R_{bca}} = \frac{5.68\ \text{V}}{25.0\ \Omega} = \boxed{0.227\ \text{A}}$$

18.10   (a)   By Ohm's law, the current in A is $\boxed{I_A = \mathcal{E}/R}$. The equivalent resistance of the series combination of bulbs B and C is $2R$. Thus, the current in each of these bulbs is $\boxed{I_B = I_C = \mathcal{E}/2R}$.

(b)   B and C have the $\boxed{\text{same brightness because they carry the same current}}$.

(c) $\boxed{A \text{ is brighter than } B \text{ or } C}$ because $\boxed{\text{it carries twice as much current}}$.

18.11   The equivalent resistance is $R_{eq} = R + R_p$, where R_p is the total resistance of the three parallel branches;

$$R_p = \left(\frac{1}{120\ \Omega} + \frac{1}{40\ \Omega} + \frac{1}{R + 5.0\ \Omega} \right)^{-1} = \left(\frac{1}{30\ \Omega} + \frac{1}{R + 5.0\ \Omega} \right)^{-1} = \frac{(30\ \Omega)(R + 5.0\ \Omega)}{R + 35\ \Omega}$$

Thus, $75\ \Omega = R + \dfrac{(30\ \Omega)(R + 5.0\ \Omega)}{R + 35\ \Omega} = \dfrac{R^2 + (65\ \Omega)R + 150\ \Omega^2}{R + 35\ \Omega}$

which reduces to $R^2 - (10\ \Omega)R - 2\ 475\ \Omega^2 = 0$ or $(R - 55\ \Omega)(R + 45\ \Omega) = 0$.

Only the positive solution is physically acceptable, so $R = \boxed{55\ \Omega}$.

18.12   (a)   The equivalent resistance of the parallel combination

is $\dfrac{1}{R_p} = \dfrac{1}{R} + \dfrac{1}{R}$ or $R_p = \dfrac{R}{2}$

and the total resistance between a and b is

$R_{ab} = R + R_p = R + \dfrac{R}{2} = \boxed{3R/2}$
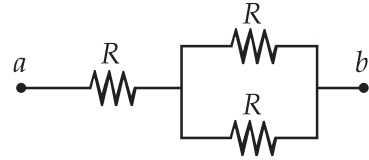
(b)   From $\mathcal{P}_{ab} = \dfrac{(\Delta V_{ab})^2}{R_{ab}}$, we have

$(\Delta V_{ab})_{max} = \sqrt{R_{ab}(\mathcal{P}_{ab})_{max}} = \sqrt{\dfrac{3R}{2}(24\ \text{W})} = \boxed{6\sqrt{R}\ \text{volts}}$

(c)   When $\Delta V_{ab} = (\Delta V_{ab})_{max}$, the current in the series resistor on the left is

$I_{ab} = \dfrac{(\Delta V_{ab})_{max}}{R_{ab}} = \dfrac{6\sqrt{R}}{3R/2} = \dfrac{4}{\sqrt{R}}$ and the current through each of the two parallel

resistors is $I_p = I_{ab}/2 = 2/\sqrt{R}$. The power delivered to the series resistor on the left

is then $\mathcal{P}_s = I_{ab}^2 R = \left(\dfrac{4}{\sqrt{R}} \right)^2 R = \boxed{16\ \text{W}}$, and the power delivered to each of the two

parallel resistors is $\mathcal{P}_p = I_p^2 R = \left(\dfrac{2}{\sqrt{R}} \right)^2 R = \boxed{4\ \text{W}}$.

18.13 The resistors in the circuit can be combined in the stages shown below to yield an equivalent resistance of $R_{ad} = (63/11)\ \Omega$.
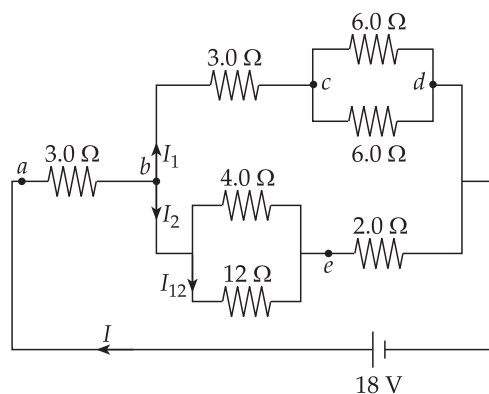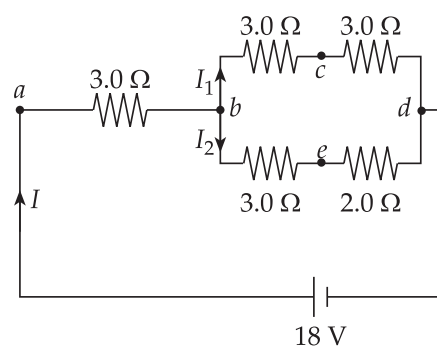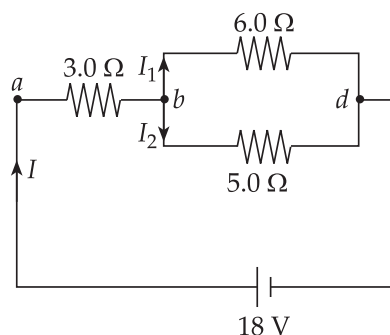

Figure 1
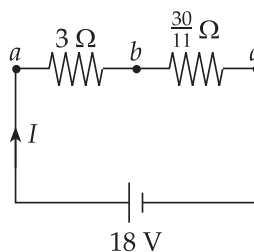
Figure 2

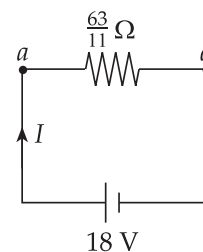**Figure 3**   **Figure 4**   **Figure 5**

From Figure 5,
$$I = \frac{(\Delta V)_{ad}}{R_{ad}} = \frac{18\ \text{V}}{(63/11)\ \Omega} = 3.14\ \text{A}$$

Then, from Figure 4,
$$(\Delta V)_{bd} = I\,R_{bd} = (3.14\ \text{A})(30/11\ \Omega) = 8.57\ \text{V}$$

Now, look at Figure 2 and observe that

$$I_2 = \frac{(\Delta V)_{bd}}{3.0\ \Omega + 2.0\ \Omega} = \frac{8.57\ \text{V}}{5.0\ \Omega} = 1.71\ \text{A}$$

so
$$(\Delta V)_{be} = I_2 R_{be} = (1.71\ \text{A})(3.0\ \Omega) = 5.14\ \text{V}$$

Finally, from Figure 1,
$$I_{12} = \frac{(\Delta V)_{be}}{R_{12}} = \frac{5.14\ \text{V}}{12\ \Omega} = \boxed{0.43\ \text{A}}$$

18.14   (a)   The resistor network connected to the battery in Figure P18.14 can be reduced to a single equivalent resistance in the following steps. The equivalent resistance of the parallel combination of the 3.00 Ω and 6.00 Ω resistors is
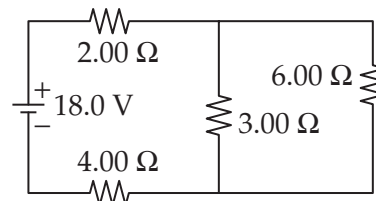
$$\frac{1}{R_p} = \frac{1}{3.00 \ \Omega} + \frac{1}{6.00 \ \Omega} = \frac{3}{6.00 \ \Omega} \quad \text{or} \quad R_p = 2.00 \ \Omega$$

This resistance is in series with the 4.00 Ω and the other 2.00 Ω resistor, giving a total equivalent resistance of $\boxed{R_{eq} = 2.00 \ \Omega + R_p + 4.00 \ \Omega = 8.00 \ \Omega}$.

(b)   The current in the 2.00 Ω resistor is the total current supplied by the battery and is equal to

$$I_{total} = \frac{\Delta V}{R_{eq}} = \frac{18.0 \ \text{V}}{8.00 \ \Omega} = \boxed{2.25 \ \text{A}}$$

(c)   The power the battery delivers to the circuit is

$$\mathcal{P} = (\Delta V) I_{total} = (18.0 \ \text{V})(2.25 \ \text{A}) = \boxed{40.5 \ \text{W}}$$

18.15   (a)   Connect $\boxed{\text{two 50-}\Omega \text{ resistors in parallel}}$ to get 25 Ω. Then connect that parallel combination $\boxed{\text{in series with a 20-}\Omega \text{ resistor}}$ for a total resistance of 45 Ω.

(b)   Connect $\boxed{\text{two 50-}\Omega \text{ resistors in parallel}}$ to get 25 Ω.

Also, $\boxed{\text{connect two 20-}\Omega \text{ resistors in parallel}}$ to get 10 Ω.

Then, $\boxed{\text{connect these two parallel combinations in series}}$ to obtain 35 Ω.

18.16   (a)   The equivalent resistance of the parallel combination between points b and e is

$$\frac{1}{R_{be}} = \frac{1}{12 \ \Omega} + \frac{1}{24 \ \Omega} \quad \text{or} \quad R_{be} = 8.0 \ \Omega$$

The total resistance between points a and e is then

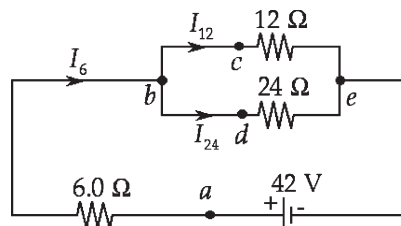$$R_{ae} = R_{ab} + R_{be} = 6.0 \ \Omega + 8.0 \ \Omega = 14 \ \Omega$$

The total current supplied by the battery (and also the current in the 6.0 Ω resistor) is

$$I_{total} = I_6 = \frac{\Delta V_{ae}}{R_{ae}} = \frac{42 \ \text{V}}{14 \ \Omega} = \boxed{3.0 \ \text{A}}$$

The potential difference between points b and e is

$$\Delta V_{be} = R_{be} I_{total} = (8.0 \ \Omega)(3.0 \ \text{A}) = 24 \ \text{V}$$

so $I_{12} = \dfrac{\Delta V_{be}}{R_{bce}} = \dfrac{24 \ \text{V}}{12 \ \Omega} = \boxed{2.0 \ \text{A}}$ and $I_{24} = \dfrac{\Delta V_{be}}{R_{bde}} = \dfrac{24 \ \text{V}}{24 \ \Omega} = \boxed{1.0 \ \text{A}}$

continued on next page

(b)   Applying the junction rule at point *b* yields $\boxed{I_6 - I_{12} - I_{24} = 0}$ **[1]**

Using the loop rule on loop *abdea* gives $+42 - 6I_6 - 24I_{24} = 0$ or $\boxed{I_6 = 7.0 - 4I_{24}}$ **[2]**

and using the loop rule on loop *bcedb* gives $-12I_{12} + 24I_{24} = 0$ or $\boxed{I_{12} = 2I_{24}}$ **[3]**

Substituting Equations [2] and [3] into [1] yields $7I_{24} = 7.0$ or $\boxed{I_{24} = 1.0 \text{ A}}$

Then, Equations [2] and [3] yield $\boxed{I_6 = 3.0 \text{ A}}$ and $\boxed{I_{12} = 2.0 \text{ A}}$

18.17   Going counterclockwise around the upper loop, applying Kirchhoff's loop rule, gives

$$+15.0 \text{ V} - (7.00)I_1 - (5.00)(2.00 \text{ A}) = 0$$

or $I_1 = \dfrac{15.0 \text{ V} - 10.0 \text{ V}}{7.00 \ \Omega} = \boxed{0.714 \text{ A}}$

From Kirchhoff's junction rule, $I_1 + I_2 - 2.00 \text{ A} = 0$

so $I_2 = 2.00 \text{ A} - I_1 = 2.00 \text{ A} - 0.714 \text{ A} = \boxed{1.29 \text{ A}}$

Going around the lower loop in a clockwise direction gives

$$+\mathcal{E} - (2.00)I_2 - (5.00)(2.00 \text{ A}) = 0$$

or $\mathcal{E} = (2.00 \ \Omega)(1.29 \text{ A}) + (5.00 \ \Omega)(2.00 \text{ A}) = \boxed{12.6 \text{ V}}$
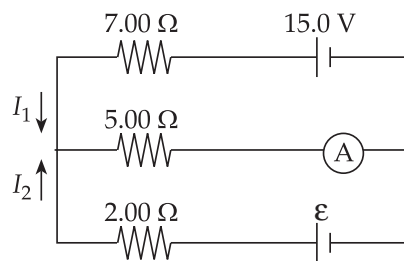
18.18   Observe that the center branch of this circuit, that is the branch containing points *a* and *b*, is not a continuous conducting path, so no current can flow in this branch. The only current in the circuit flows counterclockwise around the perimeter of this circuit. Going counterclockwise around the this outer loop and applying Kirchhoff's loop rule gives

$$-8.0 \text{ V} - (2.0 \ \Omega)I - (3.0 \ \Omega)I + 12 \text{ V} - (10 \ \Omega)I - (5.0 \ \Omega)I = 0$$

or $I = \dfrac{12 \text{ V} - 8.0 \text{ V}}{20 \ \Omega} = 0.20 \text{ A}$

Now, we start at point *b* and go around the upper panel of the circuit to point *a*, keeping track of changes in potential as they occur. This gives

$$\Delta V_{ab} = V_a - V_b = -4.0 \text{ V} + (6.0 \ \Omega)(0) - (3.0 \ \Omega)(0.20 \text{ A}) + 12 \text{ V} - (10 \ \Omega)(0.20 \text{ A}) = +5.4 \text{ V}$$

Since $\Delta V_{ab} > 0$, $\boxed{\text{point } a \text{ is 5.4 V higher in potential than point } b}$.

