15

Electric Forces and Electric Fields

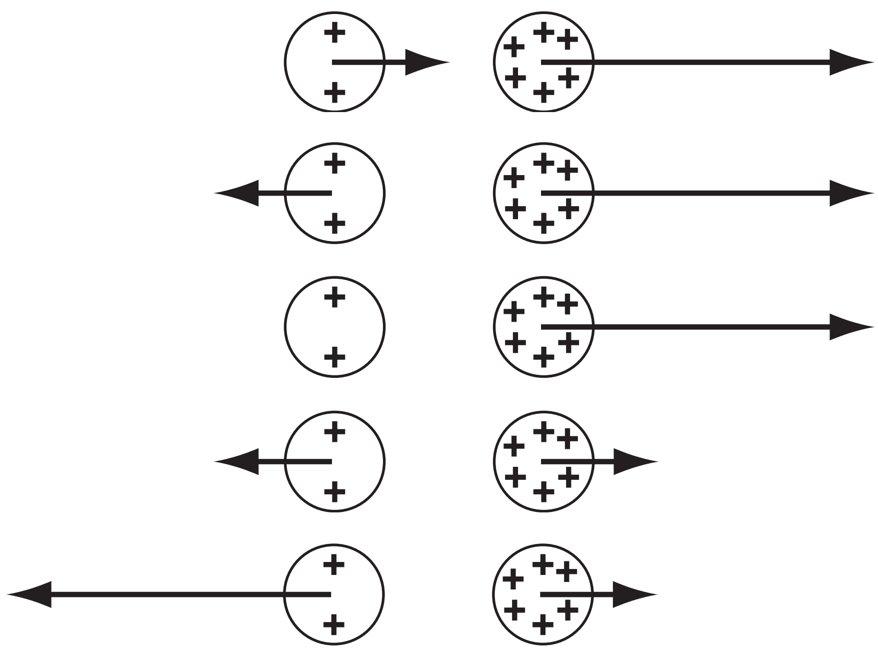
CLICKER QUESTIONS

Question L1.01a

Description: Developing an understanding of the electric force and contrasting with the electric field.

Question

The diagrams below show two uniformly charged spheres. The charge on the right sphere is 3 times as large as the charge on the left sphere. Which force diagram best represents the magnitudes and directions of the electric forces on the two spheres?



1.
2.
3.
4.
5.

Commentary

Purpose: To distinguish and relate the concepts of *electric force* and *electric field*.

Discussion: According to Newton's third law, the force exerted on the left sphere by the right one must be equal in magnitude and opposite in direction to the force exerted on the right sphere by the left. Therefore, of the listed answers, only (4) can be valid.

Remember that the electrostatic force of one charged object acting on another depends on the product of their charges, so the force of a large charge acting on a small will be the same as the force on a small charge acting on a large one.

Key Points:

- Be careful not to confuse electric forces with the electric fields causing them.
- Newton's third law applies to electromagnetic forces.

For Instructors Only

This is the first of two related questions comparing and contrasting electric forces with electric fields. Students often get these confused, so helping them to make a clear distinction is important. The question pair is an example of the "compare and contrast" tactic: by juxtaposing two very similar questions, we sensitize students to the one feature that differs, helping them to distinguish and hone related concepts; simultaneously, we help them develop their understanding of how the concepts are related, clustering the knowledge.

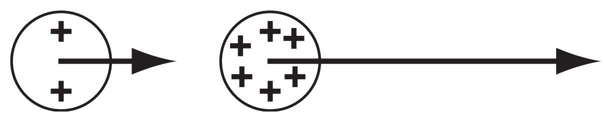
Answer (5) depicts the correct electric *fields* at those locations, so students choosing that answer are likely confusing the two concepts.

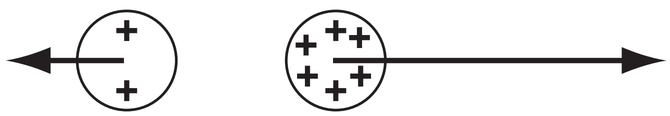
Question L1.01b

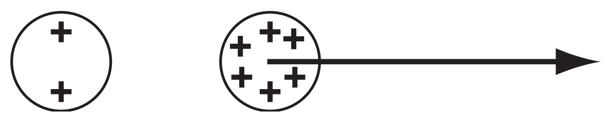
Description: Developing an understanding of the electric field and contrasting with electric force.

Question

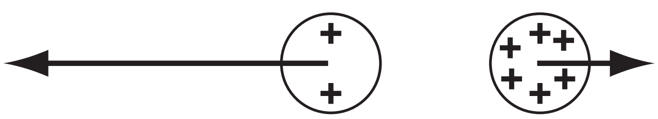
The diagrams below show two uniformly charged spheres. The charge on the right sphere is 3 times as large as the charge on the left sphere. Each arrow represents the electric field at the center of one sphere created by the other. Which choice best represents the magnitudes and directions of the electric field vectors created by one sphere at the location of the other sphere?

1. 

2. 

3. 

4. 

5. 

Commentary

Purpose: To distinguish and relate the concepts of *electric field* and *electric force*.

Discussion: According to Coulomb's law, the electric field created by a charged object is proportional to the amount of charge on it. In this situation, the field at the center of each sphere is created by the *other* sphere. The sphere with the smaller charge will therefore have a larger electric field acting on it, so (5) is the only possible answer to 1b.

The field created by a charge also gets weaker farther away from the charge. The distance between charge and field location is the same for both arrows in these diagrams, however, so this does not affect the relative force vectors we expect to see.

How can the electric fields have different magnitudes when the forces have the same magnitudes? Because the force caused by an electric field is proportional to the charge of the object *being acted upon* as well as to the magnitude of the electric field. Each force therefore depends on the product of both charges; the fields, however, each depend on only one charge.

Key Points:

- Be careful not to confuse electric fields with the electric forces they cause.
- Electric fields depend only on the charge creating them, whereas electric forces depend on the charge being acted upon as well.
- Mind your qs and Qs: when an equation depends upon a charge, make sure you understand *which* charge is meant.

For Instructors Only

This is the second of two related questions comparing and contrasting electric forces with electric fields. Students often get these confused, so helping them to make a clear distinction is important. The question pair is an example of the "compare and contrast" tactic: by juxtaposing two very similar questions, we sensitize students to the one feature that differs, helping them to distinguish and hone related concepts; simultaneously, we help them develop their understanding of how the concepts are related, clustering the knowledge.

Students choosing answer (4) might be describing the electric *forces* rather than the *fields*.

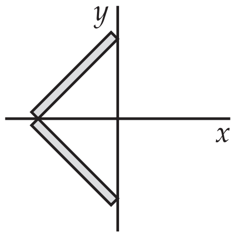
This question also reveals students' tendency to throw the variable "Q" around without being careful about whether it represents the acting or acted-upon charge. By letting students make an error because of this and then explicitly bringing the tendency and its harmful consequences to students' conscious awareness, we help them learn to overcome it.

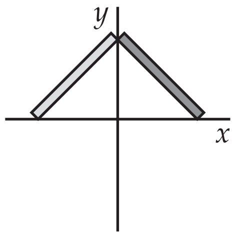
Question L1.02

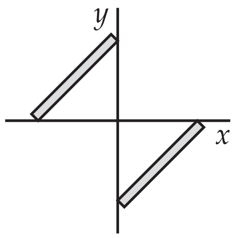
Description: Developing qualitative and graphical reasoning skills about the superposition of electric fields.

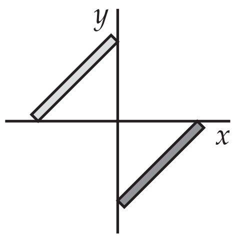
Question

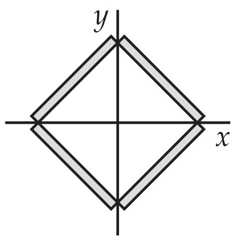
All charged rods have the same length and the same linear charge density ($+$ or $-$). Light rods are positively charged, and dark rods are negatively charged. For which arrangement below would the magnitude of the electric field at the origin be largest?

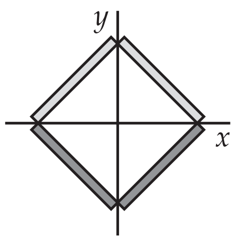
1. 

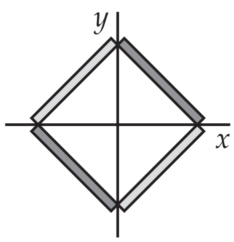
2. 

3. 

4. 

5. 

6. 

7. 

8. Impossible to determine.

Commentary

Purpose: To develop your ability to reason qualitatively about superposed electric fields.

Discussion: No equations or numbers are necessary to answer this question, only a basic understanding of vector addition, symmetry, superposition, and the direction of electric fields caused by positive or negative charges. The ability to reason qualitatively about electric fields, and in fact about anything in physics, is crucial to "understanding" and to solving problems.

Each rod will cause an electric field at the origin that points either directly away from (for positive rods) or towards (for negative rods) the center of the rod. (Any other orientation would violate symmetry.) Since each rod has the same magnitude of charge, all such fields will have the same magnitude. Now, all you need to do is draw electric field vectors at the origin due to each rod, and then add up those vectors to get the total. Note that components of vectors will frequently cancel.

For example, in (1), the two rods cause electric fields that are down and to the right, and up and to the right; adding these together results in a net field that is directly to the right, as the horizontal components add and the vertical components cancel. (2) is identical to (1), because the negative rod on the top right in (2) causes a field towards itself that is exactly the same as the field away from the bottom left rod in (1).

In (3), the fields due to the two rods cancel exactly: same magnitudes, opposite directions. The same argument applies to (5) and (7).

(6) can be thought of as a copy of (2) rotated by 90° so that its net field points downward, superposed with (added to) a mirror image of the same thing (the left half of the diagram) with a net field that also points downward. Thus, the total field for (6) must have a magnitude twice as large as that of (2). Similar arguments can be used to compare (6) to (1) and to (4). By this kind of logic, you can deduce that arrangement (6) must produce the greatest net field magnitude without ever using an equation.

Key Points:

- Graphical representations (such as vector diagrams) and qualitative reasoning can be powerful tools for solving problems without using equations.
- The concepts of symmetry and superposition can help you deduce much about a physical situation.
- To find the electric field of a complex charge arrangement, you can break it into simpler pieces, find the electric field of each, and then add up the fields to find the total field.

For Instructors Only

Students will only pay attention to qualitative reasoning and to graphical representations as thinking tools if they see problems where these approaches are clearly superior. This is such a problem.

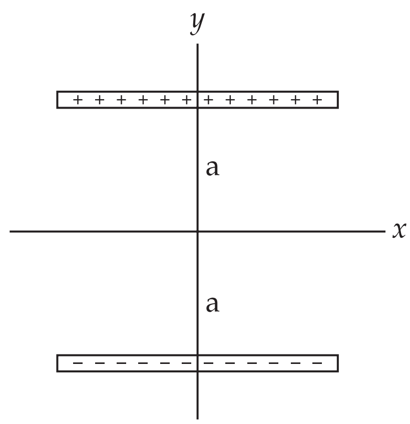
This question also reveals the degree of students' comfort and facility with vector addition, superposition, and symmetry. The instructor should devote significant time to this problem if students reveal weakness here; asking students to draw or describe the actual vector arrows for each of the rods in each situation, and visually combining them into a resultant, can be helpful.

Question L1.03

Description: Understanding superposition and the directionality of electric fields.

Question

Two uniformly charged rods are positioned horizontally as shown. The top rod is positively charged and the bottom rod is negatively charged. The total electric field at the origin:



1. is zero.
2. has both a nonzero x component and a nonzero y component.
3. points totally in the $+x$ direction.
4. points totally in the $-x$ direction.
5. points totally in the $+y$ direction.
6. points totally in the $-y$ direction.
7. points in a direction impossible to determine without doing a lot of math.

Commentary

Purpose: To improve your understanding of electric fields by reasoning about their direction using symmetry and superposition.

Discussion: The electric field obeys the principle of *superposition,* which means the total electric field at any point in space is the vector sum of the electric fields due to individual sources (i.e., distributions of charge). In this case, we can focus on each rod separately, then add up the results to find the total.

