17

Current and Resistance

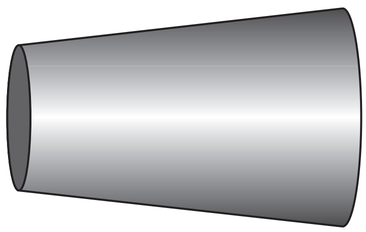
Clicker Questions

Question N1.01

Description: Developing an understanding of resistance and resistivity.

Question

An ohmic conductor is carrying a current. The cross-sectional area of the wire changes from one end of the wire to the other. Which of the following quantities vary along the wire?



A. The resistivity
B. The current
C. The current density
D. The electric field

1. A only
2. B only
3. C only
4. D only
5. A and B only
6. C and D only
7. A, B, C, and D
8. None of the above

Commentary

Purpose: To explore and relate concepts involved in electrical conduction.

Discussion: Since charge is conserved, all the current that enters one end of the "wire" must exit the other, so the current through any cross-section perpendicular to the wire's axis must be the same. Thus, quantity B does not vary.

The resistivity of a conductor is a property of the material itself (like density), and (unlike the *resistance*) does not depend upon the geometry of the conductor. So, quantity A does not vary. The current through the wire does not vary, but as the wire expands the current spreads out over a larger area (or is channeled through a smaller area, depending on the direction), so the current *density* (quantity C) will vary.

The microscopic form of Ohm's law is $\mathbf{J} = \mathbf{E}/\rho$, where **J** is the current density at a point in a conductor, ρ is its resistivity, and **E** is the electric field at that point. According to this, if the current density varies and the resistivity does not, then the electric field must also vary. So, quantity D will vary.

Thus, the best answer is (6).

Key Points:

- Conservation of charge requires that the current through a wire is the same through any cross-section of the wire, even if the wire's size or shape changes.
- Current density is current per unit area traveling through a conductor at a point; integrating the current density over an entire cross-section yields the total current.
- Electrical resistivity is an inherent property of a material, and does not depend on the amount or shape of the material.
- The current density at any point in a conductor is proportional to (and in the same direction as) the electric field at that point.

For Instructors Only

Students often find it easier to understand concepts in relation to other concepts. This question provides a good context for sorting out the distinction between current and current density, the distinction between resistance and resistivity, and the relationship between current density and electric field.

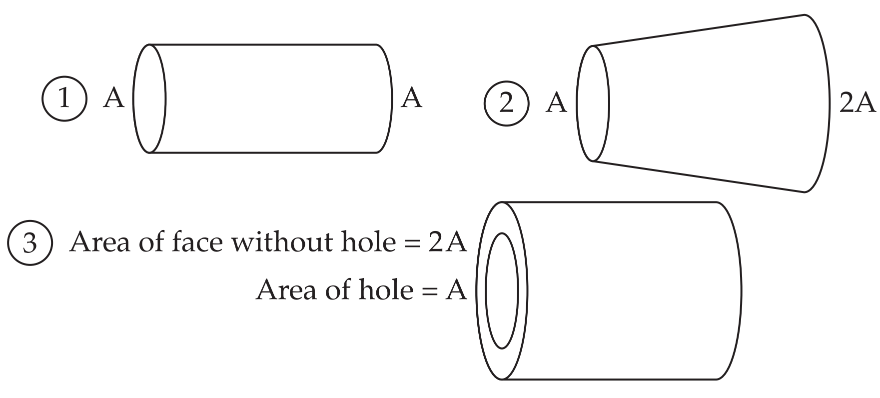
Students are more likely to be comfortable with the first three quantities than they are to understand the role of the electric field, since current flow is usually related to voltage differences.

Question N1.02

Description: Linking resistivity, resistance, and resistor geometry.

Question

Which object below has the lowest resistance? All three have length L and are made out of the same material.



1. #1
2. #2
3. #3
4. Both #1 and #3 have the same, and theirs is less than the resistance of #2.

Commentary

Purpose: To relate *resistance* to the geometry of the resistor.

Discussion: A conductor's *resistance* is a measure of how difficult it is for electrical current to flow through it. In general, the resistance of a wide wire is lower than the resistance of a narrow wire, much like water flows more easily through a wide pipe than through a narrow pipe. For a conductor of length L made out of a uniform material of resistivity ρ and having a constant cross-sectional area of A (i.e., it doesn't get wider or narrower along the direction current is flowing), resistance is given by $R = \rho L/A$.

Objects (1) and (3) both have the same length, material, and cross-sectional area, so they have the same resistance. Object (2) has a cross-section that increases from A to $2A$, so its resistance must be less than an object with constant cross-section A but less than one with constant cross-section $2A$. Thus, the best answer is (2).

Key Points:

- A uniform conductor's resistance depends on its length, cross-sectional area, and material. The *shape* of the cross-section doesn't matter.
- An object with non-uniform cross-sectional area will have a resistance greater than you would find if you just used its largest cross-section, and less than you would find if you just used its smallest cross-section.

For Instructors Only

You can analyze situation (2) in terms of the series resistance of successive slices, each a disk of increasing diameter. Analyzing (1) and (3) in terms of a large number of narrow wire-like strips in parallel can help convince students that the cross-sectional shape doesn't matter, only its area.

Analogies with water (or perhaps molasses) flowing through pipes can help students form an intuitive understanding of how resistance depends on shape, as well as of the rules for parallel and series resistances.

QUICK QUIZZES

1. (d). Negative charges moving in one direction are equivalent to positive charges moving in the opposite direction. Thus, I_a, I_b, I_c, and I_d are equivalent to the movement of 5, 3, 4, and 2 charges respectively, giving $I_d < I_b < I_c < I_a$.

2. (b). Under steady-state conditions, the current is the same in all parts of the wire. Thus, the drift velocity, given by $v_d = I/nqA$, is inversely proportional to the cross-sectional area.

3. (c), (d). Neither circuit (a) nor circuit (b) applies a difference in potential across the bulb. Circuit (a) has both lead wires connected to the same battery terminal. Circuit (b) has a low resistance path (a "short") between the two battery terminals as well as between the bulb terminals.

4. (b). The slope of the line tangent to the curve at a point is the reciprocal of the resistance at that point. Note that as ΔV increases, the slope (and hence $1/R$) increases. Thus, the resistance decreases.

5. (b). From Ohm's law, $R = \Delta V/I = 120\ \text{V}/6.00\ \text{A} = 20.0\ \Omega$.

6. (b). Consider the expression for resistance: $R = \rho \frac{L}{A} = \rho \frac{L}{\pi r^2}$. Doubling all linear dimensions increases the numerator of this expression by a factor of 2, but increases the denominator by a factor of 4. Thus, the net result is that the resistance will be reduced to one-half of its original value.

7. (a). The resistance of the shorter wire is half that of the longer wire. The power dissipated, $\mathcal{P} = (\Delta V)^2/R$ (and hence the rate of heating), will be greater for the shorter wire. Consideration of the expression $\mathcal{P} = I^2R$ might initially lead one to think that the reverse would be true. However, one must realize that the currents will not be the same in the two wires.

8. (b). $I_a = I_b > I_c = I_d > I_e = I_f$. Charges constituting the current I_a leave the positive terminal of the battery and then split to flow through the two bulbs; thus, $I_a = I_c + I_e$. Because the potential difference ΔV is the same across the two bulbs and because the power delivered to a device is $\mathcal{P} = I(\Delta V)$, the 60-W bulb with the higher power rating must carry the greater current, meaning that $I_c > I_e$. Because charge does not accumulate in the bulbs, all the charge flowing into a bulb from the left has to flow out on the right; consequently, $I_c = I_d$ and $I_e = I_f$. The two currents leaving the bulbs recombine to form the current back into the battery, $I_f + I_d = I_b$.

9. (a). The power dissipated by a resistor may be expressed as $\mathcal{P} = I^2R$, where I is the current carried by the resistor of resistance R. Since resistors connected in series carry the same current, the resistor having the largest resistance will dissipate the most power.

10. (c). Increasing the diameter of a wire increases the cross-sectional area. Thus, the cross-sectional area of A is greater than that of B, and from $R = \rho L/A$, we see that $R_A < R_B$. Since the power dissipated in a resistance may be expressed as $\mathcal{P} = (\Delta V)^2/R$, the wire having the smallest resistance dissipates the most power for a given potential difference.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The average current in a conductor is the charge passing a given point per unit time, or $I = \Delta Q/\Delta t = e(\Delta n)/\Delta t$, so the number of electrons passing this point per second is

$$\frac{\Delta n}{\Delta t} = \frac{I}{e} = \frac{1.6\ \text{C/s}}{1.6\times 10^{-19}\ \text{C/electron}} = 1.0\times 10^{19}\ \text{electron/s}$$

making (c) the correct choice.

2. The drift velocity of charge carriers in a conductor is given by $v_d = I/nqA$. Wires A and B carry the same current I. Also, they are made of the same material, so the density and charge of the charge carriers (n and q) are the same for the two wires. Since the cross-sectional area is $A = \pi r^2$, and $r_A = 2r_B$, then wire A has a cross-section 4 times that of B. This makes $v_A = v_B/4$ and the correct choice is (e).

3. Since $L_B = 2L_A$ and $r_B = 2r_A$, we find that $R_B = \rho \frac{L_B}{\pi r_B^2} = \rho \frac{(2L_A)}{\pi(4r_A^2)} = \frac{1}{2}\left(\rho \frac{L_A}{\pi r_A^2}\right) = \frac{1}{2}R$, and the correct response for this question is choice (d).