18.19   (a)   Applying Kirchhoff's loop rule, as you go clockwise around the loop, gives

$$+20.0 \text{ V} - (2\,000)I - 30.0 \text{ V} - (2\,500)I + 25.0 \text{ V} - (500)I = 0,$$

or $I = 3.00 \times 10^{-3} \text{ A} = \boxed{3.00 \text{ mA}}$

(b) Start at the grounded point and move up the left side, recording changes in potential as you go, to obtain

$$V_A = +20.0 \text{ V} - (2\ 000\ \Omega)(3.00 \times 10^{-3}\ \text{A}) - 30.0\ \text{V} - (1\ 000\ \Omega)(3.00 \times 10^{-3}\ \text{A})$$

or $V_A = \boxed{-19.0 \text{ V}}$

(c) $(\Delta V)_{1\ 500} = (1\ 500\ \Omega)(3.00 \times 10^{-3}\ \text{A}) = \boxed{4.50 \text{ V}}$

(The upper end is at the higher potential.)

18.20 Following the path of I_1 from a to b, and recording changes in potential gives

$$V_b - V_a = +24\ \text{V} - (6.0\ \Omega)(3.0\ \text{A}) = +6.0\ \text{V}$$

Now, following the path of I_2 from a to b, and recording changes in potential gives

$$V_b - V_a = -(3.0\ \Omega)I_2 = +6.0\ \text{V}, \text{ or } I_2 = \boxed{-2.0 \text{ A}}$$

Thus, I_2 is directed from $\boxed{\text{from } b \text{ toward } a}$ and has magnitude of 2.0 A.

Applying Kirchhoff's junction rule at point a gives

$$I_3 = I_1 + I_2 = 3.0\ \text{A} + (-2.0\ \text{A}) = \boxed{1.0 \text{ A}}$$
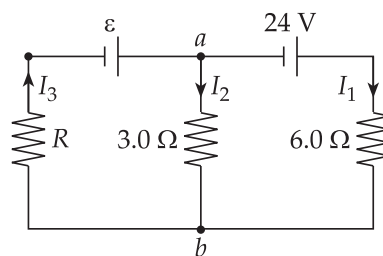
18.21 (a) Applying Kirchhoff's junction rule at point a gives

$$I_3 = I_1 + I_2 \qquad \textbf{[1]}$$

Using the loop rule on the lower loop yields

$$+12 - 12I_2 - 16I_3 = 0 \quad \text{or} \quad I_2 = 1 - \frac{4I_3}{3} \qquad \textbf{[2]}$$

Applying the loop rule to loop forming the outer perimeter of the circuit gives

$$+24 - 28I_1 - 16I_3 = 0 \quad \text{or} \quad I_1 = \frac{24 - 16I_3}{28} \qquad \textbf{[3]}$$

Substituting Equations [2] and [3] into [1] yields $I_3 = \dfrac{24 - 16I_3}{28} + 1 - \dfrac{4I_3}{3}$, and

multiplying by 84 to eliminate fractions: $84I_3 = 72 - 48I_3 + 84 - 112I_3$ which reduces
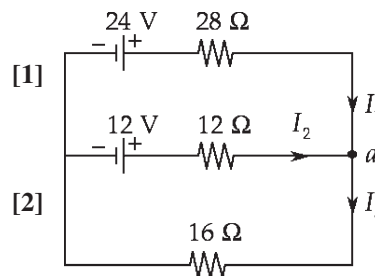
to $244I_3 = 156$ and gives $\boxed{I_3 = 0.64 \text{ A}}$.

Then, Equation [2] gives $\boxed{I_2 = 0.15 \text{ A}}$ and Equation [3] yields $\boxed{I_1 = 0.49 \text{ A}}$.

(b) The power delivered to each of the resistors in this circuit is:

$$\mathcal{P}_{28} = I_1^2 R_{28} = (0.49 \text{ A})^2 (28\ \Omega) = \boxed{6.7 \text{ W}}\ ;\ \mathcal{P}_{12} = I_2^2 R_{12} = (0.15 \text{ A})^2 (12\ \Omega) = \boxed{0.27 \text{ W}}$$

and $\mathcal{P}_{16} = I_3^2 R_{16} = (0.64 \text{ A})^2 (16\ \Omega) = \boxed{6.6 \text{ W}}$

18.22 (a) The $30.0\ \Omega$ and $50.0\ \Omega$ resistors in the upper branch are in series, and add to give a total resistance of $R_{upper} = 80.0\ \Omega$ for this path. This $80.0\ \Omega$ resistance is in parallel with the $80.0\ \Omega$ resistance of the middle branch, and the rule for combining resistors in parallel yields a total resistance of $R_{ab} = 40.0\ \Omega$ between points a and b. This resistance is in series with the $20.0\ \Omega$ resistor, so the total equivalent resistance of the circuit is
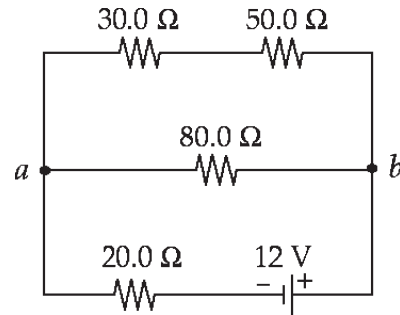
$$R_{eq} = 20.0\ \Omega + R_{ab} = 20.0\ \Omega + 40.0\ \Omega = \boxed{60.0\ \Omega}$$

(b) The current supplied to this circuit by the battery is $I_{total} = \dfrac{\Delta V}{R_{eq}} = \dfrac{12\ \text{V}}{60.0\ \Omega} = \boxed{0.20\ \text{A}}$.

(c) The power delivered by the battery is $\mathcal{P}_{total} = R_{eq}I_{total}^2 = (60.0\ \Omega)(0.20\ \text{A})^2 = \boxed{2.4\ \text{W}}$.

(d) The potential difference between points a and b is

$$\Delta V_{ab} = R_{ab}I_{total} = (40.0\ \Omega)(0.20\ \text{A}) = 8.0\ \text{V}$$

and the current in the upper branch is $I_{upper} = \dfrac{\Delta V_{ab}}{R_{upper}} = \dfrac{8.0\ \text{V}}{80.0\ \Omega} = 0.10\ \text{A}$

so the power delivered to the $50.0\ \Omega$ resistor is

$$\mathcal{P}_{50} = R_{50}I_{upper}^2 = (50.0\ \Omega)(0.10\ \text{A})^2 = \boxed{0.50\ \text{W}}$$

18.23 (a) We name the currents I_1, I_2, and I_3 as shown.

Applying Kirchhoff's loop rule to loop *abcfa*, gives $+\mathcal{E}_1 - \mathcal{E}_2 - R_2 I_2 - R_1 I_1 = 0$

or $3I_2 + 2I_1 = 10.0\ \text{mA}$

and $I_1 = 5.00\ \text{mA} - 1.50 I_2$ **[1]**

Applying the loop rule to loop *edcfe* yields

$$+\mathcal{E}_3 - R_3 I_3 - \mathcal{E}_2 - R_2 I_2 = 0$$

or $3I_2 + 4I_3 = 20.0\ \text{mA}$

and $I_3 = 5.00\ \text{mA} - 0.750 I_2$ **[2]**

Finally, applying Kirchhoff's junction rule at junction *c* gives

$$I_2 = I_1 + I_3$$ **[3]**

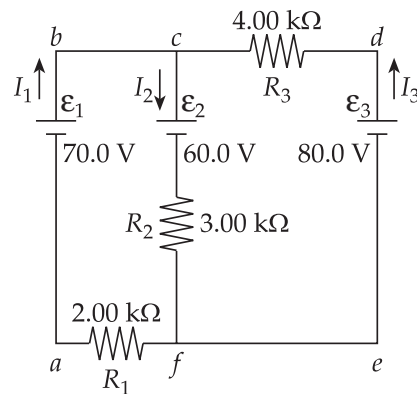Substituting Equations [1] and [2] into [3] yields

$I_2 = 5.00\ \text{mA} - 1.50 I_2 + 5.00\ \text{mA} - 0.750 I_2$ or $3.25 I_2 = 10.0\ \text{mA}$

and $\boxed{I_2 = 3.08\ \text{mA}}$. Then [1] gives $\boxed{I_1 = 0.380\ \text{mA}}$, and from [2] $\boxed{I_3 = 2.69\ \text{mA}}$.

(b) Start at point *c* and go to point *f*, recording changes in potential to obtain

$$V_f - V_c = -\mathcal{E}_2 - R_2 I_2 = -60.0\ \text{V} - (3.00 \times 10^3\ \Omega)(3.08 \times 10^{-3}\ \text{A}) = -69.2\ \text{V}$$
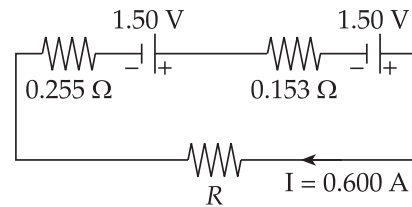
or $|\Delta V|_{cf} = \boxed{69.2\ \text{V} \text{ and point } c \text{ is at the higher potential}}$

18.24   (a)   Applying Kirchhoff's loop rule to the circuit gives

$$+3.00\ V - (0.255\ \Omega + 0.153\ \Omega + R)(0.600\ A) = 0$$

or $R = \dfrac{3.00\ V}{0.600\ A} - (0.255\ \Omega + 0.153\ \Omega) = \boxed{4.59\ \Omega}$

(b)   The total power input to the circuit is

$$\mathcal{P}_{input} = (\mathcal{E}_1 + \mathcal{E}_2)I = (1.50\ V + 1.50\ V)(0.600\ A) = 1.80\ W$$

$$\mathcal{P}_{loss} = I^2(r_1 + r_2) = (0.600\ A)^2(0.255\ \Omega + 0.153\ \Omega) = 0.147\ W$$

Thus, the fraction of the power input that is dissipated internally is

$$\frac{\mathcal{P}_{loss}}{\mathcal{P}_{input}} = \frac{0.147\ W}{1.80\ W} = \boxed{0.081\ 6\ \text{or}\ 8.16\%}$$

18.25   (a) $\boxed{\text{No.}}$ Some simplification could be made by recognizing that the 2.0 Ω and 4.0 Ω resistors are in series, adding to give a total of 6.0 Ω; and the 5.0 Ω and 1.0 Ω resistors form a series combination with a total resistance of 6.0 Ω. The circuit cannot be simplified any further, and Kirchhoff's rules must be used to analyze the circuit.