We can also use symmetry to avoid any computation. Consider the upper (positive) rod, and pick two small pieces of it, one from each side, equally spaced from the middle of the rod. These may be considered point charges. At the origin, the electric field of the left charge points down and to the right, while that of the right charge points down and to the left. By symmetry, the horizontal components must cancel and the vertical components add, resulting in an electric field that is in the $-y$ direction. Repeat this process with all the other pairs of symmetric charges in the upper rod, and superposition demands that the electric field due to the upper rod points in the $-y$ direction.

Repeating this procedure for the lower rod, the result is an electric field that also points in the $-y$ direction. Adding it to that of the upper rod yields a total electric field in the $-y$ direction, too.

Key Points:

- The electric field obeys the principle of *superposition,* which means you can subdivide any distribution of charge into smaller, more manageable pieces, then add the field due to each piece to find the total electric field.

- You can often deduce the direction of an electric field from the symmetry of the charge distribution creating it. If a charge distribution has a particular symmetry, the electric field it creates must have the same symmetry.

For Instructors Only

Students choosing answer (1) probably think that the fields from the two rods cancel.

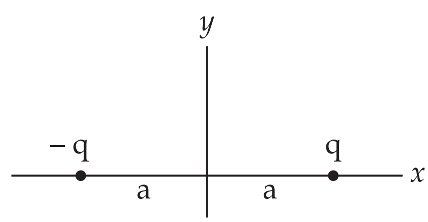
Another way to invoke symmetry is to assume that the electric field due to one of the rods has a nonzero x component. Now invert the arrangement about the y axis. The rod is the same, but if the electric field has a nonzero x component, it now points in a different direction. This is a contradiction. The only way to resolve the contradiction is to say that the x component is zero.

Question L1.05

Description: Introducing the electric dipole.

Question

Where, other than at infinity, is the electric field zero in the vicinity of the dipole shown?



1. Along the y-axis.
2. At the origin.
3. At two points, one to the right of (a, 0), the other to the left of (−a, 0).
4. At two points on the y-axis, one below the origin, one above the origin.
5. None of the above.

Commentary

Purpose: To explore the field of an *electric dipole*.

Discussion: The electric field obeys *superposition,* meaning that the total electric field due to a combination of charges is equal to the sum of the electric field due to each individual charge. Thus, the electric field due to this "electric dipole" is the sum of the electric fields due to the two point charges.

The electric field of a positive point charge points radially outward from the charge in all directions, getting weaker farther away from the charge. The electric field of a negative point charge is exactly the same, except that it points radially inward.

For the total electric field of the dipole to be zero at a particular point in space, the fields from the two point charges must cancel. That means the two fields must have the same magnitude and opposite directions. For the two fields to have the same magnitude, the point must be equidistant from the charges, which means it must be somewhere in the yz-plane. But anywhere in that plane, the field due to $+q$ has a negative x component, and the field due to $-q$ also has a negative x component. Thus, the fields cannot be in opposite directions, so they can never cancel.

Thus, the electric field of the dipole is zero nowhere except infinity: answer (5).

Key Points:

- An *electric dipole* is a combination of two charges close together, with the same magnitude but opposite sign.
- The electric field due to a combination of charges can be found through *superposition:* find the field due to each individual charge, then add up the fields (at every point in space).
- The sum of two vectors can be zero only when they have equal magnitudes and opposite directions.

For Instructors Only

Students choosing answer (1) are likely confusing the electric field with the electric potential, or at least forgetting that the electric field is a vector, rather than scalar, field.

Students choosing answer (2) may be neglecting to account for the different signs of the two charges.

Students choosing answer (3) may be remembering the answer to a different problem (e.g., if the charges differ in magnitude).

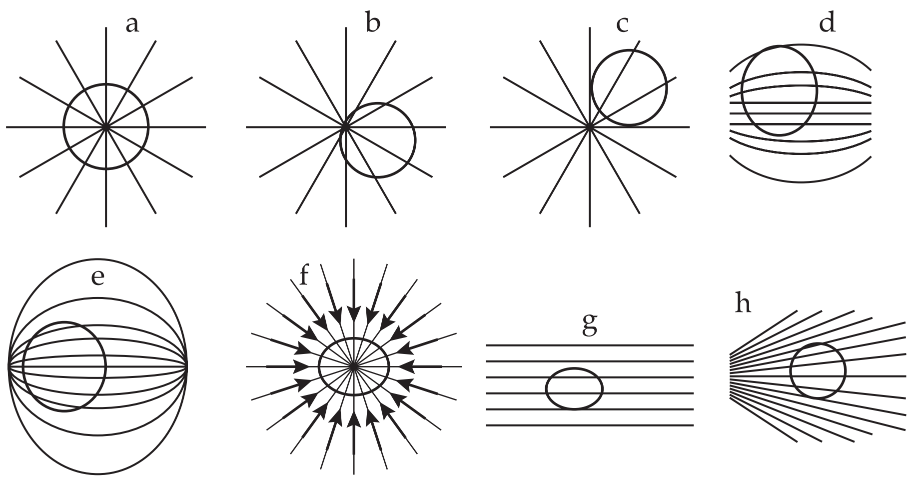
Having students draw the field lines can be helpful if many are confused. We recommend doing this before revealing which response is correct.

Question L2.01

Description: Introducing the concept of electric flux, and relating to graphical representations.

Question

The circles and ovals in the pictures below are Gaussian surfaces. All other lines are electric field lines. For which cases is the net flux through the surface *nonzero*?



1. a
2. a, b, and f
3. a, b, e, and f
4. a, b, d, e, and h
5. a and b
6. All but g
7. All of them
8. None of the above
9. Cannot be determined

Commentary

Purpose: To develop the concept of *electric flux*.

Discussion: If electric field lines enter and then leave a Gaussian surface, the net contribution to the flux through the surface is zero; the positive contribution while exiting the surface is offset by the negative contribution while entering. Thus, only surfaces that have a *source* or *sink* of electric field lines — points where lines begin or end — can have a nonzero total flux.

In cases a, b, and f, electric field lines begin or end within the surface, so the net flux is nonzero. For all the others, every line that enters also leaves, and the net flux is zero.

Key Points:

- The flux through a surface due to an electric field can be positive or negative depending on which way the field is pointing relative to the surface.
- Electric field lines that pass completely through a closed surface contribute no net flux.
- Only closed surfaces containing a source or sink of field lines have nonzero net flux.

For Instructors Only

This question can be used as a motivating and grounding context for introducing the concept of *electric flux*.

As a follow-up discussion, you can ask comparative questions about the depicted situations. For example, is the flux larger for situation a or f? Students' answer will reveal whether they appreciate that flux can be negative, or are merely counting field lines.

Question L2.02

Description: Introducing or developing the concept of electric flux.

Source: A2L: 283-400, Flux in and out of a balloon.

Question

We construct a closed Gaussian surface in the shape of a spherical balloon. Assume that a small glass bead with total charge Q is in the vicinity of the balloon. Consider the following statements:

A. If the bead is inside the balloon, the electric flux over the balloon's surface can never be 0.
B. If the bead is outside the balloon, the electric flux over the balloon surface must be 0.

Which of these statements is valid?

1. Only A is valid.
2. Only B is valid.
3. Both A and B are valid.
4. Neither one is valid.

Commentary

Purpose: To check and develop your understanding of the concept of *flux*.

Discussion: Gauss's law relates the net flux through any closed mathematical surface to the total charge enclosed by the surface. Assuming the bead is the only charge present in this situation, Gauss's law implies both statements A and B.

You can also take a more visual approach to the question. Electric field lines radiate out from a positive charge and into a negative charge. The other end of the lines goes to another charge, or to infinity if no other charges are present. The net flux through a surface can be thought of as the number of field lines passing through the surface and pointing outward, minus the number passing through and pointing inward. If the bead is inside the balloon, all the field lines radiating out from it and going to infinity must penetrate the balloon surface, resulting in a nonzero flux. So, A is true. On the other hand, if the bead is outside the balloon, any field lines radiating from it that penetrate into the balloon must come back out again on their path to infinity, resulting in zero *net* flux. So, B is true.

Key Points:

- Gauss's law relates the net electric flux through a closed surface to the total charge contained within it.
- Electric flux can be thought of as field lines passing outward through a surface (or inward, for negative flux).
- A closed surface containing no net charge must have zero net flux through it; one containing nonzero charge must have nonzero net flux.

For Instructors Only

This is a rather straightforward question about electric flux, yet it often reveals student misunderstanding and confusion.

Students sometimes accept statement A but think that the *value* of the flux depends on the location of the bead within the sphere. Therefore, asking them to talk about what happens to the net flux as the bead moves continuously from the center of the balloon towards and then through the surface can be revealing (to you) and enlightening (to them).

Lack of experience with vectors and dot products can prevent students from appreciating flux at a formal level. Having students draw the field lines does help, but only after they comprehend that the formal definition of flux is equivalent to counting field lines crossing the surface.

QUICK QUIZZES

1. (b). Object A must have a net charge because two neutral objects do not attract each other. Since object A is attracted to positively charged object B, the net charge on A must be negative.

2. (b). By Newton's third law, the two objects will exert forces having equal magnitudes but opposite directions on each other.

3. (c). The electric field at point P is due to charges *other* than the test charge. Thus, it is unchanged when the test charge is altered. However, the direction of the force this field exerts on the test change is reversed when the sign of the test charge is changed.

4. (a). If a test charge is at the center of the ring, the force exerted on the test charge by charge on any small segment of the ring will be balanced by the force exerted by charge on the diametrically opposite segment of the ring. The net force on the test charge, and hence the electric field at this location, must then be zero.

5. (c) and (d). The electron and the proton have equal magnitude charges of opposite signs. The forces exerted on these particles by the electric field have equal magnitude and opposite directions. The electron experiences an acceleration of greater magnitude than does the proton because the electron's mass is much smaller than that of the proton.

6. (a). The field is greatest at point A because this is where the field lines are closest together. The absence of lines at point C indicates that the electric field there is zero.

7. (c). When a plane area A is in a uniform electric field E, the flux through that area is $\Phi_E = EA\cos\theta$ where θ is the angle the electric field makes with the line normal to the plane of A. If A lies in the xy-plane and E is in the z direction, then $\theta = 0°$ and $\Phi_E = EA = (5.00\ \text{N/C})(4.00\ \text{m}^2) = 20.0\ \text{N}\cdot\text{m}^2/\text{C}$.

8. (b). If $\theta = 60°$ in Quick Quiz 15.7, then

$$\Phi_E = EA\cos\theta = (5.00\ \text{N/C})(4.00\ \text{m}^2)\cos(60°) = 10.0\ \text{N}\cdot\text{m}^2/\text{C}$$

9. (d). Gauss's law states that the electric flux through any closed surface is equal to the net charge enclosed divided by the permittivity of free space. For the surface shown in Figure 15.28, the net enclosed charge is $Q = -6$ C which gives $\Phi_E = Q/\epsilon_0 = -(6\ \text{C})/\epsilon_0$.

10. (b) and (d). Since the net flux through the surface is zero, Gauss's law says that the net charge enclosed by that surface must be zero as stated in (b). Statement (d) must be true because there would be a net flux through the surface if more lines entered the surface than left it (or vice versa).

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The magnitude of the electric force between two protons separated by distance r is $F = k_e e^2/r^2$, so the distance of separation must be

$$r = \sqrt{\frac{k_e e^2}{F}} = \sqrt{\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{2.3\times10^{-26}\ \text{N}}} = 0.10\ \text{m}$$

and (a) is the correct choice.

2. The magnitude of the electric field at distance r from a point charge q is $E = k_e q/r^2$, so

$$E = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})}{(5.11\times10^{-11}\ \text{m})^2} = 5.51\times10^{11}\ \text{N/C} \sim 10^{12}\ \text{N/C}$$

making (e) the best choice for this question.

3. To balance the weight of the ball, the magnitude of the upward electric force must equal the magnitude of the downward gravitational force, or $qE = mg$, which gives

$$E = \frac{mg}{q} = \frac{(5.0\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{4.0\times10^{-6}\ \text{C}} = 1.2\times10^4\ \text{N/C}$$

and the correct choice is (b).