4. The resistance of a conductor having length L' and a circular cross-section is $R = \rho \frac{L'}{A} = \rho \frac{L'}{\pi r^2} = \frac{\rho}{\pi}\left(\frac{L'}{r^2}\right)$. The value of $\frac{L'}{r^2}$ for each of the three wires is: $\frac{L}{r^2}$ for wire 1; $\frac{L}{4r^2}$ for wire 2; and $\frac{2L}{9r^2}$ for wire 3. Thus, wire 3 has the smallest resistance and choice (c) is the correct answer.

5. The kWh is $1\text{ kWh} = 1\ (\text{kW})(1\text{ h}) = (10^3\text{ W})(3\,600\text{ s}) = \left[10^3(1\text{ J/s})\right](3\,600\text{ s}) = 3.60\times10^6\text{ J}$. Thus, the combination of choice (b) has units of energy. The other combinations have units of: charge per unit time, or current for (a); energy per unit time, or power for (c); current × time, or charge for (d); and power divided by time for (e). None of these are units of energy, and choice (b) is the only correct response to the question.

6. When the potential difference across the device is 2 V, the current is 2 A so the resistance is $R = \Delta V/I = 2\text{ V}/2\text{ A} = 1\ \Omega$ and (a) is the correct choice.

7. When the potential difference across the device is 3 V, the current is 2.5 A so the resistance is $R = \Delta V/I = 3\text{ V}/2.5\text{ A} = 1.2\ \Omega$ and (b) is the correct choice.

8. The temperature variation of resistance is given by $R = R_0\left[1+\alpha(T-T_0)\right]$, where R_0 is the resistance of the conductor at the reference temperature T_0, usually 20.0°C. Using the given resistance at $T = 90.0$°C, the temperature coefficient of resistivity for this conducting material is found to be

$$\alpha = \frac{1}{T-T_0}\left(\frac{R}{R_0}-1\right) = \frac{1}{90.0^\circ\text{C}-20.0^\circ\text{C}}\left(\frac{10.55\ \Omega}{10.00\ \Omega}-1\right) = 7.86\times10^{-4}\ {}^\circ\text{C}^{-1}$$

The resistance of this conductor at $T = -20.0$°C, where

$$T-T_0 = -20.0^\circ\text{C}-(20.0^\circ\text{C}) = -40.0^\circ\text{C}$$

will be

$$R = (10.00\ \Omega)\left[1+(7.86\times10^{-4}\ {}^\circ\text{C}^{-1})(-40.0^\circ\text{C})\right] = 9.69\ \Omega$$

so (b) is the correct choice.

9. The power consumption of the set is $\mathcal{P} = (\Delta V)I = (120\text{ V})(2.5\text{ A}) = 3.0\times10^2\text{ W} = 0.30\text{ kW}$. Thus, the energy used in 8.0 h of operation is $E = \mathcal{P}\cdot t = (0.30\text{ kW})(8.0\text{ h}) = 2.4\text{ kWh}$, at a cost of $cost = (2.4\text{ kWh})(8.0\text{ cents/kWh}) = 19\text{ cents}$. The correct choice is (c).

10. The current through the resistor is $I = \Delta V/R = 1.0\text{ V}/10.0\ \Omega = 0.10\text{ A}$, and the charge passing through in a 20 s interval is $\Delta Q = I\cdot\Delta t = (0.10\text{ C/s})(20\text{ s}) = 2.0\text{ C}$. Thus, (c) is the correct choice.

11. Resistors in a parallel combination all have the same potential difference across them. Thus, from Ohm's law, $I = \Delta V/R$, the resistor with the smallest resistance carries the highest current. Choice (a) is the correct response.

12. Resistors in a series combination all carry the same current. Thus, from Ohm's law, $\Delta V = IR$, the resistor with the highest resistance has the greatest voltage drop across it. Choice (c) is the correct response.

13. One way of expressing the power dissipated by a resistor is $\mathcal{P} = (\Delta V)^2 / R$. Thus, if the potential difference across the resistor is doubled, the power will be increased by a factor of 4, to a value of 16 W, making (d) the correct choice.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. In the electrostatic case in which charges are stationary, the internal electric field must be zero. A nonzero field would produce a current (by interacting with the free electrons in the conductor), which would violate the condition of static equilibrium. In this chapter we deal with conductors that carry current, a non-electrostatic situation. The current arises because of a potential difference applied between the ends of the conductor, which produces an internal electric field.

4. The number of cars would correspond to charge Q. The rate of flow of cars past a point would correspond to current.

6. The 25 W bulb has the higher resistance. Because $R = (\Delta V)^2 / \mathcal{P}$, and both operate from 120 V, the bulb dissipating the least power has the higher resistance. The 100 W bulb carries more current, because the current is proportional to the power rating of the bulb.

8. An electrical shock occurs when your body serves as a conductor between two points having a difference in potential. The concept behind the admonition is to avoid simultaneously touching points that are at different potentials.

10. The knob is connected to a variable resistor. As you increase the magnitude of the resistance in the circuit, the current is reduced, and the bulb dims.

12. The amplitude of atomic vibrations increases with temperature, thereby scattering electrons more efficiently.

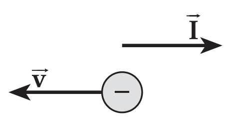
PROBLEM SOLUTIONS

17.1 The charge that moves past the cross section is $\Delta Q = I(\Delta t)$, and the number of electrons is

$$n = \frac{\Delta Q}{|e|} = \frac{I(\Delta t)}{|e|}$$

$$= \frac{(80.0 \times 10^{-3}\ \text{C/s})\left[(10.0\ \text{min})(60.0\ \text{s/min})\right]}{1.60 \times 10^{-19}\ \text{C}} = \boxed{3.00 \times 10^{20}\ \text{electrons}}$$

The negatively charged electrons move in the direction opposite to the conventional current flow.



17.2 (a) From Example 17.2 in the textbook, the density of charge carriers (electrons) in a copper wire is $n = 8.46 \times 10^{28}\ \text{electrons/m}^3$. With $A = \pi r^2$ and $|q| = e$, the drift speed of electrons in this wire is

$$v_d = \frac{I}{n|q|A} = \frac{I}{ne(\pi r^2)} = \frac{3.70\ \text{C/s}}{(8.46 \times 10^{28}\ \text{m}^{-3})(1.60 \times 10^{-19}\ \text{C})\pi(1.25 \times 10^{-3}\ \text{m})^2} = \boxed{5.57 \times 10^{-5}\ \text{m/s}}$$

(b) $\boxed{\text{The drift speed is smaller because more electrons are being conducted}}$. To create the same current, therefore, the drift speed need not be as great.

17.3 The period of the electron in its orbit is $T = 2\pi r/v$, and the current represented by the orbiting electron is

$$I = \frac{\Delta Q}{\Delta t} = \frac{|e|}{T} = \frac{v|e|}{2\pi r}$$

$$= \frac{\left(2.19\times10^{6}\ \text{m/s}\right)\left(1.60\times10^{-19}\ \text{C}\right)}{2\pi\left(5.29\times10^{-11}\ \text{m}\right)} = 1.05\times10^{-3}\ \text{C/s} = \boxed{1.05\ \text{mA}}$$

17.4 If N is the number of protons, each with charge e, that hit the target in time Δt, the average current in the beam is $I = \Delta Q/\Delta t = Ne/\Delta t$, giving

$$N = \frac{I(\Delta t)}{e} = \frac{\left(125\times10^{-6}\ \text{C/s}\right)\left(23.0\ \text{s}\right)}{1.60\times10^{-19}\ \text{C/proton}} = \boxed{1.80\times10^{16}\ \text{protons}}$$

17.5 (a) The carrier density is determined by the physical characteristics of the wire, not the current in the wire. Hence, n is $\boxed{\text{unaffected}}$.

(b) The drift velocity of the electrons is $v_d = I/nqA$. Thus, the drift velocity is $\boxed{\text{doubled}}$ when the current is doubled.

17.6 The mass of a single gold atom is

$$m_{\text{atom}} = \frac{M}{N_A} = \frac{197\ \text{g/mol}}{6.02\times10^{23}\ \text{atoms/mol}} = 3.27\times10^{-22}\ \text{g} = 3.27\times10^{-25}\ \text{kg}$$

The number of atoms deposited, and hence the number of ions moving to the negative electrode, is

$$n = \frac{m}{m_{\text{atom}}} = \frac{3.25\times10^{-3}\ \text{kg}}{3.27\times10^{-25}\ \text{kg}} = 9.93\times10^{21}$$

Thus, the current in the cell is

$$I = \frac{\Delta Q}{\Delta t} = \frac{ne}{\Delta t} = \frac{\left(9.93\times10^{21}\right)\left(1.60\times10^{-19}\ \text{C}\right)}{\left(2.78\ \text{h}\right)\left(3\,600\ \text{s/1 h}\right)} = 0.159\ \text{A} = \boxed{159\ \text{mA}}$$

17.7 The drift speed of electrons in the line is $v_d = \dfrac{I}{nqA} = \dfrac{I}{n|e|\left(\pi d^2/4\right)}$, or

$$v_d = \frac{4\left(1\,000\ \text{A}\right)}{\left(8.5\times10^{28}/\text{m}^3\right)\left(1.60\times10^{-19}\ \text{C}\right)\pi\left(0.020\ \text{m}\right)^2} = 2.3\times10^{-4}\ \text{m/s}$$