(b)   Applying Kirchhoff's junction rule at junction *a* gives

$$I_1 = I_2 + I_3 \qquad \qquad \text{[1]}$$

Using Kirchhoff's loop rule on the upper loop yields

$$+24\ V - (2.0 + 4.0)I_1 - (3.0)I_3 = 0$$

or $I_3 = 8.0\ A - 2\,I_1$ [2]

and for the lower loop,

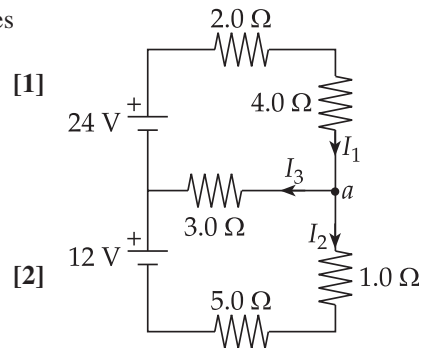$$+12\ V + (3.0)I_3 - (1.0 + 5.0)I_2 = 0$$

Using Equation [2], this reduces to

$$I_2 = \frac{12\ V + 3.0(8.0\ A - 2\,I_1)}{6.0} \quad \text{or} \quad I_2 = 6.0\ A - I_1 \qquad \text{[3]}$$

Substituting Equations [2] and [3] into [1] gives $\boxed{I_1 = 3.5\ A}$.

Then, Equation [3] gives $\boxed{I_2 = 2.5\ A}$, and [2] yields $\boxed{I_3 = 1.0\ A}$.

18.26 Using Kirchhoff's loop rule on the outer perimeter of the circuit gives

$$+12 \text{ V} - (0.01)I_1 - (0.06)I_3 = 0$$

or $I_1 = 1.2 \times 10^3 \text{ A} - 6I_3$ **[1]**

For the rightmost loop, the loop rule gives

$$+10 \text{ V} + (1.00)I_2 - (0.06)I_3 = 0$$
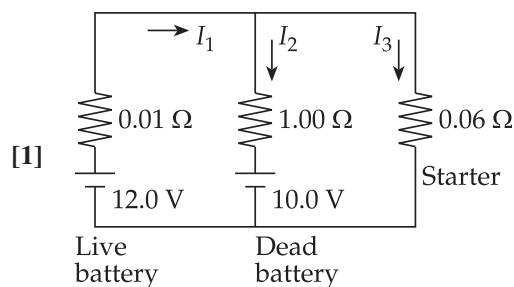
or $I_2 = 0.06 I_3 - 10 \text{ A}$ **[2]**

Applying Kirchhoff's junction rule at either junction gives

$$I_1 = I_2 + I_3$$ **[3]**

Substituting Equations [1] and [2] into [3] yields $7.06I_3 = 1\,210 \text{ A}$ and $\boxed{I_3 = 171 \text{ A (in starter)}}$.
Then Equation [2] gives $\boxed{I_2 = 0.26 \text{ A (in dead battery)}}$.

18.27 (a) $\boxed{\text{No.}}$ This multi-loop circuit does not contain any resistors in series (i.e., connected so all the current in one must pass through the other) nor in parallel (connected so the voltage drop across one is always the same as that across the other). Thus, this circuit cannot be simplified any further, and Kirchhoff's rules must be used to analyze it.

(b)   Assume currents I_1, I_2, and I_3 in the directions shown. Then, using Kirchhoff's junction rule at junction a gives

$$I_3 = I_1 + I_2$$ **[1]**

Applying Kirchhoff's loop rule on the lower loop,

$$+10.0 \text{ V} - (5.00)I_2 - (20.0)I_3 = 0$$

or $I_2 = 2.00 \text{ A} - 4I_3$ **[2]**

and for the loop around the perimeter of the circuit, $20.0 \text{ V} - 30.0 I_1 - 20.0 I_3 = 0$
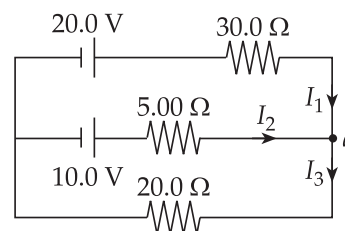
or $I_1 = 0.667 \text{ A} - 0.667 I_3$ **[3]**

Substituting Equations [2] and [3] into [1]: $I_3 = 0.667 \text{ A} - 0.667 I_3 + 2.00 \text{ A} - 4 I_3$

which reduces to $5.67 I_3 = 2.67 \text{ A}$ and gives $\boxed{I_3 = 0.471 \text{ A}}$.

Then, Equation [2] gives $\boxed{I_2 = 0.116 \text{ A}}$ and from [3] $\boxed{I_1 = 0.353 \text{ A}}$.
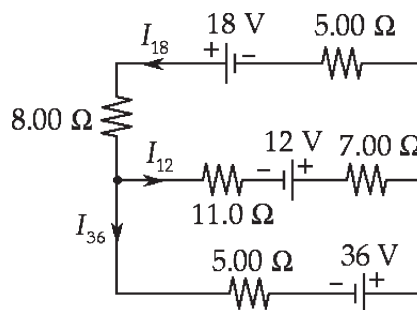
All currents are in the directions indicated in the circuit diagram given above.

18.28 (a)   Going counterclockwise around the upper loop, Kirchhoff's loop rule gives

$$-11I_{12} + 12 - 7I_{12} - 5I_{18} + 18 - 8I_{18} = 0$$

or $\boxed{18I_{12} + 13I_{18} = 30}$ **[1]**

continued on next page

(b)   Going counterclockwise around the lower loop:

$$-5I_{36} + 36 + 7I_{12} - 12 + 11I_{12} = 0$$

or $\boxed{5I_{36} - 18I_{12} = 24}$ **[2]**

(c)   Applying the junction rule at the node in the left end of the circuit gives

$\boxed{I_{18} = I_{12} + I_{36}}$ **[3]**

(d)   Solving Equation [3] for I_{36} yields $\boxed{I_{36} = I_{18} - I_{12}}$ **[4]**

(e)   Substituting Equation [4] into [2] gives $5(I_{18} - I_{12}) - 18I_{12} = 24$

or $\boxed{5I_{18} - 23I_{12} = 24}$ **[5]**

(f)   Solving Equation [5] for I_{18} yields $I_{18} = (24 + 23I_{12})/5$. Substituting this into Equation [1] and simplifying gives $389I_{12} = -162$, and $\boxed{I_{12} = -0.416 \text{ A}}$. Then, from Equation [2], $I_{18} = (30 - 18I_{12})/13$, which yields $\boxed{I_{18} = 2.88 \text{ A}}$.

(g)   Equation [4] gives $I_{36} = 2.88 \text{ A} - (-0.416 \text{ A})$, or $\boxed{I_{36} = 3.30 \text{ A}}$.

(h)   The negative sign in the answer for I_{12} means that this current flows in the opposite direction to that shown in the circuit diagram and assumed during this solution. That is, the actual current in the middle branch of the circuit flows from right to left and has a magnitude of 0.416 A.

18.29   Applying Kirchhoff's junction rule at junction *a* gives

$$I_3 = I_1 + I_2 \qquad \text{[1]}$$

Using Kirchhoff's loop rule on the leftmost loop yields

$$-3.00 \text{ V} - (4.00)I_3 - (5.00)I_1 + 12.0 \text{ V} = 0$$

so $I_1 = (9.00 \text{ A} - 4I_3)/5.00$ or $I_1 = 1.80 \text{ A} - 0.800 I_3$ **[2]**

and for the rightmost loop,

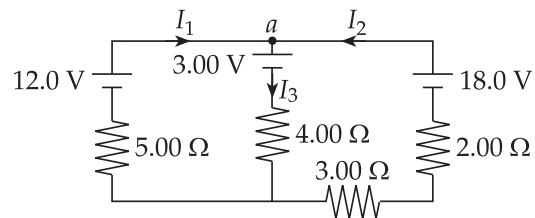$$-3.00 \text{ V} - (4.00)I_3 - (3.00 + 2.00)I_2 + 18.0 \text{ V} = 0$$

and $I_2 = (15.0 \text{ A} - 4I_3)/5.00$ or $I_2 = 3.00 \text{ A} - 0.800 I_3$ **[3]**

Substituting Equations [2] and [3] into [1] and simplifying gives $2.60I_3 = 4.80$ and $I_3 = 1.846 \text{ A}$. Then Equations [2] and [3] yield $I_1 = 0.323 \text{ A}$ and $I_2 = 1.523 \text{ A}$.

Therefore, the potential differences across the resistors are

$$\Delta V_2 = I_2(2.00 \text{ } \Omega) = \boxed{3.05 \text{ V}}, \ \Delta V_3 = I_2(3.00 \text{ } \Omega) = \boxed{4.57 \text{ V}}$$

$$\Delta V_4 = I_3(4.00 \text{ } \Omega) = \boxed{7.38 \text{ V}}, \text{ and } \Delta V_5 = I_1(5.00 \text{ } \Omega) = \boxed{1.62 \text{ V}}$$

18.30 The time constant is $\tau = RC$. Considering units, we find

$$RC \rightarrow (\text{Ohms})(\text{Farads}) = \left(\frac{\text{Volts}}{\text{Amperes}}\right)\left(\frac{\text{Coulombs}}{\text{Volts}}\right) = \left(\frac{\text{Coulombs}}{\text{Amperes}}\right)$$

$$= \left(\frac{\text{Coulombs}}{\text{Coulombs/Second}}\right) = \text{Second}$$

or $\quad \tau = RC$ has units of time.

18.31 (a) The time constant is: $\tau = RC = (75.0 \times 10^3 \ \Omega)(25.0 \times 10^{-6} \ \text{F}) = \boxed{1.88 \ \text{s}}$

(b) At $t = \tau$, $q = 0.632 Q_{max} = 0.632(C\mathcal{E}) = 0.632(25.0 \times 10^{-6} \ \text{F})(12.0 \ \text{V}) = \boxed{1.90 \times 10^{-4} \ \text{C}}$

18.32 (a) $\tau = RC = (100 \ \Omega)(20.0 \times 10^{-6} \ \text{F}) = 2.00 \times 10^{-3} \ \text{s} = \boxed{2.00 \ \text{ms}}$

(b) $Q_{max} = C\mathcal{E} = (20.0 \times 10^{-6} \ \text{F})(9.00 \ \text{V}) = 1.80 \times 10^{-4} \ \text{C} = \boxed{180 \ \mu\text{C}}$

(c) $Q = Q_{max}\left(1 - e^{-t/\tau}\right) = Q_{max}\left(1 - e^{-\tau/\tau}\right) = Q_{max}\left(1 - \frac{1}{e}\right) = \boxed{114 \ \mu\text{C}}$

18.33 $Q_{max} = C\mathcal{E} = (5.0 \times 10^{-6} \ \text{F})(30 \ \text{V}) = 1.5 \times 10^{-4} \ \text{C}$, and

$$\tau = RC = (1.0 \times 10^6 \ \Omega)(5.0 \times 10^{-6} \ \text{F}) = 5.0 \ \text{s}$$

Thus, at $t = 10 \ \text{s} = 2\tau$

$$Q = Q_{max}\left(1 - e^{-t/\tau}\right) = (1.5 \times 10^{-4} \ \text{C})(1 - e^{-2}) = \boxed{1.3 \times 10^{-4} \ \text{C}}$$

18.34 The charge on the capacitor at time t is $Q = Q_{max}\left(1 - e^{-t/\tau}\right)$, where

$$Q = C(\Delta V) \ \text{and} \ Q_{max} = C\mathcal{E}. \ \text{Thus,} \ \Delta V = \mathcal{E}\left(1 - e^{-t/\tau}\right) \ \text{or} \ e^{-t/\tau} = 1 - (\Delta V)/\mathcal{E}$$