4. From Newton's second law, the acceleration the electron will be

$$a_x = \frac{F_x}{m} = \frac{qE_x}{m} = \frac{(-1.60 \times 10^{-19}\ \text{C})(1.00 \times 10^{3}\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}} = -1.76 \times 10^{14}\ \text{m/s}^2$$

The kinematics equation $v_x^2 = v_{0x}^2 + 2a_x(\Delta x)$, with $v_x = 0$, gives the stopping distance as

$$\Delta x = \frac{-v_{0x}^2}{2a_x} = \frac{-(3.00 \times 10^{6}\ \text{m/s})^2}{2(-1.76 \times 10^{14}\ \text{m/s}^2)} = 2.56 \times 10^{-2}\ \text{m} = 2.56\ \text{cm}$$

so (a) is the correct response for this question.

5. Choosing the surface of the box as the closed surface of interest and applying Gauss's law, the net electric flux through the surface of the box is found to be

$$\Phi_E = \frac{Q_{\text{inside}}}{\epsilon_0} = \frac{(3.0 - 2.0 - 7.0 + 1.0) \times 10^{-9}\ \text{C}}{8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2} = -5.6 \times 10^{2}\ \text{N} \cdot \text{m}^2/\text{C}$$

meaning that (b) is the correct choice.

6. The ball is made of a metal, so free charges within the ball will very quickly rearrange themselves to produce electrostatic equilibrium at all points within the ball. As soon as electrostatic equilibrium exists inside the ball, the electric field is zero at all points within the ball. Thus, the correct choice is (c).

7. The displacement from the -4.00 nC charge at point $(0,\ 1.00)$ m to the point $(4.00, -2.00)$ m has components $r_x = (x_f - x_i) = +4.00$ m and $r_y = (y_f - y_i) = -3.00$ m, so the magnitude of this displacement is $r = \sqrt{r_x^2 + r_y^2} = 5.00$ m and its direction is $\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = -36.9°$. The x-component of the electric field at point $(4.00, -2.00)$ m is then

$$E_x = E\cos\theta = \frac{k_e q}{r^2}\cos\theta = \frac{(8.99 \times 10^{9}\ \text{N} \cdot \text{m}^2/\text{C}^2)(-4.00 \times 10^{-9}\ \text{C})}{(5.00\ \text{m})^2}\cos(-36.9°) = -1.15\ \text{N/C}$$

and the correct response is (d).

8. The magnitude of the electric force between charges Q_{1i} and Q_{2i}, separated by distance r_i, is $F_i = k_e Q_{1i} Q_{2i}/r_i^2$. If changes are made so $Q_{1f} = Q_{1i}$, $Q_{2f} = Q_{2i}/3$, and $r_f = 2r_i$, the magnitude of the new force will be

$$F_f = \frac{k_e Q_{1f} Q_{2f}}{r_f^2} = \frac{k_e Q_{1i}(Q_{2i}/3)}{(2r_i)^2} = \frac{1}{3(2)^2}\left(\frac{k_e Q_{1i} Q_{2i}}{r_i^2}\right) = \frac{1}{12}F_i$$

so, choice (a) is the correct answer for this question.

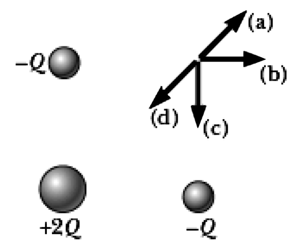
9. When a charged insulator is brought near a metallic object, free charges within the metal move around causing the metallic object to become polarized. Within the metallic object, the center of charge for the type charge opposite that on the insulator will be located closer to the charged insulator than will the center of charge for the same type as that on the insulator. This causes the attractive force between the charged insulator and the opposite type charge in the metal to exceed the magnitude of the repulsive force between the insulator and the same type charge in the metal. Thus, the net electric force between the insulator and the metallic object is one of attraction, and choice (b) is the correct answer.

10. Each of the situations described in choices (a) through (d) displays a high degree of symmetry, and as such, readily lends itself to the use of Gauss's law to determine the electric fields generated. Thus, the best answer for this question is choice (e), stating that Gauss's law can be readily applied to find the electric field in all of these contexts.

11. The outer regions of the atoms in your body and the atoms making up the ground both contain negatively charged electrons. When your body is in close proximity to the ground, these negatively charged regions exert repulsive forces on each other. Since the atoms in the solid ground are rigidly locked in position and cannot move away from your body, this repulsive force prevents your body from penetrating the ground. The best response for this question is choice (e).

12. Metal objects normally contain equal amounts of positive and negative charge and are electrically neutral. The positive charges in both metals and nonmetals are bound up in the nuclei of the atoms and cannot move about or be easily removed. However, in metals, some of the negative charges (the outer or valence electrons in the atoms) are quite loosely bound, can move about rather freely, and are easily removed from the metal. When a metal object is given a positive charge, this is accomplished by removing loosely bound electrons from the metal rather than by adding positive charge to it. Taking away the electrons to leave a net positive charge behind very slightly decreases the mass of the coin. Thus, choice (d) is the best choice for this question.

13. The positive charge $+2Q$ makes a contribution to the electric field at the upper right corner that is directed away from this charge in the direction of the arrow labeled (a). The magnitude of this contribution is $E_+ = k_e(2Q)/2s^2$, where s is the length of a side of the square. Each of the negative charges makes a contribution of magnitude $E_{-Q} = k_eQ/s^2$ directed back toward that charge. The vector sum of these two contributions due to negative charges has magnitude



$$E_- = 2E_{-Q}\cos 45° = \sqrt{2}k_eQ/s^2$$

and is directed along the diagonal of the square in the direction of the arrow labeled (d). Since $E_- > E_+$, the resultant electric field at the upper right corner of the square is in the direction of arrow (d) and has magnitude $E = E_- - E_+ = (\sqrt{2}-1)k_eQ/s^2$. The correct answer to the question is choice (d).

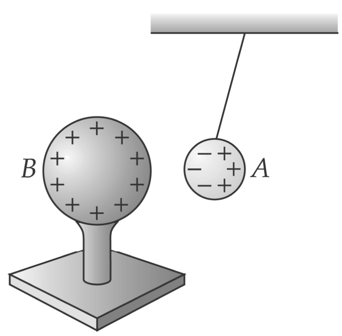
14. If the positive charge $+2Q$ at the lower left corner of the square in the above figure were removed, the field contribution E_+ discussed above would be eliminated. This would leave only $E_- = \sqrt{2}k_eQ/s^2$ as the resultant field at the upper right corner. This has a larger magnitude than the resultant field E found above, making choice (a) the correct answer.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. Conducting shoes are worn to avoid the build up of a static charge on them as the wearer walks. Rubber-soled shoes acquire a charge by friction with the floor and could discharge with a spark, possibly causing an explosive burning situation, where the burning is enhanced by the oxygen.

4. Electrons are more mobile than protons and are more easily freed from atoms than are protons.

6. No. Object A might have a charge opposite in sign to that of B, but it also might be neutral. In this latter case, object B causes object A to be polarized, pulling charge of one sign to the near face of A and pushing an equal amount of charge of the opposite sign to the far face. Then the force of attraction exerted by B on the induced charge on the near side of A is slightly larger than the force of repulsion exerted by B on the induced charge on the far side of A. Therefore, the net force on A is toward B.



8. No charge or force exists at point A because there is no particle at that point, and charge must be carried by a particle. An electric field must exist at point A. It cannot be zero because of the lack of symmetry will not allow it, regardless of the values q_1 and q_2, as long as at least one of the two is nonzero.

10. She is not shocked. She becomes part of the dome of the Van de Graaff, and charges flow onto her body. They do not jump to her body via a spark, however, so she is not shocked.

12. The antenna is similar to a lightning rod and can induce a bolt to strike it. A wire from the antenna to the ground provides a pathway for the charges to move away from the house in case a lightning strike does occur.

14. **(a)** If the charge is tripled, the flux through the surface is also tripled, because the net flux is proportional to the charge inside the surface. **(b)** The flux remains constant when the volume changes because the surface surrounds the same amount of charge, regardless of its volume. **(c)** The flux does not change when the shape of the closed surface changes. **(d)** The flux through the closed surface remains unchanged as the charge inside the surface is moved to another location inside that surface. **(e)** The flux is zero because the charge inside the surface is zero. All of these conclusions are arrived at through an understanding of Gauss's law.

PROBLEM SOLUTIONS

15.1 $$F = k_e \frac{Q_1 Q_2}{r^2} = \left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(7.5\times10^{-9}\ \mathrm{C}\right)\left(4.2\times10^{-9}\ \mathrm{C}\right)}{(1.8\ \mathrm{m})^2} = \boxed{8.7\times10^{-8}\ \mathrm{N}}$$

Since these are like charges (both positive), the force is $\boxed{\text{repulsive}}$.

15.2 Particle A exerts a force toward the right on particle B. By Newton's third law, particle B will then exert a force toward the left back on particle A. The ratio of the final magnitude of the force to the original magnitude of the force is

$$\frac{F_f}{F_i} = \frac{k_e q_1 q_2 / r_f^2}{k_e q_1 q_2 / r_i^2} = \left(\frac{r_i}{r_f}\right)^2 \qquad \text{so} \qquad F_f = F_i\left(\frac{r_i}{r_f}\right) = (2.62\ \mu\mathrm{N})\left(\frac{13.7\ \mathrm{mm}}{17.7\ \mathrm{mm}}\right)^2 = 1.57\ \mu\mathrm{N}$$

The final vector force that B exerts on A is $\boxed{1.57\ \mu\text{N directed to the left}}$.

15.3 (a) When the balls are an equilibrium distance apart, the tension in the string equals the magnitude of the repulsive electric force between the balls. Thus,

$$F = \frac{k_e q(2q)}{r^2} = 2.50\ \mathrm{N} \quad \Rightarrow \quad q^2 = \frac{(2.50\ \mathrm{N})r^2}{2k_e}$$

or $$q = \sqrt{\frac{(2.50\ \mathrm{N})(2.00\ \mathrm{m})^2}{2\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)}} = 2.36\times10^{-5}\ \mathrm{C} = \boxed{23.6\ \mu\mathrm{C}}$$

(b) The charges induce opposite charges in the bulkheads, but the induced charge in the bulkhead near ball B is greater, due to B's greater charge. Therefore, the system $\boxed{\text{moves slowly towards the bulkhead closer to ball } B}$.

15.4 (a) The two ions are both singly charged $\left(|q| = 1e\right)$, one positive and one negative. Thus,

$$|F| = \frac{k_e |q_1||q_2|}{r^2} = \frac{k_e e^2}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{\left(0.50 \times 10^{-9} \text{ m}\right)^2} = \boxed{9.2 \times 10^{-10} \text{ N}}$$

Since these are unlike charges, the force is $\boxed{\text{attractive}}$.

(b) $\boxed{\text{No.}}$ The electric force depends only on the magnitudes of the two charges and the distance between them.

15.5 (a) $F = \dfrac{k_e (2e)^2}{r^2} = \dfrac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left[4\left(1.60 \times 10^{-19} \text{ C}\right)^2\right]}{\left(5.00 \times 10^{-15} \text{ m}\right)^2} = \boxed{36.8 \text{ N}}$

(b) The mass of an alpha particle is $m = 4.002\,6$ u, where $1 \text{ u} = 1.66 \times 10^{-27}$ kg is the unified mass unit. The acceleration of either alpha particle is then

$$a = \frac{F}{m} = \frac{36.8 \text{ N}}{4.002\,6\left(1.66 \times 10^{-27} \text{ kg}\right)} = \boxed{5.54 \times 10^{27} \text{ m/s}^2}$$

15.6 The attractive force between the charged ends tends to compress the molecule. Its magnitude is

$$F = \frac{k_e (1e)^2}{r^2} = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\frac{\left(1.60 \times 10^{-19} \text{ C}\right)^2}{\left(2.17 \times 10^{-6} \text{ m}\right)^2} = 4.89 \times 10^{-17} \text{ N}$$

The compression of the "spring" is

$$x = (0.010\,0)r = (0.010\,0)\left(2.17 \times 10^{-6} \text{ m}\right) = 2.17 \times 10^{-8} \text{ m},$$

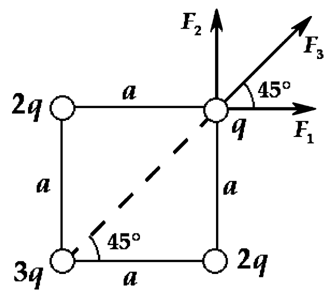
so the spring constant is $k = \dfrac{F}{x} = \dfrac{4.89 \times 10^{-17} \text{ N}}{2.17 \times 10^{-8} \text{ m}} = \boxed{2.25 \times 10^{-9} \text{ N/m}}$.