The time to travel the length of the 200-km line is then

$$\Delta t = \frac{L}{v_d} = \frac{200\times10^{3}\ \text{m}}{2.34\times10^{-4}\ \text{m/s}}\left(\frac{1\ \text{yr}}{3.156\times10^{7}\ \text{s}}\right) = \boxed{27\ \text{yr}}$$

17.8 Assuming that, on average, each aluminum atom contributes three electrons, the density of charge carriers is three times the number of atoms per cubic meter. This is

$$n = 3\left(\frac{density}{mass\ per\ atom}\right) = \frac{3\rho}{M/N_A} = \frac{3N_A\,\rho}{M},$$

or $$n = \frac{3\left(6.02\times10^{23}/\text{mol}\right)\left[\left(2.7\ \text{g/cm}^3\right)\left(10^6\ \text{cm}^3/1\ \text{m}^3\right)\right]}{26.98\ \text{g/mol}} = 1.8\times10^{29}/\text{m}^3$$

The drift speed of the electrons in the wire is then

$$v_d = \frac{I}{n|e|A} = \frac{5.0\ \text{C/s}}{\left(1.8\times10^{29}/\text{m}^3\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(4.0\times10^{-6}\ \text{m}^2\right)} = \boxed{4.3\times10^{-5}\ \text{m/s}}$$

17.9 (a) Using the periodic table on the inside back cover of the textbook, we find

$$M_{\text{Fe}} = 55.85\ \text{g/mol} = \left(55.85\ \text{g/mol}\right)\left(1\ \text{kg}/10^3\ \text{g}\right) = \boxed{55.85\times10^{-3}\ \text{kg/mol}}$$

(b) From Table 9.3, the density of iron is $\rho_{\text{Fe}} = 7.86\times10^3\ \text{kg/m}^3$, so the molar density is

$$\left(\text{molar density}\right)_{\text{Fe}} = \frac{\rho_{\text{Fe}}}{M_{\text{Fe}}} = \frac{7.86\times10^3\ \text{kg/m}^3}{55.85\times10^{-3}\ \text{kg/mol}} = \boxed{1.41\times10^5\ \text{mol/m}^3}$$

(c) The density of iron atoms is

$$\begin{aligned}\text{density of atoms} &= N_A\left(\text{molar density}\right)\\ &= \left(6.02\times10^{23}\ \frac{\text{atoms}}{\text{mol}}\right)\left(1.41\times10^5\ \frac{\text{mol}}{\text{m}^3}\right) = \boxed{8.49\times10^{28}\ \frac{\text{atoms}}{\text{m}^3}}\end{aligned}$$

(d) With two conduction electrons per iron atom, the density of charge carriers is

$$\begin{aligned}n &= \left(\text{charge carriers/atom}\right)\left(\text{density of atoms}\right)\\ &= \left(2\ \frac{\text{electrons}}{\text{atom}}\right)\left(8.49\times10^{28}\ \frac{\text{atoms}}{\text{m}^3}\right) = \boxed{1.70\times10^{29}\ \text{electrons/m}^3}\end{aligned}$$

(e) With a current of $I = 30.0\ \text{A}$ and cross-sectional area $A = 5.00\times10^{-6}\ \text{m}^2$, the drift speed of the conduction electrons in this wire is

$$v_d = \frac{I}{nqA} = \frac{30.0\ \text{C/s}}{\left(1.70\times10^{29}\ \text{m}^{-3}\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(5.00\times10^{-6}\ \text{m}^3\right)} = \boxed{2.21\times10^{-4}\ \text{m/s}}$$

17.10 From Ohm's law, $$R = \frac{\Delta V}{I} = \frac{1.20\times10^2\ \text{V}}{13.5\ \text{A}} = \boxed{8.89\ \Omega}$$

17.11 $(\Delta V)_{\max} = I_{\max} R = \left(80 \times 10^{-6} \text{ A}\right) R$

Thus, if $R = 4.0 \times 10^5\ \Omega$, $(\Delta V)_{\max} = \boxed{32 \text{ V}}$

and if $R = 2\,000\ \Omega$, $(\Delta V)_{\max} = \boxed{0.16 \text{ V}}$

17.12 The volume of the copper is

$$V = \frac{m}{density} = \frac{1.00 \times 10^{-3} \text{ kg}}{8.92 \times 10^3 \text{ kg/m}^3} = 1.12 \times 10^{-7} \text{ m}^3$$

Since $V = A \cdot L$, this gives $A \cdot L = 1.12 \times 10^{-7} \text{ m}^3$. **[1]**

(a) From $R = \dfrac{\rho L}{A}$, we find that

$$A = \left(\frac{\rho}{R}\right) L = \left(\frac{1.70 \times 10^{-8}\ \Omega \cdot \text{m}}{0.500\ \Omega}\right) L = \left(3.40 \times 10^{-8} \text{ m}\right) L$$

Inserting this expression for A into Equation [1] gives

$\left(3.40 \times 10^{-8} \text{ m}\right) L^2 = 1.12 \times 10^{-7} \text{ m}^3$, which yields $L = \boxed{1.82 \text{ m}}$

(b) From Equation [1], $A = \dfrac{\pi d^2}{4} = \dfrac{1.12 \times 10^{-7} \text{ m}^3}{L}$, or

$$d = \sqrt{\frac{4\left(1.12 \times 10^{-7} \text{ m}^3\right)}{\pi L}} = \sqrt{\frac{4\left(1.12 \times 10^{-7} \text{ m}^3\right)}{\pi (1.82 \text{ m})}}$$

$$= 2.80 \times 10^{-4} \text{ m} = \boxed{0.280 \text{ mm}}$$

17.13 (a) From Ohm's law, $R = \dfrac{\Delta V}{I} = \dfrac{1.20 \times 10^2 \text{ V}}{9.25 \text{ A}} = \boxed{13.0\ \Omega}$

(b) Using $R = \rho \dfrac{L}{A}$ and data from Table 17.1, the required length is found to be

$$L = \frac{RA}{\rho} = \frac{R\left(\pi r^2\right)}{\rho} = \frac{(13.0\ \Omega)\pi\left(0.791 \times 10^{-3} \text{ m}\right)^2}{150 \times 10^{-8}\ \Omega \cdot \text{m}} = \boxed{17.0 \text{ m}}$$

17.14 $R = \dfrac{\rho L}{A} = \dfrac{\rho L}{\pi d^2/4} = \dfrac{4\left(1.7 \times 10^{-8}\ \Omega \cdot \text{m}\right)(15 \text{ m})}{\pi\left(1.024 \times 10^{-3} \text{ m}\right)^2} = \boxed{0.31\ \Omega}$

17.15 (a) $R = \dfrac{\Delta V}{I} = \dfrac{12 \text{ V}}{0.40 \text{ A}} = \boxed{30\ \Omega}$

(b) From $R = \dfrac{\rho L}{A}$,

$$\rho = \frac{R \cdot A}{L} = \frac{(30\ \Omega)\left[\pi\left(0.40 \times 10^{-2} \text{ m}\right)^2\right]}{3.2 \text{ m}} = \boxed{4.7 \times 10^{-4}\ \Omega \cdot \text{m}}$$

17.16 Using $R = \dfrac{\rho L}{A}$ and data from Table 17.1, we have $\rho_{\text{Cu}} \dfrac{L_{\text{Cu}}}{\pi r_{\text{Cu}}^2} = \rho_{\text{Al}} \dfrac{L_{\text{AL}}}{\pi r_{\text{Al}}^2}$, which reduces to

$$\frac{r_{\text{AL}}^2}{r_{\text{Cu}}^2} = \frac{\rho_{\text{AL}}}{\rho_{\text{Cu}}} \qquad \text{and yields} \qquad \frac{r_{\text{AL}}}{r_{\text{Cu}}} = \sqrt{\frac{\rho_{\text{AL}}}{\rho_{\text{Cu}}}} = \sqrt{\frac{2.82\times10^{-8}\ \Omega\cdot\text{m}}{1.70\times10^{-8}\ \Omega\cdot\text{m}}} = \boxed{1.29}$$

17.17 The resistance is $R = \dfrac{\Delta V}{I} = \dfrac{9.11\ \text{V}}{36.0\ \text{A}} = 0.253\ \Omega$, so the resistivity of the metal is

$$\rho = \frac{R\cdot A}{L} = \frac{R\cdot\left(\pi d^2/4\right)}{L} = \frac{(0.253\ \Omega)\pi\left(2.00\times10^{-3}\ \text{m}\right)^2}{4(50.0\ \text{m})} = 1.59\times10^{-8}\ \Omega\cdot\text{m}$$

Thus, the metal is seen to be $\boxed{\text{silver}}$.