We are given that $\mathcal{E} = 12 \ \text{V}$, and at $t = 1.0 \ \text{s}$, $\Delta V = 10 \ \text{V}$

Therefore, $e^{-1.0 \ \text{s}/\tau} = 1 - \dfrac{10}{12} = \dfrac{12 - 10}{12} = \dfrac{1}{6.0}$ or $e^{+1.0 \ \text{s}/\tau} = 6.0$

Taking the natural logarithm of each side of the equation gives

$$\frac{1.0 \ \text{s}}{\tau} = \ln(6.0) \quad \text{or} \quad \tau = \frac{1.0 \ \text{s}}{\ln(6.0)} = 0.56 \ \text{s}$$

Since the time constant is $\tau = RC$, we have

$$C = \frac{\tau}{R} = \frac{0.56 \ \text{s}}{12 \times 10^3 \ \Omega} = 4.7 \times 10^{-5} \ \text{F} = \boxed{47 \ \mu\text{F}}$$

18.35   (a)   The charge remaining on the capacitor after time t is $q = Qe^{-t/\tau}$.
Thus, if $q = 0.75Q$, then $e^{-t/\tau} = 0.75$ and $-t/\tau = \ln(0.75)$,

or $t = -\tau \ln(0.75) = -(1.5 \text{ s}) \ln(0.75) = \boxed{0.43 \text{ s}}$

(b) $\tau = RC$, so $C = \dfrac{\tau}{R} = \dfrac{1.5 \text{ s}}{250 \times 10^3 \ \Omega} = 6.0 \times 10^{-6} \text{ F} = \boxed{6.0 \ \mu\text{F}}$

18.36   (a) $I_{max} = \dfrac{\mathcal{E}}{R}$, so the resistance is

$$R = \frac{\mathcal{E}}{I_{max}} = \frac{48.0 \text{ V}}{0.500 \times 10^{-3} \text{ A}} = 9.60 \times 10^4 \ \Omega$$

The time constant is $\tau = RC$, so the capacitance is found to be

$$C = \frac{\tau}{R} = \frac{0.960 \text{ s}}{9.60 \times 10^4 \ \Omega} = 1.00 \times 10^{-5} \text{ F} = \boxed{10.0 \ \mu\text{F}}$$

(b) $Q_{max} = C\mathcal{E} = (10.0 \ \mu\text{F})(48.0 \text{ V}) = 480 \ \mu\text{C}$, so the charge stored in the capacitor at
$t = 1.92$ s is

$$Q = Q_{max}\left(1 - e^{-t/\tau}\right) = (480 \ \mu\text{C})\left(1 - e^{-\frac{1.92 \text{ s}}{0.960 \text{ s}}}\right) = (480 \ \mu\text{C})(1 - e^{-2}) = \boxed{415 \ \mu\text{C}}$$

18.37   The current drawn by a single 75-W bulb connected to a 120-V source is
$I_1 = \mathcal{P}/\Delta V = 75 \text{ W}/120 \text{ V}$. Thus, the number of such bulbs that can be connected in parallel with
this source before the total current drawn will equal 30.0 A is

$$n = \frac{30.0 \text{ A}}{I_1} = (30.0 \text{ A})\left(\frac{120 \text{ V}}{75 \text{ W}}\right) = \boxed{48}$$

18.38   (a)   The equivalent resistance of the parallel combination is

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{150 \ \Omega} + \frac{1}{25 \ \Omega} + \frac{1}{50 \ \Omega}\right)^{-1} = 15 \ \Omega$$

so the total current supplied to the circuit is

$$I_{total} = \frac{\Delta V}{R} = \frac{120 \text{ V}}{15 \ \Omega} = \boxed{8.0 \text{ A}}$$

(b)   Since the appliances are connected in parallel,

the voltage across each one is $\Delta V = \boxed{120 \text{ V}}$.

(c) $I_{lamp} = \dfrac{\Delta V}{R_{lamp}} = \dfrac{120 \text{ V}}{150 \ \Omega} = \boxed{0.80 \text{ A}}$

(d) $\mathcal{P}_{heater} = \dfrac{(\Delta V)^2}{R_{heater}} = \dfrac{(120 \text{ V})^2}{25 \ \Omega} = \boxed{5.8 \times 10^2 \text{ W}}$

18.39 From $\mathcal{P} = (\Delta V)^2/R$, the resistance of the element is

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(240 \text{ V})^2}{3\,000 \text{ W}} = 19.2 \text{ }\Omega$$

When the element is connected to a 120-V source, we find that

(a) $I = \dfrac{\Delta V}{R} = \dfrac{120 \text{ V}}{19.2 \text{ }\Omega} = \boxed{6.25 \text{ A}}$, and

(b) $\mathcal{P} = (\Delta V)I = (120 \text{ V})(6.25 \text{ A}) = \boxed{750 \text{ W}}$

18.40 (a) The current drawn by each appliance operating separately is

Coffee Maker: $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{1\,200 \text{ W}}{120 \text{ V}} = \boxed{10 \text{ A}}$

Toaster: $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{1\,100 \text{ W}}{120 \text{ V}} = \boxed{9.2 \text{ A}}$

Waffle Maker: $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{1\,400 \text{ W}}{120 \text{ V}} = \boxed{12 \text{ A}}$

(b) If the three appliances are operated simultaneously, they will draw a total current of $I_{\text{total}} = (10 + 9.2 + 12) \text{ A} = 31 \text{ A}$.

(c) $\boxed{\text{No.}}$ The total current required exceeds the limit of the circuit breaker, so they cannot be operated simultaneously. In fact, with a 15 A limit, no two of these appliances could be operated at the same time without tripping the breaker.

18.41 (a) The area of each surface of this axon membrane is

$$A = L(2\pi r) = (0.10 \text{ m})\left[2\pi(10 \times 10^{-6} \text{ m})\right] = 2\pi \times 10^{-6} \text{ m}^2$$

and the capacitance is

$$C = \kappa \epsilon_0 \frac{A}{d} = 3.0(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)\left(\frac{2\pi \times 10^{-6} \text{ m}^2}{1.0 \times 10^{-8} \text{ m}}\right) = 1.67 \times 10^{-8} \text{ F}$$

In the resting state, the charge on the outer surface of the membrane is

$$Q_i = C(\Delta V)_i = (1.67 \times 10^{-8} \text{ F})(70 \times 10^{-3} \text{ V}) = 1.17 \times 10^{-9} \text{ C} \rightarrow \boxed{1.2 \times 10^{-9} \text{ C}}$$

The number of potassium ions required to produce this charge is

$$N_{\text{K}^+} = \frac{Q_i}{e} = \frac{1.17 \times 10^{-9} \text{ C}}{1.6 \times 10^{-19} \text{ C}} = \boxed{7.3 \times 10^9 \text{ K}^+ \text{ ions}}$$

and the charge per unit area on this surface is

$$\sigma = \frac{Q_i}{A} = \frac{1.17 \times 10^{-9} \text{ C}}{2\pi \times 10^{-6} \text{ m}^2}\left(\frac{1 \text{ } e}{1.6 \times 10^{-19} \text{ C}}\right)\left(\frac{10^{-20} \text{ m}^2}{1 \text{ Å}^2}\right) = \frac{1 \text{ } e}{8.6 \times 10^4 \text{ Å}^2} = \boxed{\frac{1 \text{ } e}{(290 \text{ Å})^2}}$$

This corresponds to a low charge density of one electronic charge per square of side 290 Å, compared to a normal atomic spacing of one atom per several Å^2.

(b)   In the resting state, the net charge on the inner surface of the membrane is $-Q_i = -1.17 \times 10^{-9}$ C, and the net positive charge on this surface in the excited state is

$$Q_f = C(\Delta V)_f = \left(1.67 \times 10^{-8} \text{ F}\right)\left(+30 \times 10^{-3} \text{ V}\right) = +5.0 \times 10^{-10} \text{ C}$$

The total positive charge which must pass through the membrane to produce the excited state is therefore

$$\Delta Q = Q_f - Q_i$$

$$= +5.0 \times 10^{-10} \text{ C} - \left(-1.17 \times 10^{-9} \text{ C}\right) = 1.67 \times 10^{-9} \text{ C} \rightarrow \boxed{1.7 \times 10^{-9} \text{ C}}$$

corresponding to

$$N_{Na^+} = \frac{\Delta Q}{e} = \frac{1.67 \times 10^{-9} \text{ C}}{1.6 \times 10^{-19} \text{ C}/Na^+ \text{ ion}} = \boxed{1.0 \times 10^{10} \text{ Na}^+ \text{ ions}}$$

(c)   If the sodium ions enter the axon in a time of $\Delta t = 2.0$ ms, the average current is

$$I = \frac{\Delta Q}{\Delta t} = \frac{1.67 \times 10^{-9} \text{ C}}{2.0 \times 10^{-3} \text{ s}} = 8.3 \times 10^{-7} \text{ A} = \boxed{0.83 \ \mu A}$$

(d)   When the membrane becomes permeable to sodium ions, the initial influx of sodium ions neutralizes the capacitor with no required energy input. The energy input required to charge the now neutral capacitor to the potential difference of the excited state is

$$W = \frac{1}{2}C(\Delta V)_f^2 = \frac{1}{2}\left(1.67 \times 10^{-8} \text{ F}\right)\left(30 \times 10^{-3} \text{ V}\right)^2 = \boxed{7.5 \times 10^{-12} \text{ J}}$$

18.42   The capacitance of the 10 cm length of axon was found to be $C = 1.67 \times 10^{-8}$ F in the solution of Problem 18.41.

(a)   When the membrane becomes permeable to potassium ions, these ions flow out of the axon with no energy input required until the capacitor is neutralized. To maintain this outflow of potassium ions and charge the now neutral capacitor to the resting action potential requires an energy input of

$$W = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\left(1.67 \times 10^{-8} \text{ F}\right)\left(70 \times 10^{-3} \text{ V}\right)^2 = \boxed{4.1 \times 10^{-11} \text{ J}}$$

(b)   As found in the solution of Problem 18.41, the charge on the inner surface of the membrane in the resting state is -1.17×10^{-9} C and the charge on this surface in the excited state is $+5.0 \times 10^{-10}$ C. Thus, the positive charge which must flow out of the axon as it goes from the excited state to the resting state is

$$\Delta Q = 5.0 \times 10^{-10} \text{ C} + 1.17 \times 10^{-9} \text{ C} = 1.67 \times 10^{-9} \text{ C}$$

and the average current during the 3.0 ms required to return to the resting state is

$$I = \frac{\Delta Q}{\Delta t} = \frac{1.67 \times 10^{-9} \text{ C}}{3.0 \times 10^{-3} \text{ s}} = 5.6 \times 10^{-7} \text{ A} = \boxed{0.56 \ \mu A}$$

18.43   From Figure 18.28, the duration of an action potential pulse is 4.5 ms. From the solution of Problem 18.41, the energy input required to reach the excited state is $W_1 = 7.5 \times 10^{-12}$ J. The energy input required during the return to the resting state is found in Problem 18.42 to be $W_2 = 4.1 \times 10^{-11}$ J. Therefore, the average power input required during an action potential pulse is

$$\mathcal{P} = \frac{W_{total}}{\Delta t} = \frac{W_1 + W_2}{\Delta t} = \frac{7.5 \times 10^{-12} \text{ J} + 4.1 \times 10^{-11} \text{ J}}{4.5 \times 10^{-3} \text{ s}} = 1.1 \times 10^{-8} \text{ W} = \boxed{11 \text{ nW}}$$

18.44   <u>Using a single resistor</u> → 3 distinct values: $R_1 = 2.0 \ \Omega$, $R_2 = 4.0 \ \Omega$, $R_3 = 6.0 \ \Omega$

<u>2 resistors in Series</u> → 2 additional distinct values: $R_4 = 2.0 \ \Omega + 6.0 \ \Omega = 8.0 \ \Omega$, and $R_5 = 4.0 \ \Omega + 6.0 \ \Omega = 10 \ \Omega$. Note: $2.0 \ \Omega$ and $4.0 \ \Omega$ in series duplicates R_3 above.