15.7 1.00 g of hydrogen contains Avogadro's number of atoms, each containing one proton and one electron. Thus, each charge has magnitude $|q| = N_A$. The distance separating these charges is $r = 2R_E$, where R_E is Earth's radius. Thus,

$$F = \frac{k_e \left(N_A e\right)^2}{\left(2 R_E\right)^2}$$

$$= \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\frac{\left[\left(6.02 \times 10^{23}\right)\left(1.60 \times 10^{-19} \text{ C}\right)\right]^2}{4\left(6.38 \times 10^6 \text{ m}\right)^2} = \boxed{5.12 \times 10^5 \text{ N}}$$

15.8 Refer to the sketch at the right. The magnitudes of the



$$F_1 = F_2 = k_e \frac{q(2q)}{a^2} = 2k_e \frac{q^2}{a^2} \text{ and } F_3 = k_e \frac{q(3q)}{\left(a\sqrt{2}\right)^2} = 1.50k_e \frac{q^2}{a^2}$$

The components of the resultant force on charge q are

$$F_x = F_1 + F_3 \cos 45° = (2 + 1.50\cos 45°)\, k_e \frac{q^2}{a^2} = 3.06k_e \frac{q^2}{a^2}$$

and $$F_y = F_1 + F_3 \sin 45° = (2 + 1.50\sin 45°)\, k_e \frac{q^2}{a^2} = 3.06k_e \frac{q^2}{a^2}$$

The magnitude of the resultant force is $F_e = \sqrt{F_x^2 + F_y^2} = \sqrt{2}\left(3.06k_e \frac{q^2}{a^2}\right) = \boxed{4.33k_e \frac{q^2}{a^2}}$

and it is directed at $\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}(1.00) = \boxed{45° \text{ above the horizontal}}$.

15.9 (a) The spherically symmetric charge distributions behave as if all charge was located at the centers of the spheres. Therefore, the magnitude of the $\boxed{\text{attractive}}$ force is

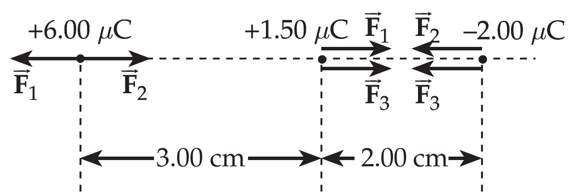
$$F = \frac{k_e q_1 |q_2|}{r^2} = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \frac{\left(12 \times 10^{-9} \text{ C}\right)\left(18 \times 10^{-9} \text{ C}\right)}{(0.30 \text{ m})^2} = \boxed{2.2 \times 10^{-5} \text{ N}}$$

(b) When the spheres are connected by a conducting wire, the net charge $q_{\text{net}} = q_1 + q_2 = -6.0 \times 10^{-9}$ C will divide equally between the two identical spheres. Thus, the force is now

$$F = \frac{k_e \left(q_{\text{net}}/2\right)^2}{r^2} = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \frac{\left(-6.0 \times 10^{-9} \text{ C}\right)^2}{4(0.30 \text{ m})^2}$$

or $F = \boxed{9.0 \times 10^{-7} \text{ N (repulsion)}}$

15.10 The forces are as shown in the sketch at the right.



$$F_1 = \frac{k_e q_1 q_2}{r_{12}^2} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(6.00\times10^{-6}\ \text{C}\right)\left(1.50\times10^{-6}\ \text{C}\right)}{\left(3.00\times10^{-2}\ \text{m}\right)^2} = 89.9\ \text{N}$$

$$F_2 = \frac{k_e q_1 |q_3|}{r_{13}^2} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(6.00\times10^{-6}\ \text{C}\right)\left(2.00\times10^{-6}\ \text{C}\right)}{\left(5.00\times10^{-2}\ \text{m}\right)^2} = 43.2\ \text{N}$$

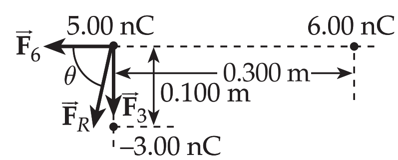
$$F_3 = \frac{k_e q_2 |q_3|}{r_{23}^2} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(1.50\times10^{-6}\ \text{C}\right)\left(2.00\times10^{-6}\ \text{C}\right)}{\left(2.00\times10^{-2}\ \text{m}\right)^2} = 67.4\ \text{N}$$

The net force on the $6\,\mu\text{C}$ charge is $F_6 = F_1 - F_2 = \boxed{46.7\ \text{N (to the left)}}$

The net force on the $1.5\,\mu\text{C}$ charge is $F_{1.5} = F_1 + F_3 = \boxed{157\ \text{N (to the right)}}$

The net force on the $-2\,\mu\text{C}$ charge is $F_{-2} = F_2 + F_3 = \boxed{111\ \text{N (to the left)}}$

15.11 In the sketch at the right, F_R is the resultant of the forces F_6 and F_3 that are exerted on the charge at the origin by the 6.00 nC and the –3.00 nC charges, respectively.



$$F_6 = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(6.00\times10^{-9}\ \text{C}\right)\left(5.00\times10^{-9}\ \text{C}\right)}{(0.300\ \text{m})^2}$$

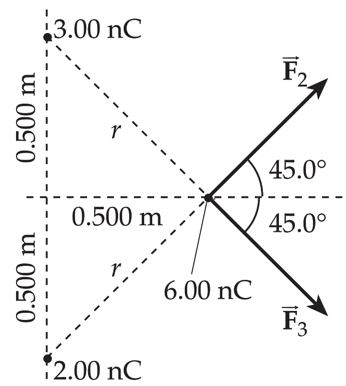
$$= 3.00\times10^{-6}\ \text{N}$$

$$F_3 = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(3.00\times10^{-9}\ \text{C}\right)\left(5.00\times10^{-9}\ \text{C}\right)}{(0.100\ \text{m})^2} = 1.35\times10^{-5}\ \text{N}$$

The resultant is $F_R = \sqrt{\left(F_6\right)^2 + \left(F_3\right)^2} = 1.38\times10^{-5}\ \text{N}$ at $\theta = \tan^{-1}\left(\frac{F_3}{F_6}\right) = 77.5°$

or $\vec{\mathbf{F}}_R = \boxed{1.38\times10^{-5}\,\text{N at } 77.5° \text{ below } -x\text{-axis}}$

15.12 Consider the arrangement of charges shown in the sketch at the right. The distance r is



$$r=\sqrt{(0.500\text{ m})^2+(0.500\text{ m})^2}=0.707\text{ m}$$

The forces exerted on the 6.00 nC charge are

$$F_2=\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(6.00\times10^{-9}\text{ C})(2.00\times10^{-9}\text{ C})}{(0.707\text{ m})^2}$$

$$=2.16\times10^{-7}\text{ N}$$

and $F_3=\left(8.99\times10^9\ \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\dfrac{(6.00\times10^{-9}\text{ C})(3.00\times10^{-9}\text{ C})}{(0.707\text{ m})^2}=3.24\times10^{-7}\text{ N}$

Thus, $\Sigma F_x=(F_2+F_3)\cos 45.0°=3.81\times10^{-7}\text{ N}$

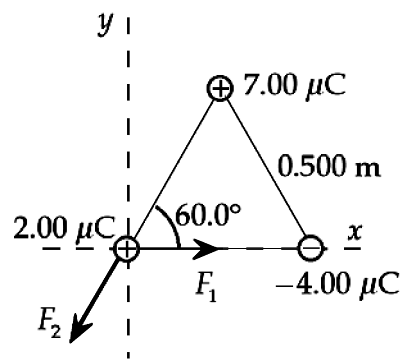
and $\Sigma F_y=(F_2-F_3)\sin 45.0°=-7.63\times10^{-8}\text{ N}$

The resultant force on the 6.00 nC charge is then

$$F_R=\sqrt{(\Sigma F_x)^2+(\Sigma F_y)^2}=3.89\times10^{-7}\text{ N at }\theta=\tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)=-11.3°$$

or $\vec{\mathbf{F}}_R=\boxed{3.89\times10^{-7}\text{ N at 11.3° below }+x\text{-axis}}$

15.13 Please see the sketch at the right.



$$F_1=\frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\text{ C})(4.00\times10^{-6}\text{ C})}{(0.500\text{ m})^2}$$

or $F_1=0.288\text{ N}$

$$F_2=\frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\text{ C})(7.00\times10^{-6}\text{ C})}{(0.500\text{ m})^2}$$

or $F_2=0.503\text{ N}$

The components of the resultant force acting on the $2.00\ \mu\text{C}$ charge are:

$$F_x=F_1-F_2\cos 60.0°=0.288\text{ N}-(0.503\text{ N})\cos 60.0°=3.65\times10^{-2}\text{ N}$$

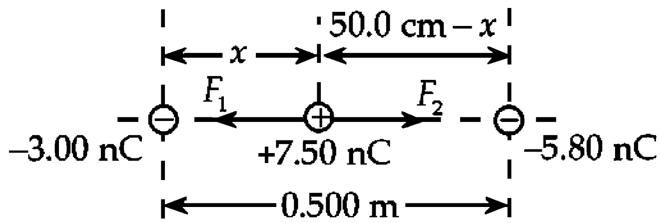
and $F_y=-F_2\sin 60.0°=-(0.503\text{ N})\sin 60.0°=-0.436\text{ N}$

The magnitude and direction of this resultant force are

$$F=\sqrt{F_x^2+F_y^2}=\sqrt{(0.0365\text{ N})^2+(0.436\text{ N})^2}=\boxed{0.438\text{ N}}$$

at $\theta=\tan^{-1}\left(\dfrac{F_y}{F_x}\right)=\tan^{-1}\left(\dfrac{-0.436\text{ N}}{0.0365\text{ N}}\right)=-85.2°$ or $\boxed{85.2°\text{ below }+x\text{-axis}}$

15.14 If the forces exerted on the positive third charge by the two negative charges are to be in opposite directions as they must, the third charge must be located on the line between the two negative charges as shown at the right.



If the two forces are to add to zero, their magnitudes must be equal. These magnitudes are

$$F_1 = k_e \frac{|q_1||q_3|}{x^2} \quad \text{and} \quad F_2 = k_e \frac{|q_2||q_3|}{(50.0 \text{ cm} - x)^2}$$

where $q_1 = -3.00$ nC and $q_2 = -5.80$ nC.

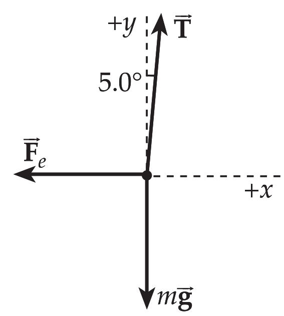
Equating and canceling common factors gives $(50.0 \text{ cm} - x)^2 = \dfrac{|q_2|}{|q_1|} x^2$

or

$$50.0 \text{ cm} - x = x\sqrt{\frac{5.80 \text{ nC}}{3.00 \text{ nC}}} = 1.39\,x \quad \text{giving} \quad 2.39\,x = 50.0 \text{ cm} \quad \text{and} \quad x = 20.9 \text{ cm}$$

Thus, the third charged should be located $\boxed{20.9 \text{ cm to the right of the} -3.00 \text{ nC charge}}$ or $\boxed{50.0 \text{ cm} - 20.9 \text{ cm} = 29.1 \text{ cm to the left of the} -5.80 \text{ nC charge}}$.