17.18 With different orientations of the block, three different values of the ratio L/A are possible. These are:

$$\left(\frac{L}{A}\right)_1 = \frac{10\ \text{cm}}{(20\ \text{cm})(40\ \text{cm})} = \frac{1}{80\ \text{cm}} = \frac{1}{0.80\ \text{m}},$$

$$\left(\frac{L}{A}\right)_2 = \frac{20\ \text{cm}}{(10\ \text{cm})(40\ \text{cm})} = \frac{1}{20\ \text{cm}} = \frac{1}{0.20\ \text{m}},$$

and $$\left(\frac{L}{A}\right)_3 = \frac{40\ \text{cm}}{(10\ \text{cm})(20\ \text{cm})} = \frac{1}{5.0\ \text{cm}} = \frac{1}{0.050\ \text{m}}$$

(a) $$I_{\max} = \frac{\Delta V}{R_{\min}} = \frac{\Delta V}{\rho(L/A)_{\min}} = \frac{(6.0\ \text{V})(0.80\ \text{m})}{1.7\times10^{-8}\ \Omega\cdot\text{m}} = \boxed{2.8\times10^{8}\ \text{A}}$$

(b) $$I_{\min} = \frac{\Delta V}{R_{\max}} = \frac{\Delta V}{\rho(L/A)_{\max}} = \frac{(6.0\ \text{V})(0.050\ \text{m})}{1.7\times10^{-8}\ \Omega\cdot\text{m}} = \boxed{1.8\times10^{7}\ \text{A}}$$

17.19 The volume of material, $V = AL_0 = \left(\pi r_0^2\right)L_0$, in the wire is constant. Thus, as the wire is stretched to decrease its radius, the length increases such that $\left(\pi r_f^2\right)L_f = \left(\pi r_0^2\right)L_0$ giving

$$L_f = \left(\frac{r_0}{r_f}\right)^2 L_0 = \left(\frac{r_0}{0.25r_0}\right)^2 L_0 = (4.0)^2 L_0 = 16L_0$$

The new resistance is then

$$R_f = \rho\frac{L_f}{A_f} = \rho\frac{L_f}{\pi r_f^2} = \rho\frac{16L_0}{\pi\left(r_0/4\right)^2} = 16(4)^2\left(\rho\frac{L_0}{\pi r_0^2}\right) = 256R_0 = 256(1.00\ \Omega) = \boxed{256\ \Omega}$$

17.20 (a) From Ohm's law, $\Delta V = IR = \left(500\times10^{-3}\ \text{A}\right)\left(1.0\times10^{6}\ \Omega\right) = \boxed{5\times10^{5}\ \text{V}}$

(b) Rubber-soled shoes and rubber gloves can increase the resistance to current and help reduce the likelihood of a serious shock.

17.21 If a conductor of length L has a uniform electric field E maintained within it, the potential difference between the ends of the conductor is $\Delta V = EL$. But, from Ohm's law, the relation between the potential difference across a conductor and the current through it is $\Delta V = IR$, where $R = \rho L/A$. Combining these relations, we obtain

$$\Delta V = EL = IR = I(\rho L/A) \qquad \text{or} \qquad \boxed{E = \rho(I/A) = \rho J}$$

17.22 Using $R = R_0[1+\alpha(T-T_0)]$ with $R_0 = 6.00\ \Omega$ at $T_0 = 20.0°\text{C}$ and $\alpha_{\text{silver}} = 3.8\times 10^{-3}\ (°\text{C})^{-1}$ (from Table 17.1 in the textbook), the resistance at $T = 34.0°\text{C}$ is

$$R = (6.00\ \Omega)\left[1 + 3.8\times 10^{-3}\ (°\text{C})^{-1}(34.0°\text{C} - 20.0°\text{C})\right] = \boxed{6.32\ \Omega}$$

17.23 From Ohm's law, $\Delta V = I_i R_i = I_f R_f$, so the current in Antarctica is

$$I_f = I_i\left(\frac{R_i}{R_f}\right) = I_i\left(\frac{R_0[1+\alpha(T_i - T_0)]}{R_0[1+\alpha(T_f - T_0)]}\right)$$

$$= (1.00\ \text{A})\left(\frac{1+\left[3.90\times 10^{-3}\ (°\text{C})^{-1}\right](58.0°\text{C} - 20.0°\text{C})}{1+\left[3.90\times 10^{-3}\ (°\text{C})^{-1}\right](-88.0°\text{C} - 20.0°\text{C})}\right) = \boxed{1.98\ \text{A}}$$

17.24 (a) Given: Aluminum wire with $\alpha = 3.90\times 10^{-3}\ (°\text{C})^{-1}$ (see Table 17.1 in textbook), and $R_0 = 30.0\ \Omega$ at $T_0 = 20.0°\text{C}$. If $R = 46.2\ \Omega$ at temperature T, solving $R = R_0[1+\alpha(T-T_0)]$ gives the final temperature as

$$T = T_0 + \frac{(R/R_0) - 1}{\alpha} = 20.0°\text{C} + \frac{(46.2\ \Omega/30.0\ \Omega) - 1}{3.90\times 10^{-3}\ (°\text{C})^{-1}} = \boxed{158°\text{C}}$$

(b) The expansion of the cross-sectional area contributes slightly more than the expansion of the length of the wire, so the answer would be slightly reduced.

17.25 For tungsten, the temperature coefficient of resistivity is $\alpha = 4.5\times 10^{-3}\ (°\text{C})^{-1}$. Thus, if $R_0 = 15\ \Omega$ at $T_0 = 20°\text{C}$, and $R = 160\ \Omega$ at the operating temperature of the filament, solving $R = R_0[1+\alpha(T-T_0)]$ for the operating temperature gives

$$T = T_0 + \frac{(R/R_0) - 1}{\alpha} = 20°\text{C} + \frac{(160\ \Omega/15\ \Omega) - 1}{4.5\times 10^{-3}\ (°\text{C})^{-1}} = \boxed{2.2\times 10^{3}\ °\text{C}}$$

17.26 For aluminum, the resistivity at room temperature is $\rho_0 = 2.82\times 10^{-8}\ \Omega\cdot\text{m}$ and the temperature coefficient of resistivity is $\alpha_{\text{Al}} = 3.9\times 10^{-3}\ (°\text{C})^{-1}$. Thus, if at some temperature, the aluminum has a resistivity of

$$\rho = 3(\rho_0)_{\text{Cu}} = 3(1.7\times 10^{-8}\ \Omega\cdot\text{m}) = 5.1\times 10^{-8}\ \Omega\cdot\text{m}$$

solving $\rho = \rho_0[1+\alpha_{\text{Al}}(T-T_0)]$ for that temperature gives

$$T = T_0 + \frac{(\rho/\rho_0) - 1}{\alpha_{\text{Al}}} = 20°\text{C} + \frac{\left(\dfrac{5.1\times 10^{-8}\ \Omega\cdot\text{m}}{2.82\times 10^{-8}\ \Omega\cdot\text{m}}\right) - 1}{3.9\times 10^{-3}\ (°\text{C})^{-1}} = \boxed{2.3\times 10^{2}\ °\text{C}}$$

17.27 At 80°C,

$$I = \frac{\Delta V}{R} = \frac{\Delta V}{R_0\left[1+\alpha\left(T-T_0\right)\right]} = \frac{5.0\text{ V}}{(200\ \Omega)\left[1+\left(-0.5\times10^{-3}\ {}^\circ\text{C}^{-1}\right)\left(80^\circ\text{C}-20^\circ\text{C}\right)\right]}$$

or $I = 2.6\times10^{-2}\text{ A} = \boxed{26\text{ mA}}$

17.28 If $R = 41.0\ \Omega$ at $T = 20^\circ\text{C}$ and $R = 41.4\ \Omega$ at $T = 29.0^\circ\text{C}$, then $R = R_0\left[1+\alpha\left(T-T_0\right)\right]$ gives the temperature coefficient of resistivity of the material making up this wire as

$$\alpha = \frac{R-R_0}{R_0\left(T-T_0\right)} = \frac{41.4\ \Omega - 41.0\ \Omega}{(41.0\ \Omega)(29.0^\circ\text{C}-20^\circ\text{C})} = \boxed{1.1\times10^{-3}\ (^\circ\text{C})^{-1}}$$

17.29 (a) The resistance at 20.0°C is

$$R_0 = \rho\frac{L}{A} = \frac{\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)(34.5\text{ m})}{\pi\left(0.25\times10^{-3}\text{ m}\right)^2} = 3.0\ \Omega$$

and the current will be $I = \dfrac{\Delta V}{R_0} = \dfrac{9.0\text{ V}}{3.0\ \Omega} = \boxed{3.0\text{ A}}$

(b) At 30.0°C,

$$R = R_0\left[1+\alpha\left(T-T_0\right)\right]$$

$$= (3.0\ \Omega)\left[1+\left(3.9\times10^{-3}\ (^\circ\text{C})^{-1}\right)\left(30.0^\circ\text{C}-20.0^\circ\text{C}\right)\right] = 3.1\ \Omega$$

Thus, the current is $I = \dfrac{\Delta V}{R} = \dfrac{9.0\text{ V}}{3.1\ \Omega} = \boxed{2.9\text{ A}}$

17.30 The resistance of the heating element when at its operating temperature is

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{1\,050\text{ W}} = 13.7\ \Omega$$

From $R = R_0\left[1+\alpha\left(T-T_0\right)\right] = \dfrac{\rho_0 L}{A}\left[1+\alpha\left(T-T_0\right)\right]$, the cross-sectional area is

$$A = \frac{\rho_0 L}{R}\left[1+\alpha\left(T-T_0\right)\right]$$

$$= \frac{\left(150\times10^{-8}\ \Omega\cdot\text{m}\right)(4.00\text{ m})}{13.7\ \Omega}\left[1+\left(0.40\times10^{-3}\ (^\circ\text{C})^{-1}\right)\left(320^\circ\text{C}-20.0^\circ\text{C}\right)\right]$$

$$A = \boxed{4.90\times10^{-7}\text{ m}^2}$$

17.31 (a) From $R = \rho L/A$, the initial resistance of the mercury is

$$R_i = \frac{\rho L_i}{A_i} = \frac{\left(9.4\times10^{-7}\ \Omega\cdot\text{m}\right)\left(1.000\ 0\ \text{m}\right)}{\pi\left(1.00\times10^{-3}\ \text{m}\right)^2/4} = \boxed{1.2\ \Omega}$$