<u>2 resistors in Parallel</u> → 3 additional distinct values:

$R_6 = 2.0 \ \Omega$ and $4.0 \ \Omega$ in parallel $= 1.3 \ \Omega$

$R_7 = 2.0 \ \Omega$ and $6.0 \ \Omega$ in parallel $= 1.5 \ \Omega$

$R_8 = 4.0 \ \Omega$ and $6.0 \ \Omega$ in parallel $= 2.4 \ \Omega$

<u>3 resistors in Series</u> → 1 additional distinct value:

$R_9 = 2.0 \ \Omega + 4.0 \ \Omega + 6.0 \ \Omega = 12 \ \Omega$

<u>3 resistors in Parallel</u> → 1 additional distinct value:

$R_{10} = 2.0 \ \Omega, \ 4.0 \ \Omega,$ and $6.0 \ \Omega$ in parallel $= 1.1 \ \Omega$

<u>1 resistor in Parallel with Series combination of the other 2:</u> → 3 additional values:

$R_{11} = \left(R_p = 2.0 \ \Omega, \ 4.0 \ \Omega, \text{ and } 6.0 \ \Omega \text{ in series} \right) = 1.7 \ \Omega$

$R_{12} = \left(R_p = 4.0 \ \Omega, \ 2.0 \ \Omega, \text{ and } 6.0 \ \Omega \text{ in series} \right) = 2.7 \ \Omega$

$R_{12} = \left(R_p = 6.0 \ \Omega, \ 2.0 \ \Omega, \text{ and } 4.0 \ \Omega \text{ in series} \right) = 3.0 \ \Omega$
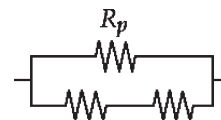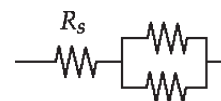

<u>1 resistor in Series with Parallel combination of the other 2:</u> → 3 additional values:

$R_{14} = \left(R_s = 2.0 \ \Omega, \ 4.0 \ \Omega \text{ and } 6.0 \ \Omega \text{ in parallel} \right) = 4.4 \ \Omega$
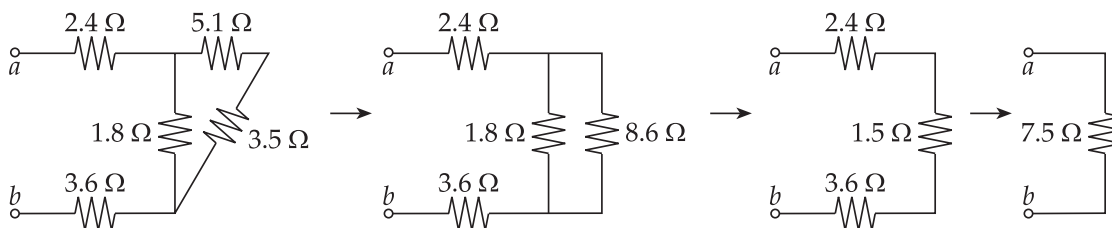
$R_{15} = \left(R_s = 4.0 \ \Omega, \ 2.0 \ \Omega, \text{ and } 6.0 \ \Omega \text{ in parallel} \right) = 5.5 \ \Omega$

$R_{16} = \left(R_s = 6.0 \ \Omega, \ 2.0 \ \Omega, \text{ and } 4.0 \ \Omega \text{ in Parallel} \right) = 7.3 \ \Omega$

Thus, 16 distinct values of resistance are possible using these three resistors.

18.45   The resistive network between a an b reduces, in the stages shown below, to an equivalent resistance of $R_{eq} = \boxed{7.5 \ \Omega}$.

18.46   (a) $R = \dfrac{\Delta V}{I} = \dfrac{6.0 \text{ V}}{3.0 \times 10^{-3} \text{ A}} = 2.0 \times 10^3 \ \Omega = \boxed{2.0 \text{ k}\Omega}$

(b)   The resistance in the circuit consists of a series combination with an equivalent resistance of $R_{eq} = 2.0 \text{ k}\Omega + 3.0 \text{ k}\Omega = 5.0 \text{ k}\Omega$. The emf of the battery is then

$\mathcal{E} = IR_{eq} = \left(3.0 \times 10^{-3} \text{ A} \right)\left(5.0 \times 10^3 \ \Omega \right) = \boxed{15 \text{ V}}$
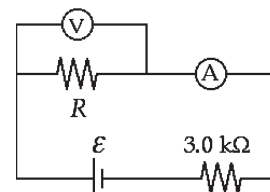
(c) $\Delta V_3 = IR_3 = \left(3.0 \times 10^{-3} \text{ A}\right)\left(3.0 \times 10^3 \text{ } \Omega\right) = \boxed{9.0 \text{ V}}$

(d)   In this solution, $\boxed{\text{we have assumed that we have ideal devices in the circuit}}$. In particular, we have assumed that the battery has negligible internal resistance, the voltmeter has an extremely large resistance and draws negligible current, and the ammeter has an extremely low resistance and a negligible voltage drop across it.

18.47   (a)   The resistors combine to an equivalent resistance of $R_{eq} = \boxed{15 \text{ } \Omega}$ as shown.
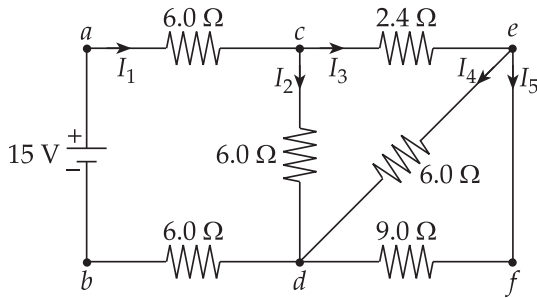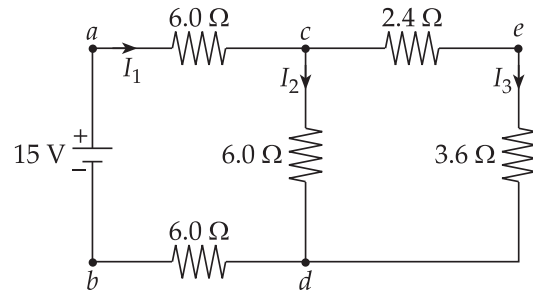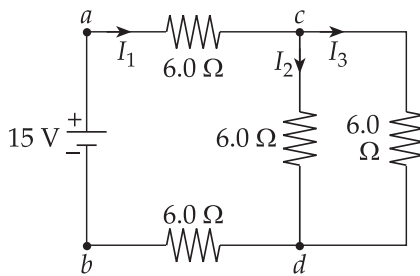

Figure 1   **Figure 2**

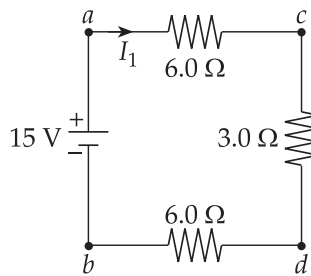**Figure 3**   **Figure 4**   **Figure 5**
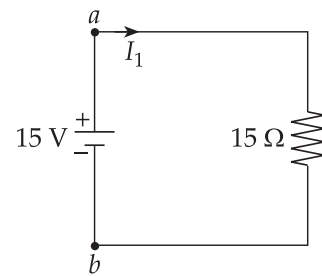
(b)   From Figure 5, $I_1 = \dfrac{\Delta V_{ab}}{R_{eq}} = \dfrac{15 \text{ V}}{15 \text{ } \Omega} = \boxed{1.0 \text{ A}}$

Then, from Figure 4,

$$\Delta V_{ac} = \Delta V_{db} = I_1\left(6.0 \text{ } \Omega\right) = 6.0 \text{ V} \text{ and } \Delta V_{cd} = I_1\left(3.0 \text{ } \Omega\right) = 3.0 \text{ V}$$

From Figure 3, $I_2 = I_3 = \dfrac{\Delta V_{cd}}{6.0 \text{ } \Omega} = \dfrac{3.0 \text{ V}}{6.0 \text{ } \Omega} = \boxed{0.50 \text{ A}}$

From Figure 2, $\Delta V_{ed} = I_3\left(3.6 \text{ } \Omega\right) = 1.8 \text{ V}$

Then, from Figure 1, $I_4 = \dfrac{\Delta V_{ed}}{6.0 \text{ } \Omega} = \dfrac{1.8 \text{ V}}{6.0 \text{ } \Omega} = \boxed{0.30 \text{ A}}$

and $I_5 = \dfrac{\Delta V_{fd}}{9.0 \text{ } \Omega} = \dfrac{\Delta V_{ed}}{9.0 \text{ } \Omega} = \dfrac{1.8 \text{ V}}{9.0 \text{ } \Omega} = \boxed{0.20 \text{ A}}$

(c)   From Figure 2, $\Delta V_{ce} = I_3\left(2.4 \text{ } \Omega\right) = \boxed{1.2 \text{ V}}$. All the other needed potential differences were calculated above in part (b). The results were

$$\Delta V_{ac} = \Delta V_{db} = \boxed{6.0 \text{ V}} \text{ ; } \Delta V_{cd} = \boxed{3.0 \text{ V}} \text{ ; and } \Delta V_{fd} = \Delta V_{ed} = \boxed{1.8 \text{ V}}$$

(d) The power dissipated in each resistor is found from $\mathcal{P} = (\Delta V)^2 / R$ with the following results:

$$\mathcal{P}_{ac} = \frac{(\Delta V)^2_{ac}}{R_{ac}} = \frac{(6.0\text{ V})^2}{6.0\ \Omega} = \boxed{6.0\text{ W}} \qquad \mathcal{P}_{ce} = \frac{(\Delta V)^2_{ce}}{R_{ce}} = \frac{(1.2\text{ V})^2}{2.4\ \Omega} = \boxed{0.60\text{ W}}$$

$$\mathcal{P}_{ed} = \frac{(\Delta V)^2_{ed}}{R_{ed}} = \frac{(1.8\text{ V})^2}{6.0\ \Omega} = \boxed{0.54\text{ W}} \qquad \mathcal{P}_{fd} = \frac{(\Delta V)^2_{fd}}{R_{fd}} = \frac{(1.8\text{ V})^2}{9.0\ \Omega} = \boxed{0.36\text{ W}}$$

$$\mathcal{P}_{cd} = \frac{(\Delta V)^2_{cd}}{R_{cd}} = \frac{(3.0\text{ V})^2}{6.0\ \Omega} = \boxed{1.5\text{ W}} \qquad \mathcal{P}_{db} = \frac{(\Delta V)^2_{db}}{R_{db}} = \frac{(6.0\text{ V})^2}{6.0\ \Omega} = \boxed{6.0\text{ W}}$$

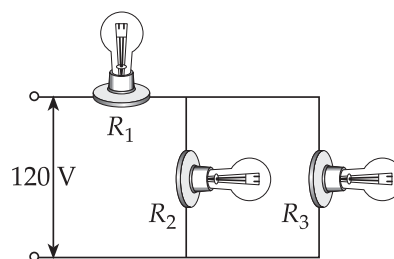**18.48** (a) From $\mathcal{P} = (\Delta V)^2 / R$, the resistance of each of the three bulbs is given by

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{60.0\text{ W}} = 240\ \Omega$$

As connected, the parallel combination of R_2 and R_3 is in series with R_1. Thus, the equivalent resistance of the circuit is

$$R_{eq} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} \right)^{-1} = 240\ \Omega + \left(\frac{1}{240\ \Omega} + \frac{1}{240\ \Omega} \right)^{-1} = 360\ \Omega$$

The total power delivered to the circuit is

$$\mathcal{P} = \frac{(\Delta V)^2}{R_{eq}} = \frac{(120\text{ V})^2}{360\ \Omega} = \boxed{40.0\text{ W}}$$

(b) The current supplied by the source is $I = \dfrac{\Delta V}{R_{eq}} = \dfrac{120\text{ V}}{360\ \Omega} = \dfrac{1}{3}$ A. Thus, the potential difference across R_1 is

$$(\Delta V)_1 = I\,R_1 = \left(\frac{1}{3}\text{ A} \right)(240\ \Omega) = \boxed{80.0\text{ V}}$$

The potential difference across the parallel combination of R_2 and R_3 is then

$$(\Delta V)_2 = (\Delta V)_3 = (\Delta V)_{\text{source}} - (\Delta V)_1 = 120\text{ V} - 80.0\text{ V} = \boxed{40.0\text{ V}}$$

18.49 (a) From $\mathcal{E} = I(r + R_{\text{load}})$, the current supplied when the headlights are the entire load is

$$I = \frac{\mathcal{E}}{r + R_{\text{load}}} = \frac{12.6\text{ V}}{(0.080 + 5.00)\ \Omega} = 2.48\text{ A}$$

The potential difference across the headlights is then

$$\Delta V = I\,R_{\text{load}} = (2.48\text{ A})(5.00\ \Omega) = \boxed{12.4\text{ V}}$$

(b)   The starter motor connects in parallel with the headlights. If I_{hl} is the current supplied to the headlights, the total current delivered by the battery is $I = I_{hl} + 35.0$ A.