15.15 Consider the free-body diagram of one of the spheres given at the right. Here, T is the tension in the string and F_e is the repulsive electrical force exerted by the other sphere.



$$\sum F_y = 0 \;\Rightarrow\; T\cos 5.0° = mg, \text{ or } T = \frac{mg}{\cos 5.0°}$$

$$\sum F_x = 0 \;\Rightarrow\; F_e = T\sin 5.0° = mg\tan 5.0°$$

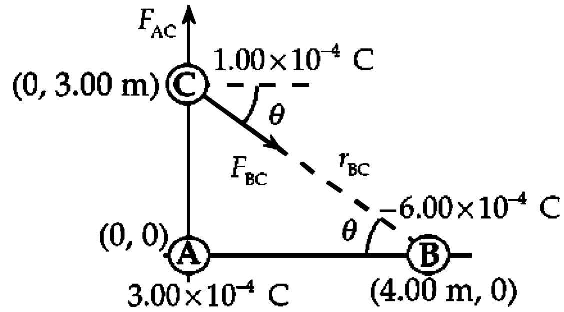
At equilibrium, the distance separating the two spheres is $r = 2\,L\sin 5.0°$.

Thus, $F_e = mg\tan 5.0°$ becomes $\dfrac{k_e q^2}{(2\,L\sin 5.0°)^2} = mg\tan 5.0°$ and yields

$$q = (2\,L\sin 5.0°)\sqrt{\frac{mg\tan 5.0°}{k_e}}$$

$$= \left[2(0.300 \text{ m})\sin 5.0°\right]\sqrt{\frac{(0.20\times10^{-3} \text{ kg})(9.80 \text{ m/s}^2)\tan 5.0°}{8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}} = \boxed{7.2 \text{ nC}}$$

15.16 In the sketch at the right,



$$r_{BC}=\sqrt{(4.00\text{ m})^2+(3.00\text{ m})^2}=5.00\text{ m}$$

and $\theta=\tan^{-1}\left(\dfrac{3.00\text{ m}}{4.00\text{ m}}\right)=36.9°$

(a) $(F_{AC})_x=\boxed{0}$

(b) $(F_{AC})_y=|F_{AC}|=k_e\dfrac{|q_A||q_C|}{r_{AC}^2}$

$$(F_{AC})_y=\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(3.00\times10^{-4}\text{ C}\right)\left(1.00\times10^{-4}\text{ C}\right)}{(3.00\text{ m})^2}=\boxed{30.0\text{ N}}$$

(c) $|F_{BC}|=k_e\dfrac{|q_B||q_C|}{r_{BC}^2}=\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\dfrac{\left(6.00\times10^{-4}\text{ C}\right)\left(1.00\times10^{-4}\text{ C}\right)}{(5.00\text{ m})^2}=\boxed{21.6\text{ N}}$

(d) $(F_{BC})_x=|F_{BC}|\cos\theta=(21.6\text{ N})\cos(36.9°)=\boxed{17.3\text{ N}}$

(e) $(F_{BC})_y=-|F_{BC}|\sin\theta=-(21.6\text{ N})\sin(36.9°)=\boxed{-13.0\text{ N}}$

(f) $(F_R)_x=(F_{AC})_x+(F_{BC})_x=0+17.3\text{ N}=\boxed{17.3\text{ N}}$

(g) $(F_R)_y=(F_{AC})_y+(F_{BC})_y=30.0-13.0\text{ N}=\boxed{17.0\text{ N}}$

(h) $F_R=\sqrt{(F_R)_x^2+(F_R)_y^2}=\sqrt{(17.3\text{ N})^2+(17.0\text{ N})^2}=\boxed{24.3\text{ N}}$

and $\varphi=\tan^{-1}\left[\dfrac{(F_R)_y}{(F_R)_x}\right]=\tan^{-1}\left(\dfrac{17.0\text{ N}}{17.3\text{ N}}\right)=\boxed{44.5°}$

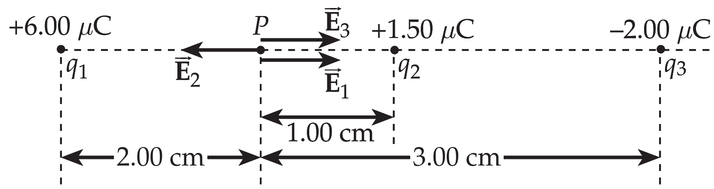
or $\boxed{\vec{\mathbf{F}}_R=24.3\text{ N at }44.5°\text{ above the }+x\text{ direction}}$

15.17 In order for the object to "float" in the electric field, the electric force exerted on the object by the field must be directed upward and have a magnitude equal to the weight of the object. Thus, $F_e=qE=mg$, and the magnitude of the electric field must be

$$E=\frac{mg}{|q|}=\frac{\left(3.80\ \cancel{\text{g}}\right)\left(9.80\text{ m/s}^2\right)}{18\times10^{-6}\text{ C}}\left(\frac{1\text{ kg}}{10^3\ \cancel{\text{g}}}\right)=\boxed{2.07\times10^3\text{ N/C}}$$

The electric force on a negatively charged object is in the direction opposite to that of the electric field. Since the electric force must be directed upward, the electric field must be $\boxed{\text{directed downward.}}$

15.18 (a) Taking to the right as positive, the resultant electric field at point P is given by



$$E_R = E_1 + E_3 - E_2$$

$$= \frac{k_e|q_1|}{r_1^2} + \frac{k_e|q_3|}{r_3^2} - \frac{k_e|q_2|}{r_2^2}$$

$$= \left(8.99\times10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\left[\frac{6.00\times10^{-6}\ \mathrm{C}}{(0.020\,0\ \mathrm{m})^2} + \frac{2.00\times10^{-6}\ \mathrm{C}}{(0.030\,0\ \mathrm{m})^2} - \frac{1.50\times10^{-6}\ \mathrm{C}}{(0.010\,0\ \mathrm{m})^2}\right]$$

This gives $E_R = +2.00\times10^7\ \mathrm{N/C}$

or $\vec{\mathbf{E}}_R = \boxed{2.00\times10^7\ \mathrm{N/C}\ \text{to the right}}$

(b) $\vec{\mathbf{F}} = q\vec{\mathbf{E}}_R = (-2.00\times10^{-6}\ \mathrm{C})(2.00\times10^7\ \mathrm{N/C}) = -40.0\ \mathrm{N}$

or $\vec{\mathbf{F}} = \boxed{40.0\ \mathrm{N}\ \text{to the left}}$

15.19 We shall treat the concentrations as point charges. Then, the resultant field consists of two contributions, one due to each concentration.

The contribution due to the positive charge at 3 000 m altitude is

$$E_+ = k_e\frac{|q|}{r^2} = \left(8.99\times10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{(40.0\ \mathrm{C})}{(1\,000\ \mathrm{m})^2} = 3.60\times10^5\ \mathrm{N/C}\ (\text{downward})$$

The contribution due to the negative charge at 1 000 m altitude is

$$E_- = k_e\frac{|q|}{r^2} = \left(8.99\times10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{(40.0\ \mathrm{C})}{(1\,000\ \mathrm{m})^2} = 3.60\times10^5\ \mathrm{N/C}\ (\text{downward})$$

The resultant field is then

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_+ + \vec{\mathbf{E}}_- = \boxed{7.20\times10^5\ \mathrm{N/C}\ (\text{downward})}$$

15.20 (a) The magnitude of the force on the electron is $F = |q|E = eE$, and the acceleration is

$$a = \frac{F}{m_e} = \frac{eE}{m_e} = \frac{(1.60\times10^{-19}\ \mathrm{C})(300\ \mathrm{N/C})}{9.11\times10^{-31}\ \mathrm{kg}} = \boxed{5.27\times10^{13}\ \mathrm{m/s^2}}$$

(b) $v = v_0 + at = 0 + (5.27\times10^{13}\ \mathrm{m/s^2})(1.00\times10^{-8}\ \mathrm{s}) = \boxed{5.27\times10^5\ \mathrm{m/s}}$

15.21 Note that at the point midway between the two charges ($x = 2.00$ m, $y = 0$), the field contribution $\vec{\mathbf{E}}_1$ (due to the negative charge at the origin) and the contribution $\vec{\mathbf{E}}_2$ (due to the positive charge at $x = 4.00$ m) are both in the negative x-direction. The magnitude of the resultant field at this point is therefore

$$E_{\text{net}} = E_1 + E_2 = k_e\left(\frac{|q_1|}{r_1^2} + \frac{q_2}{r_2^2}\right) = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{5.00\times10^{-9}\ \text{C}}{(2.00\ \text{m})^2} + \frac{7.00\times10^{-9}\ \text{C}}{(2.00\ \text{m})^2}\right] = 27.0\ \text{N/C}$$

Thus, $\vec{\mathbf{E}}_{\text{net}} = \boxed{27.0\ \text{N/C in the negative } x \text{ direction}}$.

15.22 The force an electric field exerts on a positive change is in the direction of the field. Since this force must serve as a retarding force and bring the proton to rest, the force and hence the field must be $\boxed{\text{in the direction opposite to the proton's velocity}}$.

The work-energy theorem, $W_{\text{net}} = KE_f - KE_i$, gives the magnitude of the field as

$$-(qE)\Delta x = 0 - KE_i \quad \text{or} \quad E = \frac{KE_i}{q(\Delta x)} = \frac{3.25\times10^{-15}\ \text{J}}{\left(1.60\times10^{-19}\ \text{C}\right)(1.25\ \text{m})} = \boxed{1.63\times10^4\ \text{N/C}}$$

15.23 (a) $$a = \frac{F}{m} = \frac{qE}{m_p} = \frac{\left(1.60\times10^{-19}\ \text{C}\right)(640\ \text{N/C})}{1.673\times10^{-27}\ \text{kg}} = \boxed{6.12\times10^{10}\ \text{m/s}^2}$$

(b) $$t = \frac{\Delta v}{a} = \frac{1.20\times10^6\ \text{m/s}}{6.12\times10^{10}\ \text{m/s}^2} = 1.96\times10^{-5}\ \text{s} = \boxed{19.6\ \mu\text{s}}$$

(c) $$\Delta x = \frac{v_f^2 - v_0^2}{2a} = \frac{\left(1.20\times10^6\ \text{m/s}\right)^2 - 0}{2\left(6.12\times10^{10}\ \text{m/s}^2\right)} = \boxed{11.8\ \text{m}}$$

(d) $$KE_f = \frac{1}{2}m_p v_f^2 = \frac{1}{2}\left(1.673\times10^{-27}\ \text{kg}\right)\left(1.20\times10^6\ \text{m/s}\right)^2 = \boxed{1.20\times10^{-15}\ \text{J}}$$

15.24 (a) Please refer to the solution of Problem 15.13 earlier in this chapter. There it is shown that the resultant electric force experienced by the 2.00 μC located at the origin is $\vec{\mathbf{F}} = 0.438$ N at 85.2° below the $+x$-axis. Since the electric field at a location is defined as the force per unit charge experienced by a test charge placed in that location, the electric field at the origin in the charge configuration of Figure P15.13 is

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_0} = \frac{0.438\ \text{N}}{2.00\times10^{-6}\ \text{C}}\ \text{at} - 85.2° = \boxed{2.19\times10^5\ \text{N/C at } 85.2°\ \text{below } +x\text{-axis}}$$

(b) The electric force experienced by the charge at the origin is directly proportional to the magnitude of that charge. Thus, doubling the magnitude of this charge would $\boxed{\text{double the magnitude of the electric force}}$. However, the electric field is the force per unit charge and $\boxed{\text{the field would be unchanged if the charge was doubled}}$. This is easily seen in the calculation of part (a) above. Doubling the magnitude of the charge at the origin would double both the numerator and the denominator of the ratio $\vec{\mathbf{F}}/q_0$, but the value of the ratio (i.e., the electric field) would be unchanged.