(b) Since the volume of mercury is constant, $V = A_f \cdot L_f = A_i \cdot L_i$ gives the final cross-sectional area as $A_f = A_i \cdot \left(L_i/L_f\right)$. Thus, the final resistance is given by $R_f = \dfrac{\rho L_f}{A_f} = \dfrac{\rho L_f^2}{A_i \cdot L_i}$. The fractional change in the resistance is then

$$\Delta = \frac{R_f - R_i}{R_i} = \frac{R_f}{R_i} - 1 = \frac{\rho L_f^2/\left(A_i\cdot L_i\right)}{\rho L_i/A_i} - 1 = \left(\frac{L_f}{L_i}\right)^2 - 1$$

$$\Delta = \left(\frac{100.04}{100.00}\right)^2 - 1 = \boxed{8.0\times10^{-4}}\ \text{or}\ \boxed{\text{a }0.080\%\text{ increase}}$$

17.32 The resistance at 20.0°C is

$$R_0 = \frac{R}{1+\alpha\left(T-T_0\right)} = \frac{200.0\ \Omega}{1+\left[3.92\times10^{-3}\ (^\circ\text{C})^{-1}\right]\left(0^\circ\text{C}-20.0^\circ\text{C}\right)} = 217\ \Omega$$

Solving $R = R_0\left[1+\alpha\left(T-T_0\right)\right]$ for T gives the temperature of the melting potassium as

$$T = T_0 + \frac{R-R_0}{\alpha R_0} = 20.0^\circ\text{C} + \frac{253.8\ \Omega - 217\ \Omega}{\left[3.92\times10^{-3}\ (^\circ\text{C})^{-1}\right]\left(217\ \Omega\right)} = \boxed{63.3^\circ\text{C}}$$

17.33 (a) The power consumed by the device is $\mathcal{P} = I\left(\Delta V\right)$, so the current must be

$$I = \frac{\mathcal{P}}{\Delta V} = \frac{1.00\times10^3\ \text{W}}{1.20\times10^2\ \text{V}} = \boxed{8.33\ \text{A}}$$

(b) From Ohm's law, the resistance is $R = \dfrac{\Delta V}{I} = \dfrac{1.20\times10^2\ \text{V}}{8.33\ \text{A}} = \boxed{14.4\ \Omega}$

17.34 (a) The energy used by a 100-W bulb in 24 h is

$$E = \mathcal{P}\cdot\Delta t = \left(100\ \text{W}\right)\left(24\ \text{h}\right) = \left(0.100\ \text{kW}\right)\left(24\ \text{h}\right) = 2.4\ \text{kWh}$$

and the cost of this energy, at a rate of \$0.12 per kilowatt-hour is

$$cost = E\cdot rate = \left(2.4\ \text{kWh}\right)\left(\$0.12/\text{kWh}\right) = \boxed{\$0.29}$$

(b) The energy used by the oven in 5.0 h is

$$E = \mathcal{P}\cdot\Delta t = \left[I\left(\Delta V\right)\right]\cdot\Delta t = \left[\left(20.0\ \text{C/s}\right)\left(220\ \text{J/C}\right)\left(\frac{1\ \text{kW}}{10^3\ \text{J/s}}\right)\right]\left(5.0\ \text{h}\right) = 22\ \text{kWh}$$

and the cost of this energy, at a rate of \$0.12 per kilowatt-hour is

$$cost = E\cdot rate = \left(22\ \text{kWh}\right)\left(\$0.12/\text{kWh}\right) = \boxed{\$2.6}$$

17.35 The power required is $\mathcal{P} = I(\Delta V) = (0.350\ \text{A})(6.0\ \text{V}) = \boxed{2.1\ \text{W}}$

17.36 (a) The power loss in the line is

$$\mathcal{P}_{\text{loss}} = I^2R = (1\,000\ \text{A})^2\left[(0.31\ \Omega/\text{km})(160\ \text{km})\right] = 5.0\times10^7\ \text{W} = \boxed{50\ \text{MW}}$$

(b) The total power transmitted is

$$\mathcal{P}_{\text{input}} = (\Delta V)I = \left(700\times10^3\ \text{V}\right)(1\,000\ \text{A}) = 7.0\times10^8\ \text{W} = 700\ \text{MW}$$

Thus, the fraction of the total transmitted power represented by the line losses is

$$\textit{fraction loss} = \frac{\mathcal{P}_{\text{loss}}}{\mathcal{P}_{\text{input}}} = \frac{50\ \text{MW}}{700\ \text{MW}} = 0.071\ \text{ or }\ \boxed{7.1\%}$$

17.37 The energy required to bring the water to the boiling point is

$$E = mc(\Delta T) = (0.500\ \text{kg})(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(100^\circ\text{C} - 23.0^\circ\text{C}) = 1.61\times10^5\ \text{J}$$

The power input by the heating element is

$$\mathcal{P}_{\text{input}} = (\Delta V)I = (120\ \text{V})(2.00\ \text{A}) = 240\ \text{W} = 240\ \text{J/s}$$

Therefore, the time required is

$$t = \frac{E}{\mathcal{P}_{\text{input}}} = \frac{1.61\times10^5\ \text{J}}{240\ \text{J/s}} = 672\ \text{s}\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{11.2\ \text{min}}$$

17.38 (a) $E = \mathcal{P}\cdot t = (90\ \text{W})(1\ \text{h}) = (90\ \text{J/s})(3\,600\ \text{s}) = \boxed{3.2\times10^5\ \text{J}}$

(b) The power consumption of the color set is

$$\mathcal{P} = (\Delta V)I = (120\ \text{V})(2.50\ \text{A}) = 300\ \text{W}$$

Therefore, the time required to consume the energy found in (a) is

$$t = \frac{E}{\mathcal{P}} = \frac{3.2\times10^5\ \text{J}}{300\ \text{J/s}} = 1.1\times10^3\ \text{s}\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{18\ \text{min}}$$

17.39 The energy input required is

$$E = mc(\Delta T) = (1.50\ \text{kg})(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(50.0^\circ\text{C} - 10.0^\circ\text{C}) = 2.51\times10^5\ \text{J}$$

and, if this is to be added in $\Delta t = 10.0\ \text{min} = 600\ \text{s}$, the power input needed is

$$\mathcal{P} = \frac{E}{\Delta t} = \frac{2.51\times10^5\ \text{J}}{600\ \text{s}} = 419\ \text{W}$$

The power input to the heater may be expressed as $\mathcal{P} = (\Delta V)^2/R$, so the needed resistance is

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\ \text{V})^2}{419\ \text{W}} = \boxed{34.4\ \Omega}$$

17.40 (a) At the operating temperature,

$$\mathcal{P} = (\Delta V)I = (120\ \text{V})(1.53\ \text{A}) = \boxed{184\ \text{W}}$$

(b) From $R = R_0\left[1+\alpha(T-T_0)\right]$, the temperature T is given by $T = T_0 + \dfrac{R-R_0}{\alpha R_0}$. The resistances are given by Ohm's law as

$$R = \frac{(\Delta V)}{I} = \frac{120\ \text{V}}{1.53\ \text{A}},\ \text{and}\ R_0 = \frac{(\Delta V)_0}{I_0} = \frac{120\ \text{V}}{1.80\ \text{A}}$$

Therefore, the operating temperature is

$$T = 20.0°\text{C} + \frac{(120/1.53)-(120/1.80)}{\left(0.400\times10^{-3}\ (°\text{C})^{-1}\right)(120/1.80)} = \boxed{461°\text{C}}$$

17.41 The resistance per unit length of the cable is

$$\frac{R}{L} = \frac{\mathcal{P}/I^2}{L} = \frac{\mathcal{P}/L}{I^2} = \frac{2.00\ \text{W/m}}{(300\ \text{A})^2} = 2.22\times10^{-5}\ \ \Omega/\text{m}$$

From $R = \rho L/A$, the resistance per unit length is also given by $R/L = \rho/A$. Hence, the cross-sectional area is $\pi r^2 = A = \dfrac{\rho}{R/L}$, and the required radius is

$$r = \sqrt{\frac{\rho}{\pi(R/L)}} = \sqrt{\frac{1.7\times10^{-8}\ \Omega\cdot\text{m}}{\pi\left(2.22\times10^{-5}\ \ \Omega/\text{m}\right)}} = 0.016\ \text{m} = \boxed{1.6\ \text{cm}}$$

17.42 (a) The rating of the 12-V battery is $I\cdot\Delta t = 55\ \text{A}\cdot\text{h}$. Thus, the stored energy is

$$\text{Energy stored} = \mathcal{P}\cdot\Delta t = (\Delta V)I\cdot\Delta t = (12\ \text{V})(55\ \text{A}\cdot\text{h}) = 660\ \text{W}\cdot\text{h} = \boxed{0.66\ \text{kWh}}$$

(b) $cost = (0.66\ \text{kWh})(\$0.12/\text{kWh}) = \$0.079 = \boxed{7.9\ \text{cents}}$

17.43 $\mathcal{P} = (\Delta V)I = \left(75\times10^{-3}\ \text{V}\right)\left(0.20\times10^{-3}\ \text{A}\right) = 1.5\times10^{-5}\ \text{W} = 15\times10^{-6}\ \text{W} = \boxed{15\ \mu\text{W}}$