The terminal potential difference of the battery is $\Delta V = \mathcal{E} - I r$, so the total current is $I = (\mathcal{E} - \Delta V)/r$ while the current to the headlights is $I_{hl} = \Delta V/5.00$ Ω. Thus, $I = I_{hl} + 35.0$ A becomes

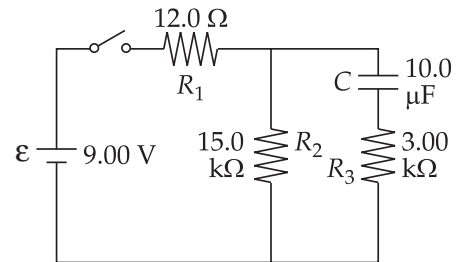$$\frac{\mathcal{E} - \Delta V}{r} = \frac{\Delta V}{5.00\ \Omega} + 35.0\ \text{A}$$

which yields

$$\Delta V = \frac{\mathcal{E} - (35.0\ \text{A})r}{1 + r/(5.00\ \Omega)} = \frac{12.6\ \text{V} - (35.0\ \text{A})(0.080\ \Omega)}{1 + (0.080\ \Omega)/(5.00\ \Omega)} = \boxed{9.65\ \text{V}}$$

18.50   (a)   After steady-state conditions have been reached, there is no current in the branch containing the capacitor.

Thus, for R_3: $\boxed{I_{R_3} = 0\ \text{(steady-state)}}$

For the other two resistors, the steady-state current is simply determined by the 9.00-V emf across the 12.0-kΩ and 15.0-kΩ resistors in series:

For R_1 and R_2:

$$I_{(R_1 + R_2)} = \frac{\mathcal{E}}{R_1 + R_2} = \frac{9.00\ \text{V}}{(12.0\ \text{k}\Omega + 15.0\ \text{k}\Omega)} = \boxed{333\ \mu\text{A}\ \text{(steady-state)}}$$

(b)   When the steady state has been reached, the potential difference across C is the same as the potential difference across R_2 because there is no change in potential across R_3. Therefore, the charge on the capacitor is

$$Q = C(\Delta V)_{R_2}$$

$$= C(I R_2) = (10.0\ \mu\text{F})(333 \times 10^{-6}\ \text{A})(15.0 \times 10^3\ \Omega) = \boxed{50.0\ \mu\text{C}}$$

18.51   (a)   When switch S is open, all three bulbs are in series and the equivalent resistance is $R_{eq}^{open} = R + R + R = \boxed{3R}$.

When the switch is closed, bulb C is shorted across and no current will flow through that bulb. This leaves bulbs A and B in series with an equivalent resistance of $R_{eq}^{closed} = R + R = \boxed{2R}$.
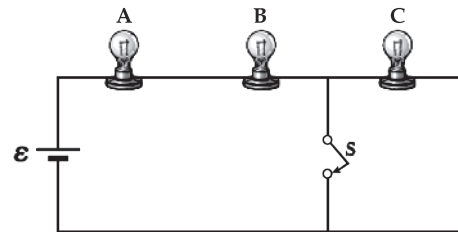
(b)   With the switch open, the power delivered by the battery is $\mathcal{P}_{open} = \dfrac{\mathcal{E}^2}{R_{eq}^{open}} = \boxed{\dfrac{\mathcal{E}^2}{3R}}$, and with the switch closed, $\mathcal{P}_{closed} = \mathcal{E}^2/R_{eq}^{closed} = \boxed{\mathcal{E}^2/2R}$.

(c)   When the switch is open, the three bulbs have equal brightness. When S is closed, $\boxed{\text{bulb C goes out, while A and B remain equal at a greater brightness}}$ than they had when the switch was open.

18.52 With the switch open, the circuit may be reduced as follows:
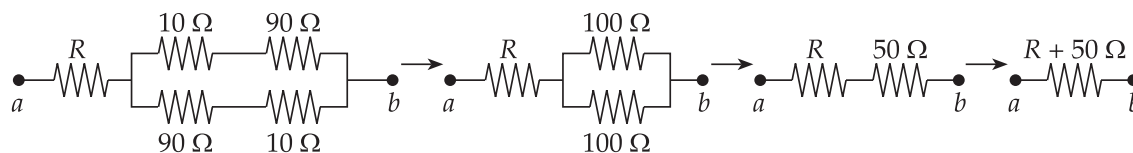
With the switch closed, the circuit reduces as shown below:
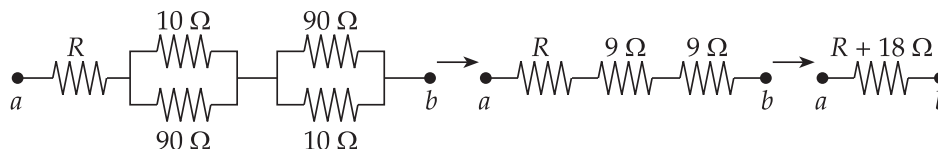
Since the equivalent resistance with the switch closed is one-half that when the switch is open, we have

$$R + 18\ \Omega = \frac{1}{2}(R + 50\ \Omega),\ \text{which yields}\ \boxed{R = 14\ \Omega}$$

18.53 When a generator with emf $\mathcal{E}$ and internal resistance r supplies current I, its terminal voltage is $\Delta V = \mathcal{E} - I\,r$.

If $\Delta V = 110\ \text{V}$ when $I = 10.0\ \text{A}$, then $\qquad\qquad 110\ \text{V} = \mathcal{E} - (10.0\ \text{A})r$ **[1]**

Given that $\Delta V = 106\ \text{V}$ when $I = 30.0\ \text{A}$, yields $\qquad 106\ \text{V} = \mathcal{E} - (30.0\ \text{A})r$ **[2]**

Subtracting Equation [2] from [1] gives $4.0\ \text{V} = (20.0\ \text{A})r$, or $\boxed{r = 0.20\ \Omega}$. Then, Equation [1] yields $\boxed{\mathcal{E} = 112\ \text{V}}$.

18.54 At time t, the charge on the capacitor will be $Q = Q_{\max}\left(1 - e^{-t/\tau}\right)$, where

$$\tau = RC = \left(2.0 \times 10^{6}\ \Omega\right)\left(3.0 \times 10^{-6}\ \text{F}\right) = 6.0\ \text{s}$$

When $Q = 0.90\,Q_{\max}$, this gives $0.90 = 1 - e^{-t/\tau}$

or $\qquad e^{-t/\tau} = 0.10 \qquad$ Thus, $-\dfrac{t}{\tau} = \ln(0.10)$

giving $t = -(6.0\ \text{s})\ln(0.10) = \boxed{14\ \text{s}}$

18.55 (a) For the first measurement, the equivalent circuit is as shown in Figure 1. From this,

$$R_{ab} = R_1 = R_y + R_y = 2\,R_y$$

so $\quad R_y = \dfrac{1}{2}R_1$ **[1]**

For the second measurement, the equivalent circuit is shown in Figure 2. This gives

$$R_{ac} = R_2 = \frac{1}{2}R_y + R_x$$ **[2]**

Substitute [1] into [2] to obtain

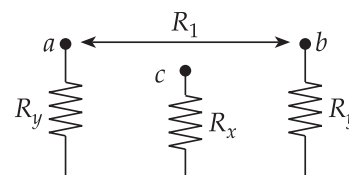$$R_2 = \frac{1}{2}\left(\frac{1}{2}R_1\right) + R_x,\ \text{or}\ \boxed{R_x = R_2 - \frac{1}{4}R_1}$$
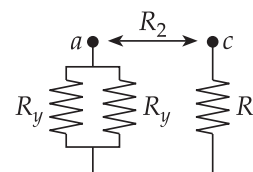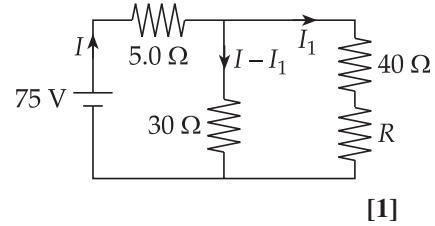

Figure 1

Figure 2

(b)   If $R_1 = 13\ \Omega$ and $R_2 = 6.0\ \Omega$, then $\boxed{R_x = 2.8\ \Omega}$.

Since this exceeds the limit of $2.0\ \Omega$, the antenna is $\boxed{\text{inadequately grounded}}$.

18.56   Assume a set of currents as shown in the circuit diagram at the right. Applying Kirchhoff's loop rule to the leftmost loop gives

$$+75 - (5.0)I - (30)(I - I_1) = 0$$

or $7I - 6I_1 = 15$ **[1]**

For the rightmost loop, the loop rule gives

$$-(40 + R)I_1 + (30)(I - I_1) = 0, \text{ or } I = \left(\frac{7}{3} + \frac{R}{30}\right)I_1$$ **[2]**

Substituting Equation [1] into [2] and simplifying gives

$$310\,I_1 + 7\left(I_1 R\right) = 450$$ **[3]**

Also, it is known that $P_R = I_1^2 R = 20$ W, so $I_1 R = \dfrac{20\ \text{W}}{I_1}$ **[4]**

Substitution of Equation [4] into [3] yields

$$310\,I_1 + \frac{140}{I_1} = 450 \qquad \text{or} \qquad 310\,I_1^2 - 450\,I_1 + 140 = 0$$

Using the quadratic formula: $I_1 = \dfrac{-(-450) \pm \sqrt{(-450)^2 - 4(310)(140)}}{2(310)}$,

yielding $I_1 = 1.0$ A   and $I_1 = 0.452$ A. Then, from $R = \dfrac{20\ \text{W}}{I_1^2}$, we find two possible values for the resistance R. These are: $\boxed{R = 20\ \Omega \quad \text{or} \quad R = 98\ \Omega}$

18.57   When connected in series, the equivalent resistance is $R_{\text{eq}} = R_1 + R_2 + \cdots + R_n = n\,R$. Thus, the current is $I_s = (\Delta V)/R_{\text{eq}} = (\Delta V)/n\,R$, and the power consumed by the series configuration is

$$P_s = I_s^2\,R_{\text{eq}} = \frac{(\Delta V)^2}{(n\,R)^2}(n\,R) = \frac{(\Delta V)^2}{n\,R}$$

For the parallel connection, the power consumed by each individual resistor is $P_1 = \dfrac{(\Delta V)^2}{R}$, and the total power consumption is

$$P_p = nP_1 = \frac{n(\Delta V)^2}{R}$$

Therefore, $\dfrac{P_s}{P_p} = \dfrac{(\Delta V)^2}{n\,R} \cdot \dfrac{R}{n(\Delta V)^2} = \dfrac{1}{n^2}$ or $\boxed{P_s = \dfrac{1}{n^2}P_p}$

18.58 Consider a battery of emf $\mathcal{E}$ connected between points a and b as shown. Applying Kirchhoff's loop rule to loop *acbea* gives

$$-(1.0)I_1 - (1.0)(I_1 - I_3) + \mathcal{E} = 0$$

or $I_3 = 2I_1 - \mathcal{E}$ **[1]**

Applying the loop rule to loop *adbea* gives

$$-(3.0)I_2 - (5.0)(I_2 + I_3) + \mathcal{E} = 0$$

or $8I_2 + 5I_3 = \mathcal{E}$ **[2]**

For loop *adca*, the loop rule yields

$$-(3.0)I_2 + (1.0)I_3 + (1.0)I_1 = 0 \text{ or } I_2 = \frac{I_1 + I_3}{3}$$ **[3]**

Substituting Equation [1] into [3] gives $I_2 = I_1 - \mathcal{E}/3$.   **[4]**

Now, substitute Equations [1] and [4] into [2] to obtain $18I_1 = \frac{26}{3}\mathcal{E}$, which reduces to $I_1 = \frac{13}{27}\mathcal{E}$.