15.25 The alpha particle (with a charge $+2e$ and mass of 6.63×10^{-27} kg) will experience a constant electric force in the $-y$-direction (the direction of the electric field) once it enters the tube. This force gives the alpha particle a constant acceleration with components

$$a_x = 0 \qquad \text{and} \qquad a_y = \frac{F_e}{m} = \frac{qE_y}{m} = \frac{\left(3.20\times10^{-19}\ \text{C}\right)\left(-4.50\times10^{-4}\ \text{N/C}\right)}{6.64\times10^{-27}\ \text{kg}} = -2.17\times10^{4}\ \text{m/s}^2$$

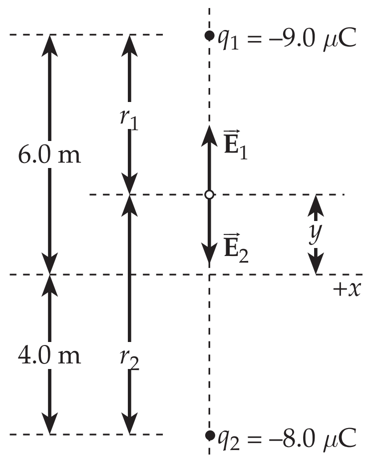
The particle will strike the tube wall when its vertical displacement is $\Delta y = -0.500$ cm, and from $\Delta y = v_{0y}t + a_y t^2/2$ with $v_{0y} = 0$, the elapsed time when this occurs is

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2\left(-0.500\times10^{-2}\ \text{m}\right)}{-2.17\times10^{4}\ \text{m/s}^2}} = 6.79\times10^{-4}\ \text{s}$$

During this time, the distance the alpha particle travels parallel to the axis of the tube is

$$\Delta x = v_{0x}t + \frac{1}{2}a_x t^2 = (1\,250\ \text{m/s})\left(6.79\times10^{-4}\ \text{s}\right) + 0 = \boxed{0.849\ \text{m}}$$

15.26 If the resultant field is to be zero, the contributions of the two charges must be equal in magnitude and must have opposite directions. This is only possible at a point on the line between the two negative charges.

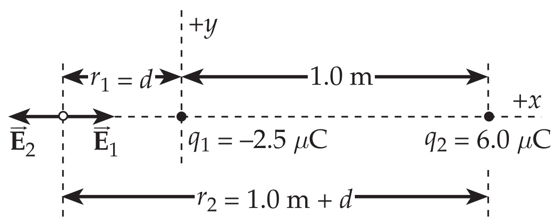


Assume the point of interest is located on the y-axis at $-4.0\ \text{m} < y < 6.0\ \text{m}$. Then, for equal magnitudes,

$$\frac{k_e|q_1|}{r_1^2} = \frac{k_e|q_2|}{r_2^2} \quad \text{or} \quad \frac{9.0\ \mu\text{C}}{(6.0\ \text{m} - y)^2} = \frac{8.0\ \mu\text{C}}{(y + 4.0\ \text{m})^2}$$

Solving for y gives $y + 4.0\ \text{m} = \sqrt{\frac{8}{9}}(6.0 - y)$, or $y = \boxed{+0.85\ \text{m}}$.

15.27 If the resultant field is zero, the contributions from the two charges must be in opposite directions and also have equal magnitudes. Choose the line connecting the charges as the x-axis, with the origin at the $-2.5\ \mu$C charge. Then, the two contributions will have opposite directions only in the regions $x < 0$ and $x > 1.0$ m. For the magnitudes to be equal, the point must be nearer the smaller charge. Thus, the point of zero resultant field is on the x-axis at $x < 0$.



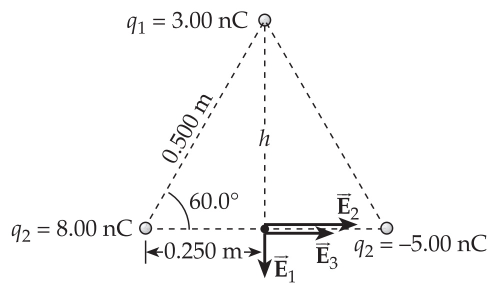
Requiring equal magnitudes gives $\dfrac{k_e|q_1|}{r_1^2} = \dfrac{k_e|q_2|}{r_2^2}$ or $\dfrac{2.5\ \mu\text{C}}{d^2} = \dfrac{6.0\ \mu\text{C}}{(1.0\ \text{m} + d)^2}$

Thus, $(1.0\ \text{m} + d)\sqrt{\dfrac{2.5}{6.0}} = d$

Solving for d yields

$$d = 1.8\ \text{m}, \qquad \text{or} \qquad \boxed{1.8\ \text{m to the left of the } -2.5\ \mu\text{C charge}}$$

15.28 The altitude of the triangle is



$$h = (0.500 \text{ m})\sin 60.0° = 0.433 \text{ m}$$

and the magnitudes of the fields due to each of the charges are

$$E_1 = \frac{k_e q_1}{h^2} = \frac{(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(3.00\times 10^{-9}\text{ C})}{(0.433\text{ m})^2}$$

$$= 144 \text{ N/C}$$

$$E_2 = \frac{k_e|q_2|}{r_2^2} = \frac{(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(8.00\times 10^{-9}\text{ C})}{(0.250\text{ m})^2} = 1.15\times 10^3 \text{ N/C}$$

and $E_3 = \dfrac{k_e|q_3|}{r_3^2} = \dfrac{(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times 10^{-9}\text{ C})}{(0.250\text{ m})^2} = 719 \text{ N/C}$

Thus, $\Sigma E_x = E_2 + E_3 = 1.87\times 10^3 \text{ N/C}$ and $\Sigma E_y = -E_1 = -144 \text{ N/C}$ giving

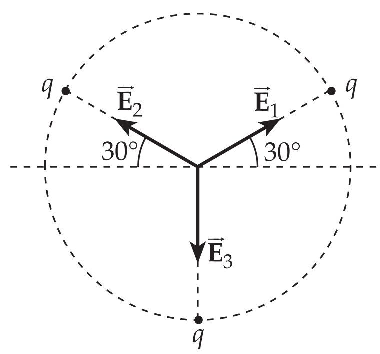
$$E_R = \sqrt{(\Sigma E_x)^2 + (\Sigma E_y)^2} = 1.88\times 10^3 \text{ N/C}$$

and

$$\theta = \tan^{-1}(\Sigma E_y/\Sigma E_x) = \tan^{-1}(-0.0769) = -4.40°$$

Hence $\boxed{\vec{\mathbf{E}}_R = 1.88\times 10^3 \text{ N/C at } 4.40° \text{ below the } +x\text{-axis}}$

15.29 From the symmetry of the charge distribution, students should recognize that the resultant electric field at the center is



$$\boxed{\vec{\mathbf{E}}_R = 0}$$

If one does not recognize this intuitively, consider:

$$\vec{\mathbf{E}}_R = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3, \text{ so}$$

$$E_x = E_{1x} - E_{2x} = \frac{k_e|q|}{r^2}\cos 30° - \frac{k_e|q|}{r^2}\cos 30° = 0$$

and

$$E_y = E_{1y} + E_{2y} - E_3 = \frac{k_e|q|}{r^2}\sin 30° + \frac{k_e|q|}{r^2}\sin 30° - \frac{k_e|q|}{r^2} = 0$$

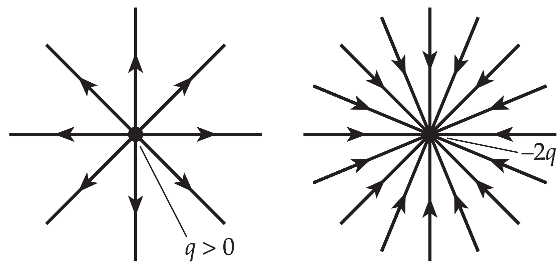
Thus, $E_R = \sqrt{E_x^2 + E_y^2} = \boxed{0}$

15.30 The magnitude of q_2 is three times the magnitude of q_1 because 3 times as many lines emerge from q_2 as enter q_1. $|q_2| = 3|q_1|$

(a) Then, $\boxed{q_1/q_2 = -1/3}$

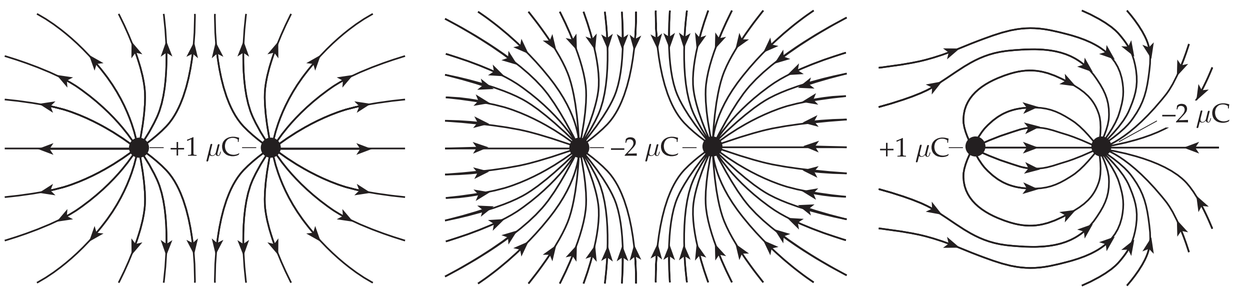
(b) $\boxed{q_2 > 0}$ because lines emerge from it, and $\boxed{q_1 < 0}$ because lines terminate on it.

15.31 Note in the sketches at the right that electric field lines originate on positive charges and terminate on negative charges. The density of lines is twice as great for the $-2q$ charge in (b) as it is for the $1q$ charge in (a).



(a) **(b)**

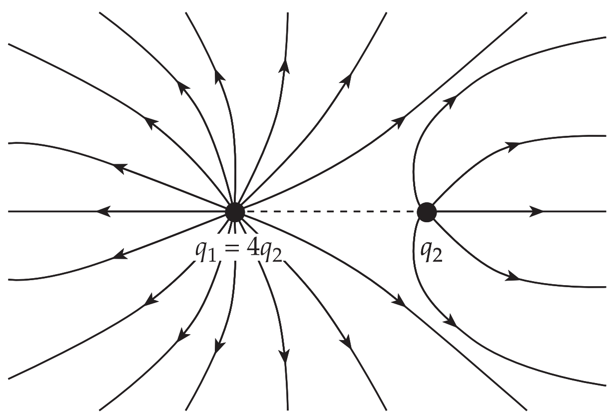
15.32 Rough sketches for these charge configurations are shown below.



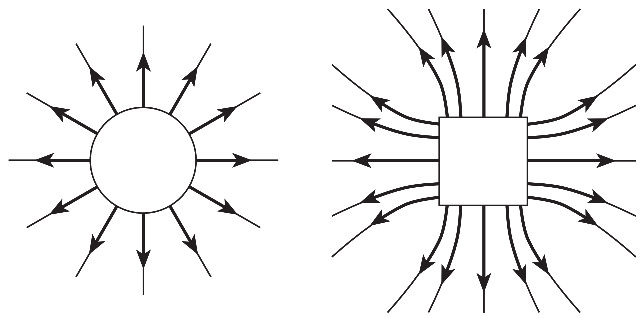
(a) **(b)** **(c)**

15.33 (a) The sketch for (a) is shown at the right. Note that four times as many lines should leave q_1 as emerge from q_2 although, for clarity, this is not shown in this sketch.

(b) The field pattern looks the same here as that shown for (a) with the exception that the arrows are reversed on the field lines.



15.34 (a) In the sketch for (a) at the right, note that there are no lines inside the sphere. On the outside of the sphere, the field lines are uniformly spaced and radially outward.



(a) (b)

(b) In the sketch for (b) above, note that the lines are perpendicular to the surface at the points where they emerge. They should also be symmetrical about the symmetry axes of the cube. The field is zero inside the cube.

15.35 (a) $\boxed{\text{Zero}}$ net charge on each surface of the sphere.

(b) The negative charge lowered into the sphere repels $\boxed{-5\mu\text{C on the outside}}$ surface, and leaves $\boxed{+5\ \mu\text{C on the inside}}$ surface of the sphere.

(c) The negative charge lowered inside the sphere neutralizes the inner surface, leaving $\boxed{\text{zero charge on the inside}}$. This leaves $\boxed{-5\mu\text{C on the outside}}$ surface of the sphere.

(d) When the object is removed, the sphere is left with $\boxed{-5.00\ \mu\text{C on the outside}}$ surface and $\boxed{\text{zero charge on the inside}}$.