17.44 (a) $E = \mathcal{P}\cdot t = (40.0\ \text{W})(14.0\ \text{d})(24.0\ \text{h/d}) = 1.34\times10^4\ \text{Wh} = 13.4\ \text{kWh}$

$$cost = E\cdot(rate) = (13.4\ \text{kWh})(\$0.120/\text{kWh}) = \boxed{\$1.61}$$

(b) $E = \mathcal{P}\cdot t = (0.970\ \text{kW})(3.00\ \text{min})(1\ \text{h}/60\ \text{min}) = 4.85\times10^{-2}\ \text{kWh}$

$$cost = E\cdot(rate)$$

$$= \left(4.85\times10^{-2}\ \text{kWh}\right)(\$0.120/\text{kWh}) = \$0.005\ 83 = \boxed{0.583\ \text{cents}}$$

(c) $E = \mathcal{P}\cdot t = (5.20\ \text{kW})(40.0\ \text{min})(1\ \text{h}/60\ \text{min}) = 3.47\ \text{kWh}$

$$cost = E\cdot(rate) = (3.47\ \text{kWh})(\$0.120/\text{kWh}) = \$0.416 = \boxed{41.6\ \text{cents}}$$

17.45 The energy saved is

$$\Delta E = \left(\mathcal{P}_{\text{high}} - \mathcal{P}_{\text{low}}\right)\cdot t = (40\text{ W} - 11\text{ W})(100\text{ h}) = 2.9\times10^{3}\text{ Wh} = 2.9\text{ kWh}$$

and the monetary savings is

$$savings = \Delta E \cdot rate = (2.9\text{ kWh})(\$0.080/\text{kWh}) = \$0.23 = \boxed{23\text{ cents}}$$

17.46 The power required to warm the water to 100°C in 4.00 min is

$$\mathcal{P} = \frac{\Delta Q}{\Delta t} = \frac{mc(\Delta T)}{\Delta t} = \frac{(0.250\text{ kg})(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(100^\circ\text{C} - 20^\circ\text{C})}{(4.00\text{ min})(60\text{ s/1 min})} = 3.5\times10^{2}\text{ W}$$

The required resistance (at 100°C) of the heating element is then

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{3.5\times10^{2}\text{ W}} = 41\ \Omega$$

so the resistance at 20°C would be

$$R_0 = \frac{R}{1+\alpha(T-T_0)} = \frac{41\ \Omega}{1+\left(0.4\times10^{-3}\ {}^\circ\text{C}^{-1}\right)(100^\circ\text{C} - 20^\circ\text{C})} = 40\ \Omega$$

We find the needed dimensions of a nichrome wire for this heating element from $R_0 = \rho_0\, L/A = \rho_0\, L/\left(\pi d^2/4\right) = 4\rho_0\, L/\pi d^2$, where L is the length of the wire and d is its diameter. This gives

$$d^2 = \left[\frac{4\rho_0}{\pi R_0}\right]L = \left[\frac{4\left(150\times10^{-8}\ \Omega\cdot\text{m}\right)}{\pi(40\ \Omega)}\right]L = \left(4.8\times10^{-8}\text{ m}\right)L$$

Thus, $\boxed{\text{any combination of length and diameter satisfying the relation } d^2 = \left(4.8\times10^{-8}\text{ m}\right)L}$ will be suitable. A typical combination might be

$$L = 3.0\text{ m} \qquad \text{and} \qquad d = \sqrt{\left(4.8\times10^{-8}\text{ m}\right)(3.0\text{ m})} = 3.8\times10^{-4}\text{ m} = 0.38\text{ mm}$$

$\boxed{\text{Yes}}$, such heating elements could easily be made from less than 0.5 cm^3 of nichrome. The volume of material required for the typical wire given above is

$$V = AL = \left(\frac{\pi d^2}{4}\right)L = \frac{\pi\left(3.8\times10^{-4}\text{ m}\right)^2}{4}(3.0\text{ m}) = \left(3.4\times10^{-7}\text{ m}^3\right)\left(\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right) = 0.34\text{ cm}^3$$

17.47 The energy that must be added to the water is

$$E = mc(\Delta T) = (200\text{ kg})\left(4\,186\ \frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)(80^\circ\text{C} - 15^\circ\text{C})\left(\frac{1\text{ kWh}}{3.60\times10^{6}\text{ J}}\right) = 15\text{ kWh}$$

and the cost is $cost = E\cdot rate = (15\text{ kWh})(\$0.080/\text{kWh}) = \boxed{\$1.2}$

17.48 (a) For tungsten, Table 17.1 from the textbook gives the resistivity at $T_0 = 20.0°\text{C} = 293\text{ K}$ as $\rho_0 = 5.6 \times 10^{-8}\ \Omega \cdot \text{m}$ and the temperature coefficient of resistivity as $\alpha = 4.5 \times 10^{-3}\ (°\text{C})^{-1} = 4.5 \times 10^{-3}\ \text{K}^{-1}$. Thus, for a tungsten wire having a radius of 1.00 mm and a length of 15.0 cm, the resistance at $T_0 = 293\text{ K}$ is

$$R_0 = \rho_0 \frac{L}{A} = \rho_0 \frac{L}{(\pi r^2)} = (5.6 \times 10^{-8}\ \Omega \cdot \text{m}) \frac{(15.0 \times 10^{-2}\ \text{m})}{\pi (1.00 \times 10^{-3}\ \text{m})^2} = \boxed{2.7 \times 10^{-3}\ \Omega}$$

(b) From Stefan's law, the radiated power is $\mathcal{P} = \sigma A e T^4$, where A is the area of the radiating surface. Note that since we are computing the radiated power, not the net energy gained or lost as a result of radiation, the ambient temperature is not needed here. In the case of a wire, this is the cylindrical surface area $A = 2\pi r L$. The temperature of the wire when it is radiating a power of $\mathcal{P} = 75.0\text{ W}$ must be

$$T = \left[\frac{\mathcal{P}}{\sigma A e}\right]^{1/4} = \left[\frac{75.0\text{ W}}{(5.669 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)2\pi(1.00 \times 10^{-3}\ \text{m})(0.150\text{ m})(0.320)}\right]^{1/4}$$

or $T = \boxed{1.45 \times 10^3\ \text{K}}$

(c) Assuming a linear temperature variation of resistance, the resistance of the wire at this temperature is

$$R = R_0\left[1 + \alpha(T - T_0)\right] = (2.7 \times 10^{-3}\ \Omega)\left[1 + (4.5 \times 10^{-3}\ \text{K}^{-1})(1.45 \times 10^3\ \text{K} - 293\text{ K})\right]$$

giving $R = \boxed{1.7 \times 10^{-2}\ \Omega}$

(d) The voltage drop across the wire when it is radiating 75.0 W and has the resistance found in part (c) above is given by $\mathcal{P} = (\Delta V)^2/R$ as

$$\Delta V = \sqrt{R \cdot \mathcal{P}} = \sqrt{(1.7 \times 10^{-2}\ \Omega)(75.0\text{ W})} = \boxed{1.1\text{ V}}$$

(e) $\boxed{\text{Tungsten bulbs release very little of the energy consumed in the form of visible light}}$, making them inefficient sources of light.

17.49 The battery is rated to deliver the equivalent of 60.0 amperes of current (i.e., 60.0 C/s) for 1 hour. This is

$$Q = I \cdot \Delta t = (60.0\text{ A})(1\text{ h}) = (60.0\text{ C/s})(3\,600\text{ s}) = \boxed{2.16 \times 10^5\ \text{C}}$$

17.50 The energy available in the battery is

$$\text{Energy stored} = \mathcal{P} \cdot t = (\Delta V) I \cdot t = (\Delta V)(I \cdot t) = (12\ \text{V})(90\ \text{A} \cdot \text{h}) = 1.1 \times 10^{3}\ \text{W} \cdot \text{h}$$

The two headlights together consume a total power of $\mathcal{P} = 2(36\ \text{W}) = 72\ \text{W}$, so the time required to completely discharge the battery is

$$\Delta t = \frac{\text{Energy stored}}{\mathcal{P}} = \frac{1.1 \times 10^{3}\ \text{W} \cdot \text{h}}{72\ \text{W}} = \boxed{15\ \text{h}}$$

17.51 Assuming a constant resistance, the power consumed by the device is proportional to the square of the applied voltage, $\mathcal{P} = (\Delta V)^2 / R$. Thus,

$$\frac{\mathcal{P}_2}{\mathcal{P}_1} = \frac{(\Delta V_2)^2 / R}{(\Delta V_1)^2 / R} \qquad \text{or} \qquad \mathcal{P}_2 = \mathcal{P}_1 \left(\frac{\Delta V_2}{\Delta V_1}\right)^2 = (15\ \text{W})\left(\frac{6.0\ \text{V}}{9.0\ \text{V}}\right)^2 = \boxed{6.7\ \text{W}}$$

17.52 The temperature variation of resistance is described by $R = R_0\left[1 + \alpha(T - T_0)\right]$, where R_0 is the resistance at $T_0 = 20°\text{C}$. Thus, if an aluminum wire $\left[\alpha = 3.9 \times 10^{-3}\ (°\text{C})^{-1}\right]$ has $R = 2R_0$, we have $2 = 1 + \alpha(T - 20°\text{C})$, and its temperature must be

$$T = 20°\text{C} + \frac{1}{\alpha} = 20°\text{C} + \frac{1}{3.9 \times 10^{-3}\ (°\text{C})^{-1}} = \boxed{2.8 \times 10^{2}\ °\text{C}}$$

17.53 From $\mathcal{P} = (\Delta V)^2 / R$, the total resistance needed is

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(20\ \text{V})^2}{48\ \text{W}} = 8.3\ \Omega$$

Thus, from $R = \rho L / A$, the length of wire required is

$$L = \frac{R \cdot A}{\rho} = \frac{(8.3\ \Omega)(4.0 \times 10^{-6}\ \text{m}^2)}{3.0 \times 10^{-8}\ \Omega \cdot \text{m}} = 1.1 \times 10^{3}\ \text{m} = \boxed{1.1\ \text{km}}$$

17.54 The resistance of the 4.0 cm length of wire between the feet is

$$R = \frac{\rho L}{A} = \frac{(1.7 \times 10^{-8}\ \Omega \cdot \text{m})(0.040\ \text{m})}{\pi(0.011\ \text{m})^2} = 1.79 \times 10^{-6}\ \Omega,$$

so the potential difference is

$$\Delta V = IR = (50\ \text{A})(1.79 \times 10^{-6}\ \Omega) = 8.9 \times 10^{-5}\ \text{V} = \boxed{89\ \mu\text{V}}$$

17.55 Ohm's law gives the resistance as $R = (\Delta V)/I$. From $R = \rho L/A$, the resistivity is given by $\rho = R\cdot(A/L)$. The results of these calculations for each of the three wires are summarized in the table below.