Then, Equation [4] gives $I_2 = \left[\frac{13}{27} - \frac{9}{27}\right]\mathcal{E} = \frac{4}{27}\mathcal{E}$, and [1] yields $I_3 = -\frac{1}{27}\mathcal{E}$.

Then, applying Kirchhoff's junction rule at junction a gives

$$I = I_1 + I_2 = \frac{13}{27}\mathcal{E} + \frac{4}{27}\mathcal{E} = \frac{17}{27}\mathcal{E}. \text{ Therefore, } R_{ab} = \frac{\mathcal{E}}{I} = \frac{\mathcal{E}}{(17\mathcal{E}/27)} = \boxed{\frac{27}{17}\ \Omega}.$$

18.59 (a) and (b). With R the value of the load resistor, the current in a series circuit composed of a 12.0 V battery, an internal resistance of $10.0\ \Omega$, and a load resistor is
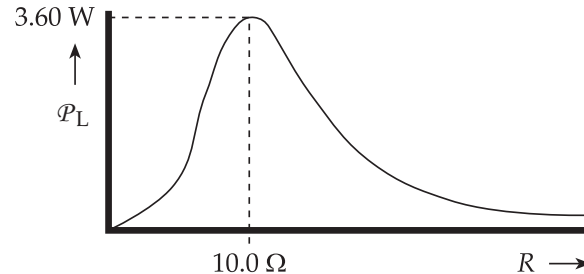
$$I = \frac{12.0\ \text{V}}{R + 10.0\ \Omega}$$

and the power delivered to the load resistor is
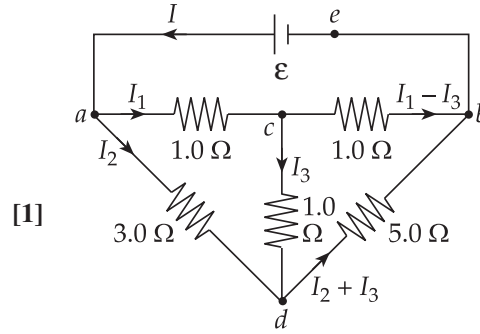
$$\mathcal{P}_L = I^2 R = \boxed{\frac{(144\ \text{V}^2)R}{(R + 10.0\ \Omega)^2}}$$

Some typical data values for the graph are

$R\ (\Omega)$	$\mathcal{P}_L(\text{W})$
1.00	1.19
5.00	3.20
10.0	3.60
15.0	3.46
20.0	3.20
25.0	2.94
30.0	2.70

The curve peaks at $\mathcal{P}_L = 3.60\ \text{W}$ at a load resistance of $R = 10.0\ \Omega$.

18.60 The total resistance in the circuit is

$$R = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} = \left(\frac{1}{2.0 \text{ k}\Omega} + \frac{1}{3.0 \text{ k}\Omega} \right)^{-1} = 1.2 \text{ k}\Omega$$

and the total capacitance is $C = C_1 + C_2 = 2.0 \text{ }\mu\text{F} + 3.0 \text{ }\mu\text{F} = 5.0 \text{ }\mu\text{F}$.

Thus, $Q_{\text{max}} = C\,\mathcal{E} = (5.0 \text{ }\mu\text{F})(120 \text{ V}) = 600 \text{ }\mu\text{C}$

and $\tau = RC = (1.2 \times 10^3 \text{ }\Omega)(5.0 \times 10^{-6} \text{ F}) = 6.0 \times 10^{-3} \text{ s} = \dfrac{6.0 \text{ s}}{1\,000}$

The total stored charge at any time t is then

$$Q = Q_1 + Q_2 = Q_{\text{max}}\left(1 - e^{-t/\tau}\right) \quad \text{or} \quad Q_1 + Q_2 = (600 \text{ }\mu\text{C})\left(1 - e^{-1\,000\,t/6.0\text{ s}}\right) \qquad \text{[1]}$$

Since the capacitors are in parallel with each other, the same potential difference exists across both at any time.

Therefore, $\qquad (\Delta V)_C = \dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2}, \qquad \text{or} \qquad Q_2 = \left(\dfrac{C_2}{C_1} \right)Q_1 = 1.5\,Q_1 \qquad \text{[2]}$
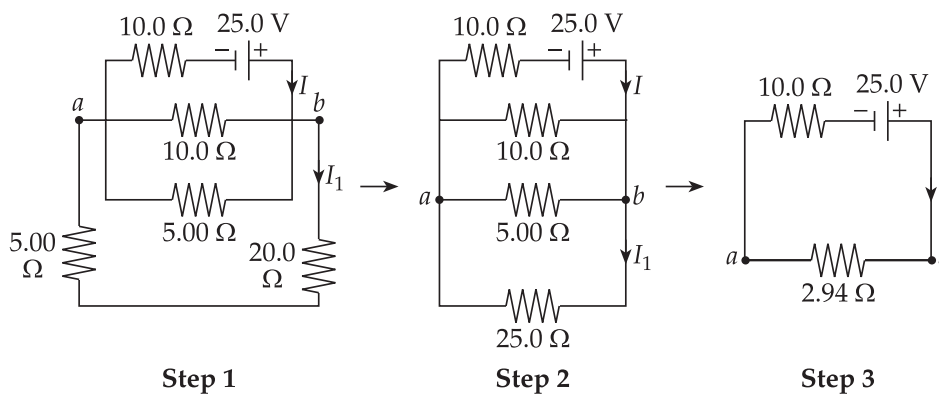
Substituting Equation [2] into [1] gives

$$2.5Q_1 = (600 \text{ }\mu\text{C})\left(1 - e^{-1\,000\,t/6.0\text{ s}}\right) \qquad \text{and} \qquad Q_1 = \boxed{(240 \text{ }\mu\text{C})\left(1 - e^{-1\,000\,t/6.0\text{ s}}\right)}$$

Then, Equation [2] yields

$$Q_2 = 1.5(240 \text{ }\mu\text{C})\left(1 - e^{-1\,000\,t/6.0\text{ s}}\right) = \boxed{(360 \text{ }\mu\text{C})\left(1 - e^{-1\,000\,t/6.0\text{ s}}\right)}$$

18.61 (a) Using the rules for combining resistors in series and parallel, the circuit reduces as shown below:
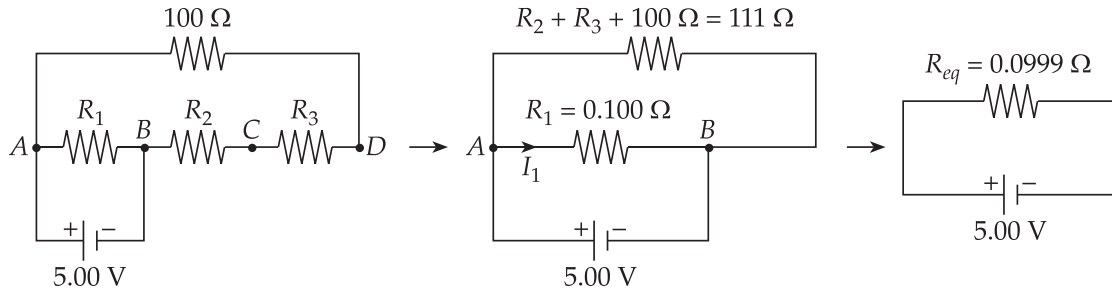

Step 1   **Step 2**   **Step 3**

From the figure of Step 3, observe that

$$I = \frac{25.0 \text{ V}}{12.94 \text{ A}} = 1.93 \text{ A} \qquad \text{and} \qquad \Delta V_{ab} = I(2.94 \text{ }\Omega) = (1.93 \text{ A})(2.94 \text{ }\Omega) = \boxed{5.68 \text{ V}}$$

(b) From the figure of Step 1, observe that $\quad I_1 = \dfrac{\Delta V_{ab}}{25.0 \text{ }\Omega} = \dfrac{5.68 \text{ V}}{25.0 \text{ }\Omega} = \boxed{0.227 \text{ A}}$
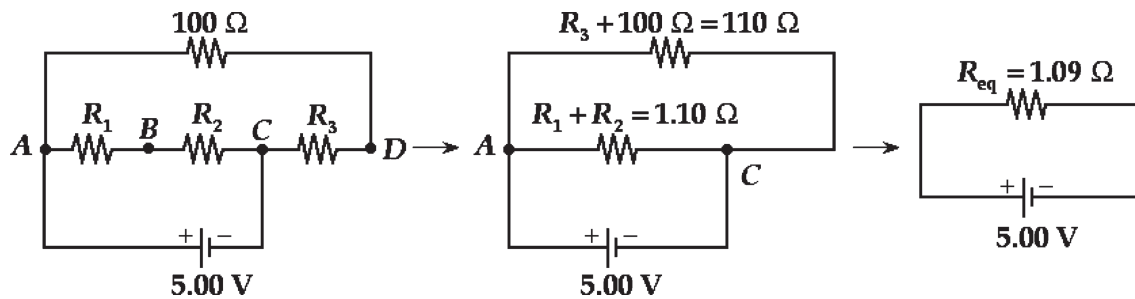
18.62 (a) When the power supply is connected to points A and B, the circuit reduces as shown below to an equivalent resistance of $R_{eq} = \boxed{0.099\ 9\ \Omega}$.

100 Ω

R_1 R_2 R_3

A B C D

5.00 V

$R_2 + R_3 + 100\ \Omega = 111\ \Omega$

$R_1 = 0.100\ \Omega$ B

A I_1

5.00 V

$R_{eq} = 0.0999\ \Omega$

5.00 V

From the center figure above, observe that $I_{R_1} = I_1 = \dfrac{5.00\ \text{V}}{0.100\ \Omega} = \boxed{50.0\ \text{A}}$

and $I_{R_2} = I_{R_3} = I_{100} = \dfrac{5.00\ \text{V}}{111\ \Omega} = 0.045\ 0\ \text{A} = \boxed{45.0\ \text{mA}}$

(b) When the power supply is connected to points A and C, the circuit reduces as shown below to an equivalent resistance of $R_{eq} = \boxed{1.09\ \Omega}$.