15.36 (a) The dome is a closed conducting surface. Therefore, the electric field is $\boxed{\text{zero}}$ everywhere inside it.

At the surface and outside of this spherically symmetric charge distribution, the field is as if all the charge were concentrated at the center of the sphere.

(b) At the surface,

$$E = \frac{k_e q}{R^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.0\times10^{-4}\ \text{C}\right)}{(1.0\ \text{m})^2} = \boxed{1.8\times10^6\ \text{N/C}}$$

(c) Outside the spherical dome, $E = \dfrac{k_e q}{r^2}$. Thus, at $r = 4.0$ m,

$$E = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.0\times10^{-4}\ \text{C}\right)}{(4.0\ \text{m})^2} = \boxed{1.1\times10^5\ \text{N/C}}$$

15.37 For a uniformly charged sphere, the field is strongest at the surface.

Thus, $E_{\text{max}} = \dfrac{k_e q_{\text{max}}}{R^2}$,

or $$q_{\text{max}} = \frac{R^2 E_{\text{max}}}{k_e} = \frac{(2.0\ \text{m})^2\left(3.0\times10^6\ \text{N/C}\right)}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{1.3\times10^{-3}\ \text{C}}$$

15.38 If the weight of the drop is balanced by the electric force, then $mg = |q|E = eE$ or the mass of the drop must be

$$m = \frac{eE}{g} = \frac{\left(1.6\times10^{-19}\ \text{C}\right)\left(3\times10^{4}\ \text{N/C}\right)}{9.8\ \text{m/s}^2} \approx 5\times10^{-16}\ \text{kg}$$

But, $m = \rho V = \rho\left(\frac{4}{3}\pi r^3\right)$ and the radius of the drop is $r = \left[\frac{3m}{4\pi\rho}\right]^{1/3}$

$$r = \left[\frac{3\left(5\times10^{-16}\ \text{kg}\right)}{4\pi\left(858\ \text{kg/m}^3\right)}\right]^{1/3} = 5.2\times10^{-7}\ \text{m} \qquad \text{or} \qquad r \sim \boxed{1\ \mu\text{m}}$$

15.39 (a) $F_e = ma = \left(1.67\times10^{-27}\ \text{kg}\right)\left(1.52\times10^{12}\ \text{m/s}^2\right) = \boxed{2.54\times10^{-15}\ \text{N}}$ in the direction of the acceleration, or radially outward.

(b) The direction of the field is the direction of the force on a positive charge (such as the proton). Thus, the field is directed $\boxed{\text{radially outward}}$. The magnitude of the field is

$$E = \frac{F_e}{q} = \frac{2.54\times10^{-15}\ \text{N}}{1.60\times10^{-19}\ \text{C}} = \boxed{1.59\times10^{4}\ \text{N/C}}$$

15.40 The flux through an area is $\Phi_E = EA\cos\theta$, where θ is the angle between the direction of the field E and the line perpendicular to the area A.

(a) $\Phi_E = EA\cos\theta = \left(6.2\times10^{5}\ \text{N/C}\right)\left(3.2\ \text{m}^2\right)\cos 0° = \boxed{2.0\times10^{6}\ \text{N}\cdot\text{m}^2/\text{C}}$

(b) In this case, $\theta = 90°$ and $\Phi_E = \boxed{0}$.

15.41 The area of the rectangular plane is $A = (0.350\ \text{m})(0.700\ \text{m}) = 0.245\ \text{m}^2$.

(a) When the plane is parallel to the yz-plane, $\theta = 0°$, and the flux is

$$\Phi_E = EA\cos\theta = \left(3.50\times10^{3}\ \text{N/C}\right)\left(0.245\ \text{m}^2\right)\cos 0° = \boxed{858\ \text{N}\cdot\text{m}^2/\text{C}}$$

(b) When the plane is parallel to the x-axis, $\theta = 90°$ and $\Phi_E = \boxed{0}$.

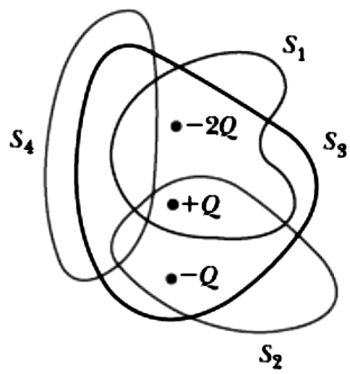
(c) $\Phi_E = EA\cos\theta = \left(3.50\times10^{3}\ \text{N/C}\right)\left(0.245\ \text{m}^2\right)\cos 40° = \boxed{657\ \text{N}\cdot\text{m}^2/\text{C}}$

15.42 (a) Gauss's law states that the electric flux through any closed surface equals the net charge enclosed divided by ϵ_0. We choose to consider a closed surface in the form of a sphere, centered on the center of the charged sphere and having a radius infinitesimally larger than that of the charged sphere. The electric field then has a uniform magnitude and is perpendicular to our surface at all points on that surface. The flux through the chosen closed surface is therefore $\Phi_E = EA = E\left(4\pi r^2\right)$, and Gauss's law gives

$$\begin{aligned} Q_{\text{inside}} &= \epsilon_0\,\Phi_E = 4\pi\,\epsilon_0\,Er^2 \\ &= 4\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(575\ \text{N/C}\right)\left(0.230\ \text{m}\right)^2 = 3.38\times10^{-9}\ \text{C} = \boxed{3.38\ \text{nC}} \end{aligned}$$

(b) Since the electric field displays spherical symmetry, you can conclude that the charge distribution generating that field is $\boxed{\text{spherically symmetric}}$. Also, since the electric field lines are directed outward away from the sphere, the sphere must contain a $\boxed{\text{net positive charge}}$.

15.43 From Gauss's law, the electric flux through any closed surface is equal to the net charge enclosed divided by $\in_0$. Thus, the flux through each surface (with a positive flux coming outward from the enclosed interior and a negative flux going inward toward that interior) is



For S_1: $\Phi_E = Q_{\text{net}}/\in_0 = (+Q-2Q)/\in_0 = \boxed{-Q/\in_0}$

For S_2: $\Phi_E = Q_{\text{net}}/\in_0 = (+Q-Q)/\in_0 = \boxed{0}$

For S_3: $\Phi_E = Q_{\text{net}}/\in_0 = (-2Q+Q-Q)/\in_0 = \boxed{-2Q/\in_0}$

For S_4: $\Phi_E = Q_{\text{net}}/\in_0 = (0)/\in_0 = \boxed{0}$

15.44 The electric field has constant magnitude and is everywhere perpendicular to the bottom of the car. Thus, the electric flux through the bottom of the car is

$$\Phi_E = EA = \left(1.80\times10^4\ \text{N/C}\right)\left[(5.50\ \text{m})(2.00\ \text{m})\right] = \boxed{1.98\times10^5\ \text{N}\cdot\text{m}^2/\text{C}}$$

15.45 We choose a spherical Gaussian surface, concentric with the charged spherical shell and of radius r. Then, $\Sigma EA\cos\theta = E\left(4\pi r^2\right)\cos 0° = 4\pi r^2 E$.

(a) For $r > a$ (that is, outside the shell), the total charge enclosed by the Gaussian surface is $Q = +q - q = 0$. Thus, Gauss's law gives $4\pi r^2 E = 0$, or $\boxed{E=0}$.

(b) Inside the shell, $r < a$, and the enclosed charge is $Q = +q$.

Therefore, from Gauss's law, $4\pi r^2 E = \dfrac{q}{\in_0}$, or $E = \dfrac{q}{4\pi \in_0 r^2} = \dfrac{k_e q}{r^2}$

The field for $r < a$ is $\boxed{\vec{\mathbf{E}} = \dfrac{k_e q}{r^2}\ \text{directed radially outward}}$.

15.46 (a) The surface of the cube is a closed surface which surrounds a total charge of $Q = 1.70\times10^2\ \mu\text{C}$. Thus, by Gauss's law, the electric flux through the whole surface of the cube is

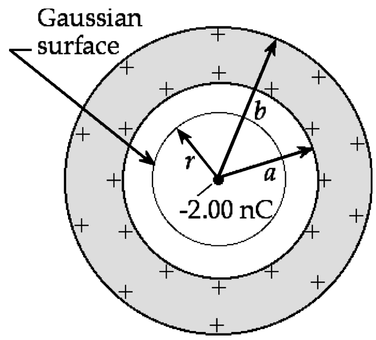
$$\Phi_E = \frac{Q_{\text{inside}}}{\in_0} = \frac{1.70\times10^{-4}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{1.92\times10^7\ \text{N}\cdot\text{m}^2/\text{C}}$$

(b) Since the charge is located at the center of the cube, the six faces of the cube are symmetrically positioned around the location of the charge. Thus, one-sixth of the flux passes through each of the faces, or

$$\Phi_{\text{face}} = \frac{\Phi_E}{6} = \frac{1.92\times10^7\ \text{N}\cdot\text{m}^2/\text{C}}{6} = \boxed{3.20\times10^6\ \text{N}\cdot\text{m}^2/\text{C}}$$

(c) The answer to part (b) would change because the charge could now be at different distances from each face of the cube, but the answer to part (a) would be unchanged because the flux through the entire closed surface depends only on the total charge inside the surface.

15.47 Note that with the point charge $-2.00\ \text{nC}$ positioned at the center of the spherical shell, we have complete spherical symmetry in this situation. Thus, we can expect the distribution of charge on the shell, as well as the electric fields both inside and outside of the shell, to also be spherically symmetric.



(a) We choose a spherical Gaussian surface, centered on the center of the conducting shell, with radius $r = 1.50\ \text{m} < a$ as shown at the right. Gauss's law gives

$$\Phi_E = EA = E\left(4\pi r^2\right) = \frac{Q_{\text{inside}}}{\epsilon_0} \quad \text{or} \quad E = \frac{Q_{\text{inside}}}{4\pi \epsilon_0\, r^2} = \frac{k_e Q_{\text{center}}}{r^2}$$

so $$E = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(-2.00\times10^{-9}\ \text{C}\right)}{\left(1.50\ \text{m}\right)^2}$$

and $E = \boxed{-7.99\ \text{N/C}}$. The negative sign means that the field is $\boxed{\text{radial inward}}$.

(b) All points at $r = 2.20\ \text{m}$ are in the range $a < r < b$, and hence are located within the conducting material making up the shell. Under conditions of electrostatic equilibrium, the field is $\boxed{E = 0}$ at all points inside a conducting material.

(c) If the radius of our Gaussian surface is $r = 2.50\ \text{m} > b$, Gauss's law (with total spherical symmetry) leads to $E = \dfrac{Q_{\text{inside}}}{4\pi \epsilon_0\, r^2} = \dfrac{k_e Q_{\text{inside}}}{r^2}$ just as in part (a). However, now $Q_{\text{inside}} = Q_{\text{shell}} + Q_{\text{center}} = +3.00\ \text{nC} - 2.00\ \text{nC} = +1.00\ \text{nC}$. Thus, we have

$$E = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(+1.00\times10^{-9}\ \text{C}\right)}{\left(2.50\ \text{m}\right)^2} = \boxed{+1.44\ \text{N/C}}$$

with the positive sign telling us that the field is $\boxed{\text{radial outward}}$ at this location.

(d) Under conditions of electrostatic equilibrium, all excess charge on a conductor resides entirely on its surface. Thus, the sum of the charge on the inner surface of the shell and that on the outer surface of the shell is $Q_{\text{shell}} = +3.00\ \text{nC}$. To see how much of this is on the inner surface, consider our Gaussian surface to have a radius r that is infinitesimally larger than a. Then, all points on the Gaussian surface lie within the conducting material, meaning that $E = 0$ at all points and the total flux through the surface is $\Phi_E = 0$. Gauss's law then states that $Q_{\text{inside}} = Q_{\substack{\text{inner}\\ \text{surface}}} + Q_{\text{center}} = 0$,

or

$$Q_{\substack{\text{inner}\\ \text{surface}}} = -Q_{\text{center}} = -\left(-2.00\ \text{nC}\right) = \boxed{+2.00\ \text{nC}}$$

The charge on the outer surface must be

$$Q_{\substack{\text{outer}\\ \text{surface}}} = Q_{\text{shell}} - Q_{\substack{\text{inner}\\ \text{surface}}} = 3.00\ \text{nC} - 2.00\ \text{nC} = \boxed{+1.00\ \text{nC}}$$

15.48 Please review Example 15.8 in your textbook. There it is shown that the electric field due to a nonconducting plane sheet of charge parallel to the *xy*-plane has a constant magnitude given by $E_z = |\sigma_{\text{sheet}}|/2\epsilon_0$, where σ_{sheet} is the uniform charge per unit area on the sheet. This field is everywhere perpendicular to the *xy*-plane, is directed away from the sheet if it has a positive charge density, and is directed toward the sheet if it has a negative charge density.