$L(\mathrm{m})$	$R(\Omega)$	$\rho(\Omega\cdot\mathrm{m})$
0.540	10.4	1.41×10^{-6}
1.028	21.1	1.50×10^{-6}
1.543	31.8	1.50×10^{-6}

The average value found for the resistivity is

$$\rho_{\text{av}} = \frac{\Sigma\rho_i}{3} = \boxed{1.47\times10^{-6}\ \Omega\cdot\mathrm{m}}$$

which differs from the value of $\rho = 150\times10^{-8}\ \Omega\cdot\mathrm{m} = 1.50\times10^{-6}\ \Omega\cdot\mathrm{m}$ given in Table 17.1 by $\boxed{2.0\%}$.

17.56 At temperature *T*, the resistance of the carbon wire is $R_c = R_{0c}\left[1+\alpha_c\left(T-T_0\right)\right]$, and that of the nichrome wire is $R_n = R_{0n}\left[1+\alpha_n\left(T-T_0\right)\right]$. When the wires are connected end to end, the total resistance is

$$R = R_c + R_n = \left(R_{0c}+R_{0n}\right) + \left(R_{0c}\alpha_c + R_{0n}\alpha_n\right)\left(T-T_0\right)$$

If this is to have a constant value of $10.0\ \mathrm{k}\Omega$ as the temperature changes, it is necessary that

$$R_{0c} + R_{0n} = 10.0\ \mathrm{k}\Omega \qquad \textbf{[1]}$$

and $$R_{0c}\alpha_c + R_{0n}\alpha_n = 0 \qquad \textbf{[2]}$$

From equation [1], $R_{0c} = 10.0\ \mathrm{k}\Omega - R_{0n}$, and substituting into Equation [2] gives

$$\left(10.0\ \mathrm{k}\Omega - R_{0n}\right)\left[-0.50\times10^{-3}\ (^\circ\mathrm{C})^{-1}\right] + R_{0n}\left[0.40\times10^{-3}\ (^\circ\mathrm{C})^{-1}\right] = 0$$

Solving this equation gives $R_{0n} = \boxed{5.6\ \mathrm{k}\Omega\ \ (\text{nichrome wire})}$

Then, $R_{0c} = 10.0\ \mathrm{k}\Omega - 5.6\ \mathrm{k}\Omega = \boxed{4.4\ \mathrm{k}\Omega\ \ (\text{carbon wire})}$

17.57 (a) The total power you now use while cooking breakfast is

$$\mathcal{P} = (1\,200 + 500)\ \text{W} = 1.70\ \text{kW}$$

The cost to use this power for 0.500 h each day for 30.0 days is

$$cost = \left[\mathcal{P} \times (\Delta t)\right] \times rate = \left[(1.70\ \text{kW})\left(0.500\ \frac{\text{h}}{\text{day}}\right)(30.0\ \text{days})\right](\$0.120/\text{kWh}) = \boxed{\$3.06}$$

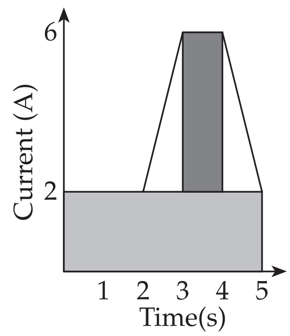
(b) If you upgraded, the new power requirement would be:

$$\mathcal{P} = (2\,400 + 500)\ \text{W} = 2\,900\ \text{W}$$

and the required current would be $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{2\,900\ \text{W}}{110\ \text{V}} = 26.4\ \text{A} > 20\ \text{A}$

$\boxed{\text{No}}$, your present circuit breaker cannot handle the upgrade.

17.58 (a) The charge passing through the conductor in the interval $0 \le t \le 5.0\ \text{s}$ is represented by the area under the I vs. t graph given in Figure P17.58. This area consists of two rectangles and two triangles. Thus,



$$\Delta Q = A_{\text{rectangle 1}} + A_{\text{rectangle 2}} + A_{\text{triangle 1}} + A_{\text{triangle 2}}$$

$$= (5.0\ \text{s} - 0)(2.0\ \text{A} - 0) + (4.0\ \text{s} - 3.0\ \text{s})(6.0\ \text{A} - 2.0\ \text{A})$$
$$+ \frac{1}{2}(3.0\ \text{s} - 2.0\ \text{s})(6.0\ \text{A} - 2.0\ \text{A}) + \frac{1}{2}(5.0\ \text{s} - 4.0\ \text{s})(6.0\ \text{A} - 2.0\ \text{A})$$

$$\Delta Q = \boxed{18\ \text{C}}$$

(b) The constant current that would pass the same charge in 5.0 s is

$$I = \frac{\Delta Q}{\Delta t} = \frac{18\ \text{C}}{5.0\ \text{s}} = \boxed{3.6\ \text{A}}$$

17.59 (a) From $\mathcal{P} = (\Delta V)I$, the current is $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{8.00 \times 10^3\ \text{W}}{12.0\ \text{V}} = \boxed{667\ \text{A}}$.

(b) The time before the stored energy is depleted is

$$t = \frac{E_{\text{storage}}}{\mathcal{P}} = \frac{2.00 \times 10^7\ \text{J}}{8.00 \times 10^3\ \text{J/s}} = 2.50 \times 10^3\ \text{s}$$

Thus, the distance traveled is

$$d = v \cdot t = (20.0\ \text{m/s})(2.50 \times 10^3\ \text{s}) = 5.00 \times 10^4\ \text{m} = \boxed{50.0\ \text{km}}$$

17.60 The volume of aluminum available is

$$V = \frac{mass}{density} = \frac{115 \times 10^{-3}\ \text{kg}}{2.70 \times 10^{3}\ \text{kg/m}^3} = 4.26 \times 10^{-5}\ \text{m}^3$$

(a) For a cylinder whose height equals the diameter, the volume is

$$V = \left(\frac{\pi d^2}{4}\right)d = \frac{\pi d^3}{4}$$

and the diameter is $d = \left(\frac{4V}{\pi}\right)^{1/3} = \left[\frac{4\left(4.26 \times 10^{-5}\ \text{m}^3\right)}{\pi}\right]^{1/3} = 0.037\ 85\ \text{m}$

The resistance between ends is then

$$R = \frac{\rho L}{A} = \frac{\rho d}{\left(\pi d^2/4\right)} = \frac{4\,\rho}{\pi\, d} = \frac{4\left(2.82 \times 10^{-8}\ \Omega \cdot \text{m}\right)}{\pi\left(0.037\ 85\ \text{m}\right)} = \boxed{9.49 \times 10^{-7}\ \Omega}$$

(b) For a cube, $V = L^3$, so the length of an edge is

$$L = (V)^{1/3} = \left(4.26 \times 10^{-5}\ \text{m}\right)^{1/3} = 0.034\ 9\ \text{m}$$

The resistance between opposite faces is

$$R = \frac{\rho L}{A} = \frac{\rho L}{L^2} = \frac{\rho}{L} = \frac{2.82 \times 10^{-8}\ \Omega \cdot \text{m}}{0.034\ 9\ \text{m}} = \boxed{8.07 \times 10^{-7}\ \Omega}$$

17.61 The current in the wire is $I = \frac{\Delta V}{R} = \frac{15.0\ \text{V}}{0.100\ \Omega} = 150\ \text{A}$

Then, from $v_d = I/nqA$, the density of free electrons is

$$n = \frac{I}{v_d e\left(\pi r^2\right)} = \frac{150\ \text{A}}{\left(3.17 \times 10^{-4}\ \text{m/s}\right)\left(1.60 \times 10^{-19}\ \text{C}\right)\pi\left(5.00 \times 10^{-3}\ \text{m}\right)^2}$$

or $n = \boxed{3.77 \times 10^{28}/\text{m}^3}$

17.62 Each speaker has a resistance of $R = 4.00\ \Omega$ and can handle 60.0 W of power. From $\mathcal{P} = I^2 R$, the maximum safe current is

$$I_{\max} = \sqrt{\frac{\mathcal{P}}{R}} = \sqrt{\frac{60.0\ \text{W}}{4.00\ \Omega}} = 3.87\ \text{A}$$

Thus, the system $\boxed{\text{is not adequately protected}}$ by a 4.00 A fuse.