100 Ω

R_1 B R_2 C R_3

A D

5.00 V

$R_3 + 100\ \Omega = 110\ \Omega$

$R_1 + R_2 = 1.10\ \Omega$

A C

5.00 V

$R_{eq} = 1.09\ \Omega$

5.00 V

From the center figure above, observe that $I_{R_1} = I_{R_2} = I_1 = \dfrac{5.00\ \text{V}}{1.10\ \Omega} = \boxed{4.55\ \text{A}}$

and $I_{R_3} = I_{100} = \dfrac{5.00\ \text{V}}{110\ \Omega} = 0.045\ 5\ \text{A} = \boxed{45.5\ \text{mA}}$
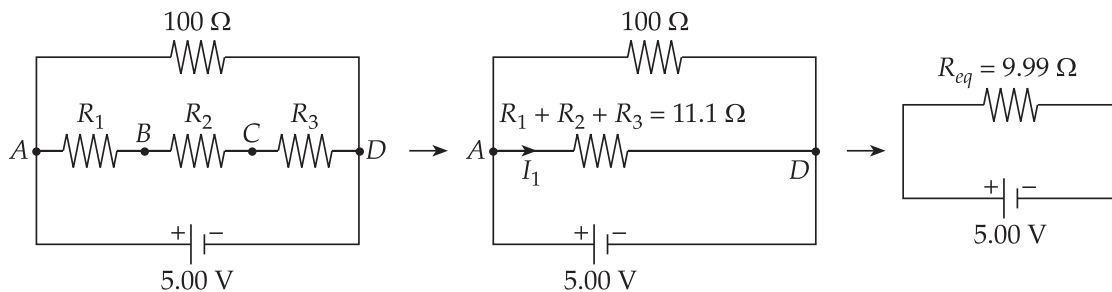
(c) When the power supply is connected to points A and D, the circuit reduces as shown below to an equivalent resistance of $R_{eq} = \boxed{9.99\ \Omega}$.

100 Ω

R_1 B R_2 C R_3

A D

5.00 V

100 Ω

$R_1 + R_2 + R_3 = 11.1\ \Omega$

A I_1 D

5.00 V

$R_{eq} = 9.99\ \Omega$

5.00 V

From the center figure above, observe that $I_{R_1} = I_{R_2} = I_{R_3} = I_1 = \dfrac{5.00\ \text{V}}{11.1\ \Omega} = \boxed{0.450\ \text{A}}$

and $I_{100} = \dfrac{5.00\ \text{V}}{100\ \Omega} = 0.050\ 0\ \text{A} = \boxed{50.0\ \text{mA}}$

18.63   In the circuit diagram at the right, note that all points labeled *a* are at the same potential and equivalent to each other. Also, all points labeled *c* are equivalent.

To determine the voltmeter reading, go from point *e* to point *d* along the path *ecd*, keeping track of all changes in potential to find:

$$\Delta V_{ed} = V_d - V_e = -4.50 \text{ V} + 6.00 \text{ V} = \boxed{+1.50 \text{ V}}$$

Apply Kirchhoff's loop rule around loop *abcfa* to find

$$-(6.00 \ \Omega)I + (6.00 \ \Omega)I_3 = 0 \quad \text{or} \quad I_3 = I \qquad \qquad \textbf{[1]}$$

Apply Kirchhoff's loop rule around loop *abcda* to find

$$-(6.00 \ \Omega)I + 6.00 \text{ V} - (10.0 \ \Omega)I_2 = 0 \qquad \text{or} \qquad I_2 = 0.600 \text{ A} - 0.600I \qquad \textbf{[2]}$$

Apply Kirchhoff's loop rule around loop *abcea* to find

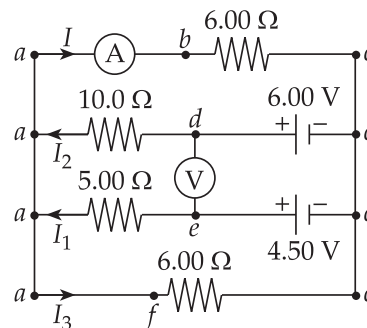$$-(6.00 \ \Omega)I + 4.50 \text{ V} - (5.00 \ \Omega)I_1 = 0 \qquad \text{or} \qquad I_1 = 0.900 \text{ A} - 1.20I \qquad \textbf{[3]}$$

Finally, apply Kirchhoff's junction rule at either point *a* or point *c* to obtain

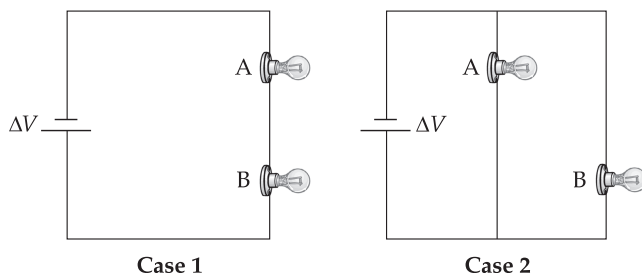$$I + I_3 = I_1 + I_2 \qquad \qquad \textbf{[4]}$$

Substitute Equations [1], [2], and [3] into Equation [4] to obtain the current through the ammeter. This gives

$$I + I = 0.900 \text{ A} - 1.20I + 0.600 \text{ A} - 0.600I$$

or $3.80I = 1.50 \text{ A}$ and $I = 1.50 \text{ A}/3.80 = \boxed{0.395 \text{ A}}$

18.64   In the figure given below, note that all bulbs have the same resistance, *R*.

(a)   In the series situation, Case 1, the same current I_1 flows through both bulbs. Thus, the same power, $\mathcal{P}_1 = I_1^2 R$, is supplied to each bulb. Since the brightness of a bulb is proportional to the power supplied to it, they will have the same brightness. We conclude that the $\boxed{\text{bulbs have the same current, power supplied, and brightness}}$.

(b)   In the parallel case, Case 2, the same potential difference ΔV is maintained across each of the bulbs. Thus, the $\boxed{\text{same current}}$ $I_2 = \Delta V/R$ will flow in each branch of this parallel circuit. This means that, again, the $\boxed{\text{same power}}$ $\mathcal{P}_2 = I_2^2 R$ is supplied to each bulb, and the two bulbs will have $\boxed{\text{equal brightness}}$.

(c) The total resistance of the single branch of the series circuit (Case 1) is $2R$. Thus, the current in this case is $I_1 = \Delta V/2R$. Note that this is one half of the current I_2 that flows through each bulb in the parallel circuit (Case 2). Since the power supplied is proportional to the square of the current, the power supplied to each bulb in Case 2 is four times that supplied to each bulb in Case 1. Thus, the bulbs in $\boxed{\text{Case 2}}$ are much brighter than those in Case 1.

(d) If either bulb goes out in Case 1, the only conducting path of the circuit is broken and all current ceases. Thus, in the series case, $\boxed{\text{the other bulb must also go out}}$. If one bulb goes out in Case 2, there is still a continuous conducting path through the other bulb. Neglecting any internal resistance of the battery, the battery continues to maintain the same potential difference ΔV across this bulb as was present when both bulbs were lit. Thus, in the parallel case, $\boxed{\text{the second bulb remains lit}}$ with $\boxed{\text{unchanged current and brightness}}$ when one bulb fails.

18.65 (a) The equivalent capacitance of this parallel combination is

$$C_{eq} = C_1 + C_2 = 3.00\ \mu\text{F} + 2.00\ \mu\text{F} = 5.00\ \mu\text{F}$$

When fully charged by a 12.0-V battery, the total stored charge before the switch is closed is

$$Q_0 = C_{eq}(\Delta V) = (5.00\ \mu\text{F})(12.0\ \text{V}) = 60.0\ \mu\text{C}$$

Once the switch is closed, the time constant of the resulting RC circuit is

$$\tau = RC_{eq} = (5.00 \times 10^2\ \Omega)(5.00\ \mu\text{F}) = 2.50 \times 10^{-3}\ \text{s} = 2.50\ \text{ms}$$

Thus, at $t = 1.00$ ms after closing the switch, the remaining total stored charge is

$$q = Q_0 e^{-t/\tau} = (60.0\ \mu\text{C})e^{-1.00\ \text{ms}/2.50\ \text{ms}} = (60.0\ \mu\text{C})e^{-0.400} = 40.2\ \mu\text{C}$$

The potential difference across the parallel combination of capacitors is then

$$\Delta V = \frac{q}{C_{eq}} = \frac{40.2\ \mu\text{C}}{5.00\ \mu\text{F}} = 8.04\ \text{V}$$

and the charge remaining on the 3.00 μF capacitor will be

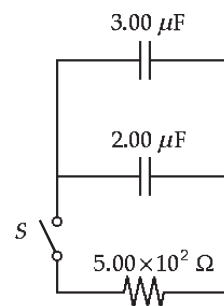$$q_3 = C_3(\Delta V) = (3.00\ \mu\text{F})(8.04\ \text{V}) = \boxed{24.1\ \mu\text{C}}$$

(b) The charge remaining on the 2.00 μF at this time is

$$q_2 = q - q_3 = 40.2\ \mu\text{C} - 24.1\ \mu\text{C} = \boxed{16.1\ \mu\text{C}}$$

or, alternately, $\quad q_2 = C_2(\Delta V) = (2.00\ \mu\text{F})(8.04\ \text{V}) = \boxed{16.1\ \mu\text{C}}$

(c) Since the resistor is in parallel with the capacitors, it has the same potential difference across it as do the capacitors at all times. Thus, Ohm's law gives

$$I = \frac{\Delta V}{R} = \frac{8.04\ \text{V}}{5.00 \times 10^2\ \Omega} = 1.61 \times 10^{-2}\ \text{A} = \boxed{16.1\ \text{mA}}$$

18.66 The resistor network shown at the right does not contain any obvious combinations of resistors in series or parallel. Thus, the method of replacing such combinations by single resistors to simplify the network and find the equivalent resistance is not useful here. Still, the problem is easily solved if one takes note of, and utilizes, the symmetry present in this network.
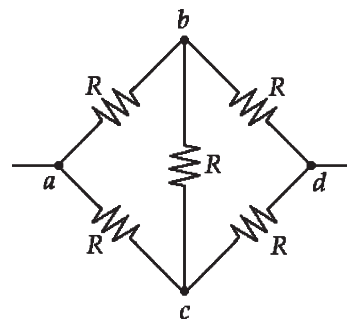
Imagine a battery of emf $\mathcal{E}$ connected to points a and d, supplying current I which enters the network at a and exits at d. When the current reaches a, it has two identical paths that it could follow. Since the network is totally symmetric about a horizontal line drawn from a to d, the current will split equally with $I/2$ flowing from a to b and $I/2$ flowing from a to c. This means $\Delta V_{ab} = \Delta V_{ac} = (I/2)R$, so the potential difference between points b and c will be zero. Therefore, no current will flow through the vertical resistor between these two points, and currents of $I/2$ follow each of the paths b to d and c to d. There, they merge forming current I again, which returns to the battery.

The potential difference between points a and d is $\mathcal{E} = IR_{eq}$. But it may also be written as

$\mathcal{E} = \Delta V_{ac} + \Delta V_{cd} = \left(\dfrac{I}{2}\right)R + \left(\dfrac{I}{2}\right)R = IR$. Since both are equal to the emf of the battery, we see that

$IR_{eq} = IR$, and conclude that the equivalent resistance of this network is $\boxed{R_{eq} = R}$.

18.67 (a) With 4.0×10^3 cells, each with an emf of 150 mV, connected in series, the total terminal potential difference is

$$\Delta V = \left(4.0 \times 10^3\right)\left(150 \times 10^{-3}\ \text{V}\right) = 6.0 \times 10^2\ \text{V}$$

When delivering a current of $I = 1.0$ A, the power output is

$$\mathcal{P} = I(\Delta V) = (1.0\ \text{A})\left(6.0 \times 10^2\ \text{V}\right) = \boxed{6.0 \times 10^2\ \text{W}}$$

(b) The energy released in one shock is

$$E_1 = \mathcal{P}(\Delta t)_1 = \left(6.0 \times 10^2\ \text{W}\right)\left(2.0 \times 10^{-3}\ \text{s}\right) = \boxed{1.2\ \text{J}}$$

(c) The energy released in 300 such shocks is $E_{total} = 300E_1 = 300(1.2\ \text{J}) = 3.6 \times 10^2\ \text{J}$. For a 1.0-kg object to be given a gravitational potential energy of this magnitude, the height the object must be lifted above the reference level is

$$h = \frac{PE_g}{mg} = \frac{3.6 \times 10^2\ \text{J}}{(1.0\ \text{kg})\left(9.80\ \text{m/s}^2\right)} = \boxed{37\ \text{m}}$$