17.63 The cross-sectional area of the conducting material is $A = \pi\left(r_{outer}^2 - r_{inner}^2\right)$. Thus,

$$R = \frac{\rho L}{A} = \frac{\left(3.5\times10^5\ \Omega\cdot\text{m}\right)\left(4.0\times10^{-2}\ \text{m}\right)}{\pi\left[\left(1.2\times10^{-2}\ \text{m}\right)^2 - \left(0.50\times10^{-2}\ \text{m}\right)^2\right]} = 3.7\times10^7\ \Omega = \boxed{37\ \text{M}\Omega}$$

17.64 The volume of the material is

$$V = \frac{mass}{density} = \frac{50.0\ \text{g}}{7.86\ \text{g/cm}^3}\left(\frac{1\ \text{m}^3}{10^6\ \text{cm}^3}\right) = 6.36\times10^{-6}\ \text{m}^3$$

Since $V = A\cdot L$, the cross-sectional area of the wire is $A = V/L$.

(a) From $R = \dfrac{\rho L}{A} = \dfrac{\rho L}{V/L} = \dfrac{\rho L^2}{V}$, the length of the wire is given by

$$L = \sqrt{\frac{R\cdot V}{\rho}} = \sqrt{\frac{(1.5\ \Omega)\left(6.36\times10^{-6}\ \text{m}^3\right)}{11\times10^{-8}\ \Omega\cdot\text{m}}} = \boxed{9.3\ \text{m}}$$

(b) The cross-sectional area of the wire is $A = \dfrac{\pi d^2}{4} = \dfrac{V}{L}$. Thus, the diameter is

$$d = \sqrt{\frac{4V}{\pi L}} = \sqrt{\frac{4\left(6.36\times10^{-6}\ \text{m}^3\right)}{\pi(9.3\ \text{m})}} = 9.3\times10^{-4}\ \text{m} = \boxed{0.93\ \text{mm}}$$

17.65 The power the beam delivers to the target is

$$\mathcal{P} = (\Delta V)I = \left(4.0\times10^6\ \text{V}\right)\left(25\times10^{-3}\ \text{A}\right) = 1.0\times10^5\ \text{W}$$

The mass of cooling water that must flow through the tube each second if the rise in the water temperature is not to exceed 50°C is found from $\mathcal{P} = (\Delta m/\Delta t)c(\Delta T)$ as

$$\frac{\Delta m}{\Delta t} = \frac{\mathcal{P}}{c(\Delta T)} = \frac{1.0\times10^5\ \text{J/s}}{(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(50^\circ\text{C})} = \boxed{0.48\ \text{kg/s}}$$

17.66 (a) At temperature T, the resistance is $R = \dfrac{\rho L}{A}$, where $\rho = \rho_0\left[1+\alpha\left(T-T_0\right)\right]$,

$L = L_0\left[1+\alpha'\left(T-T_0\right)\right]$, and $A = A_0\left[1+\alpha'\left(T-T_0\right)\right]^2 \approx A_0\left[1+2\alpha'\left(T-T_0\right)\right]$

Thus,

$$R = \left(\frac{\rho_0 L_0}{A_0}\right)\frac{\left[1+\alpha\left(T-T_0\right)\right]\cdot\left[1+\alpha'\left(T-T_0\right)\right]}{\left[1+2\alpha'\left(T-T_0\right)\right]} = \boxed{\frac{R_0\left[1+\alpha\left(T-T_0\right)\right]\cdot\left[1+\alpha'\left(T-T_0\right)\right]}{\left[1+2\alpha'\left(T-T_0\right)\right]}}$$

continued on next page

(b) $$R_0 = \frac{\rho_0 L_0}{A_0} = \frac{\left(1.70\times10^{-8}\ \Omega\cdot\text{m}\right)\left(2.00\ \text{m}\right)}{\pi\left(0.100\times10^{-3}\right)^2} = 1.082\ \Omega$$

Then $R = R_0\left[1+\alpha\left(T-T_0\right)\right]$ gives

$$R = \left(1.082\ \Omega\right)\left[1+\left(3.90\times10^{-3}/^\circ\text{C}\right)\left(80.0^\circ\text{C}\right)\right] = \boxed{1.420\ \Omega}$$

The more complex formula gives

$$R = \frac{\left(1.420\ \Omega\right)\cdot\left[1+\left(17\times10^{-6}/^\circ\text{C}\right)\left(80.0^\circ\text{C}\right)\right]}{\left[1+2\left(17\times10^{-6}/^\circ\text{C}\right)\left(80.0^\circ\text{C}\right)\right]} = \boxed{1.418\ \Omega}$$

Note: Some rules for handing significant figures have been deliberately violated in this solution in order to illustrate the very small difference in the results obtained with these two expressions.

17.67 **Note** that all potential differences in this solution have a value of $\Delta V = 120\ \text{V}$.
First, we shall do a symbolic solution for many parts of the problem, and then enter the specified numeric values for the cases of interest.

From the marked specifications on the cleaner, its internal resistance (assumed constant) is

$$R_i = \frac{\left(\Delta V\right)^2}{\mathcal{P}_1} \quad \text{where } \mathcal{P}_1 = 535\ \text{W} \qquad \textbf{Equation [1]}$$

If each of the two conductors in the extension cord has resistance R_c, the total resistance in the path of the current (outside of the power source) is

$$R_t = R_i + 2R_c \qquad \textbf{Equation [2]}$$

so the current which will exist is $I = \Delta V/R_t$ and the power that is delivered to the cleaner is

$$\mathcal{P}_{\text{delivered}} = I^2 R_i = \left(\frac{\Delta V}{R_t}\right)^2 R_i = \left(\frac{\Delta V}{R_t}\right)^2 \frac{\left(\Delta V\right)^2}{\mathcal{P}_1} = \frac{\left(\Delta V\right)^4}{R_t^2\mathcal{P}_1} \qquad \textbf{Equation [3]}$$

The resistance of a copper conductor of length L and diameter d is

$$R_c = \rho_{\text{Cu}}\frac{L}{A} = \rho_{\text{Cu}}\frac{L}{\left(\pi d^2/4\right)} = \frac{4\rho_{\text{Cu}}L}{\pi d^2}$$

Thus, if $R_{c,\,\text{max}}$ is the maximum allowed value of R_c, the minimum acceptable diameter of the conductor is

$$d_{\min} = \sqrt{\frac{4\rho_{\text{Cu}}L}{\pi R_{c,\,\text{max}}}} \qquad \textbf{Equation [4]}$$

continued on next page

(a) If $R_c = 0.900\ \Omega$, then from Equations [2] and [1],

$$R_t = R_i + 2(0.900\ \Omega) = \frac{(\Delta V)^2}{\mathcal{P}_1} + 1.80\ \Omega = \frac{(120\ \text{V})^2}{535\ \text{W}} + 1.80\ \Omega$$

and, from Equation [3], the power delivered to the cleaner is

$$\mathcal{P}_{\text{delivered}} = \frac{(120\ \text{V})^4}{\left[\dfrac{(120\ \text{V})^2}{535\ \text{W}} + 1.80\ \Omega\right]^2 (535\ \text{W})} = \boxed{470\ \text{W}}$$

(b) If the minimum acceptable power delivered to the cleaner is $\mathcal{P}_{\text{min}}$, then Equations [2] and [3] give the maximum allowable total resistance as

$$R_{t,\,\text{max}} = R_i + 2R_{c,\,\text{max}} = \sqrt{\frac{(\Delta V)^4}{\mathcal{P}_{\text{min}}\mathcal{P}_1}} = \frac{(\Delta V)^2}{\sqrt{\mathcal{P}_{\text{min}}\mathcal{P}_1}}$$

so

$$R_{c,\,\text{max}} = \frac{1}{2}\left[\frac{(\Delta V)^2}{\sqrt{\mathcal{P}_{\text{min}}\mathcal{P}_1}} - R_i\right] = \frac{1}{2}\left[\frac{(\Delta V)^2}{\sqrt{\mathcal{P}_{\text{min}}\mathcal{P}_1}} - \frac{(\Delta V)^2}{\mathcal{P}_1}\right] = \frac{(\Delta V)^2}{2}\left[\frac{1}{\sqrt{\mathcal{P}_{\text{min}}\mathcal{P}_1}} - \frac{1}{\mathcal{P}_1}\right]$$

When $\mathcal{P}_{\text{min}} = 525$ W, then $R_{c,\,\text{max}} = \dfrac{(120\ \text{V})^2}{2}\left[\dfrac{1}{\sqrt{(525\ \text{W})(535\ \text{W})}} - \dfrac{1}{535\ \text{W}}\right] = 0.128\ \Omega$

and, from Equation [4], $d_{\text{min}} = \sqrt{\dfrac{4(1.7\times10^{-8}\ \Omega\cdot\text{m})(15.0\ \text{m})}{\pi(0.128\ \Omega)}} = \boxed{1.60\ \text{mm}}$

When $\mathcal{P}_{\text{min}} = 532$ W, then $R_{c,\,\text{max}} = \dfrac{(120\ \text{V})^2}{2}\left[\dfrac{1}{\sqrt{(532\ \text{W})(535\ \text{W})}} - \dfrac{1}{535\ \text{W}}\right] = 0.037\ 9\ \Omega$

and $d_{\text{min}} = \sqrt{\dfrac{4(1.7\times10^{-8}\ \Omega\cdot\text{m})(15.0\ \text{m})}{\pi(0.037\ 9\ \Omega)}} = \boxed{2.93\ \text{mm}}$