In this problem, we have two plane sheets of charge, both parallel to the *xy*-plane and separated by a distance of 2.00 cm. The upper sheet has charge density $\sigma_{\text{sheet}} = -2\sigma$, while the lower sheet has $\sigma_{\text{sheet}} = +\sigma$. Taking upward as the positive *z*-direction, the fields due to each of the sheets in the three regions of interest are:

	Lower sheet (at *z* = 0)	**Upper sheet (at *z* = 2.00 cm)**
Region	Electric Field	Electric Field
$z < 0$	$E_z = -\dfrac{\lvert+\sigma\rvert}{2\epsilon_0} = -\dfrac{\sigma}{2\epsilon_0}$	$E_z = +\dfrac{\lvert-2\sigma\rvert}{2\epsilon_0} = +\dfrac{\sigma}{\epsilon_0}$
$0 > z > 2.00$ cm	$E_z = +\dfrac{\lvert+\sigma\rvert}{2\epsilon_0} = +\dfrac{\sigma}{2\epsilon_0}$	$E_z = +\dfrac{\lvert-2\sigma\rvert}{2\epsilon_0} = +\dfrac{\sigma}{\epsilon_0}$
$z > 2.00$ cm	$E_z = +\dfrac{\lvert+\sigma\rvert}{2\epsilon_0} = +\dfrac{\sigma}{2\epsilon_0}$	$E_z = -\dfrac{\lvert-2\sigma\rvert}{2\epsilon_0} = -\dfrac{\sigma}{\epsilon_0}$

When both plane sheets of charge are present, the resultant electric field in each region is the vector sum of the fields due to the individual sheets for that region.

(a) For $z < 0$: $$E_z = E_{z,\text{lower}} + E_{z,\text{upper}} = -\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{\epsilon_0} = \boxed{+\frac{\sigma}{2\epsilon_0}}$$

(b) For $0 < z < 2.00$ cm: $$E_z = E_{z,\text{lower}} + E_{z,\text{upper}} = +\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{\epsilon_0} = \boxed{+\frac{3\sigma}{2\epsilon_0}}$$

(c) For $z > 2.00$ cm: $$E_z = E_{z,\text{lower}} + E_{z,\text{upper}} = +\frac{\sigma}{2\epsilon_0} - \frac{\sigma}{\epsilon_0} = \boxed{-\frac{\sigma}{2\epsilon_0}}$$

15.49 The radius of each sphere is small in comparison to the distance to the nearest neighboring charge (the other sphere). Thus, we shall assume that the charge is uniformly distributed over the surface of each sphere and, in its interaction with the other charge, treat it as though it were a point charge. In this model, we then have two identical point charges, of magnitude 35.0 mC, separated by a total distance of 310 m (the length of the cord plus the radius of each sphere). Each of these charges repels the other with a force of magnitude

$$F_e = k_e \frac{Q^2}{r^2} = \left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(35.0 \times 10^{-3} \text{ C}\right)^2}{\left(3.10 \times 10^2 \text{ m}\right)^2} = 115 \text{ N}$$

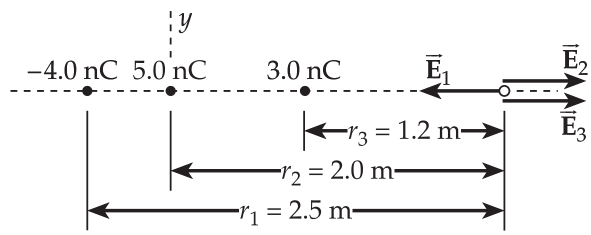
Thus, to counterbalance this repulsion and hold each sphere in equilibrium, the cord must have a tension of $\boxed{115 \text{ N}}$ so it will exert a 115 N on that sphere, directed toward the other sphere.

15.50 (a) As shown in Example 15.8 in the textbook, the electric field due to a nonconducting plane sheet of charge has a constant magnitude of $E = \sigma/2\in_0$, where σ is the uniform charge per unit area on the sheet. The direction of the field at all locations is perpendicular to the plane sheet and directed away from the sheet if σ is positive, and toward the sheet if σ is negative. Thus, if $\sigma = +5.20 \ \mu\text{C/m}^2$, the magnitude of the electric field at all distances greater than zero from the plane (including the distance of 8.70 cm) is

$$E = \frac{\sigma}{2\in_0} = \frac{+5.20 \times 10^{-6} \text{ C/m}^2}{2\left(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{2.94 \times 10^5 \text{ N/C}}$$

(b) The field does not vary with distance as long as the distance is small compared with the dimensions of the plate.

15.51 The three contributions to the resultant electric field at the point of interest are shown in the sketch at the right.



The magnitude of the resultant field is

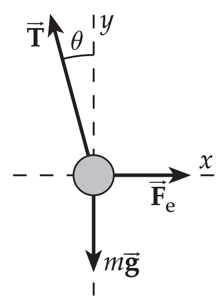
$$E_R = -E_1 + E_2 + E_3$$

$$E_R = -\frac{k_e|q_1|}{r_1^2} + \frac{k_e|q_2|}{r_2^2} + \frac{k_e|q_3|}{r_3^2} = k_e\left[-\frac{|q_1|}{r_1^2} + \frac{|q_2|}{r_2^2} + \frac{|q_3|}{r_3^2}\right]$$

$$E_R = \left(8.99 \times 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[-\frac{4.0 \times 10^{-9} \text{ C}}{(2.5 \text{ m})^2} + \frac{5.0 \times 10^{-9} \text{ C}}{(2.0 \text{ m})^2} + \frac{3.0 \times 10^{-9} \text{ C}}{(1.2 \text{ m})^2}\right]$$

$$E_R = +24 \text{ N/C, or } \vec{\mathbf{E}}_R = \boxed{24 \text{ N/C in the } +x \text{ direction}}$$

15.52 Consider the free-body diagram shown at the right.



$$\Sigma F_y = 0 \ \Rightarrow \ T\cos\theta = mg \quad \text{or} \quad T = \frac{mg}{\cos\theta}$$

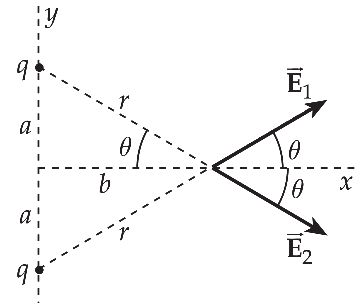
$$\Sigma F_x = 0 \ \Rightarrow \ F_e = T\sin\theta = mg\tan\theta$$

Since $F_e = qE$, we have

$$qE = mg\tan\theta, \text{ or } q = \frac{mg\tan\theta}{E}$$

$$q = \frac{\left(2.00 \times 10^{-3} \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)\tan 15.0°}{1.00 \times 10^3 \text{ N/C}} = 5.25 \times 10^{-6} \text{ C} = \boxed{5.25 \ \mu\text{C}}$$

15.53 (a) At a point on the x-axis, the contributions by the two charges to the resultant field have equal magnitudes given by $E_1 = E_2 = \dfrac{k_e q}{r^2}$.



The components of the resultant field are

$$E_y = E_{1y} - E_{2y} = \left(\frac{k_e q}{r^2}\right)\sin\theta - \left(\frac{k_e q}{r^2}\right)\sin\theta = 0$$

and $$E_x = E_{1x} + E_{2x} = \left(\frac{k_e q}{r^2}\right)\cos\theta + \left(\frac{k_e q}{r^2}\right)\cos\theta = \left[\frac{k_e(2q)}{r^2}\right]\cos\theta$$

Since $\dfrac{\cos\theta}{r^2} = \dfrac{b/r}{r^2} = \dfrac{b}{r^3} = \dfrac{b}{\left(a^2+b^2\right)^{3/2}}$, the resultant field is

$$\vec{\mathbf{E}}_R = \boxed{\frac{k_e(2q)b}{\left(a^2+b^2\right)^{3/2}} \text{ in the } +x \text{ direction}}$$

(b) Note that the result of part (a) may be written as $E_R = \dfrac{k_e(Q)b}{\left(a^2+b^2\right)^{3/2}}$ where $Q = 2q$ is the total charge in the charge distribution generating the field.

In the case of a uniformly charged circular ring, consider the ring to consist of a very large number of pairs of charges uniformly spaced around the ring. Each pair consists of two identical charges located diametrically opposite each other on the ring. The total charge of pair number i is Q_i. At a point on the axis of the ring, this pair of charges generates an electric field contribution that is parallel to the axis and has magnitude $E_i = \dfrac{k_e b Q_i}{\left(a^2+b^2\right)^{3/2}}$.

The resultant electric field of the ring is the summation of the contributions by all pairs of charges, or

$$E_R = \Sigma E_i = \left[\frac{k_e b}{\left(a^2+b^2\right)^{3/2}}\right]\Sigma Q_i = \frac{k_e b Q}{\left(a^2+b^2\right)^{3/2}}$$

where $Q = \Sigma Q_i$ is the total charge on the ring.

$$\vec{\mathbf{E}}_R = \boxed{\frac{k_e Q b}{\left(a^2+b^2\right)^{3/2}} \text{ in the } +x \text{ direction}}$$

15.54 It is desired that the electric field exert a retarding force on the electrons, slowing them down and bringing them to rest. For the retarding force to have maximum effect, it should be anti-parallel to the direction of the electrons motion. Since the force an electric field exerts on negatively charged particles (such as electrons) is in the direction opposite to the field, $\boxed{\text{the electric field should be in the direction of the electron's motion}}$.

The work a retarding force of magnitude $F_e = |q|E = eE$ does on the electrons as they move distance d is $W = F_e d\cos 180° = -F_e d = -eEd$. The work-energy theorem $(W = \Delta KE)$ then gives

$$-eEd = KE_f - KE_i = 0 - K$$

and the magnitude of the electric field required to stop the electrons in distance d is

$$E = \frac{-K}{-ed} \qquad \text{or} \qquad \boxed{E = K/ed}$$

15.55 (a) Without the electric field present, the ball comes to equilibrium when the upward force exerted on the ball by the spring equals the downward gravitational force acting on it, or when $kx = mg$. The distance the ball stretches the spring before reaching the equilibrium position in this case is

$$x = \frac{mg}{k} = \frac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{845\text{ N/m}} = 4.64 \times 10^{-2}\text{ m} = \boxed{4.64\text{ cm}}$$

(b) With the electric field present, the positively charged ball experiences an upward electric force in addition to the upward spring force and downward gravitational force. When the ball comes to equilibrium, with the spring stretched a distance x', the total upward force equals the magnitude of the downward force, or $qE + kx' = mg$. The distance between the unstretched position and the new equilibrium position of the ball is

$$x' = \frac{mg - qE}{k} = \frac{(4.00\text{ kg})(9.80\text{ m/s}^2) - (0.050\,0\text{ C})(355\text{ N/C})}{845\text{ N/m}} = 2.54 \times 10^{-2}\text{ m} = \boxed{2.54\text{ cm}}$$

15.56 (a) The downward electrical force acting on the ball is

$$F_e = qE = (2.00 \times 10^{-6}\text{ C})(1.00 \times 10^{5}\text{ N/C}) = 0.200\text{ N}$$

The total downward force acting on the ball is then

$$F = F_e + mg = 0.200\text{ N} + (1.00 \times 10^{-3}\text{ kg})(9.80\text{ m/s}^2) = 0.210\text{ N}$$

Thus, the ball will behave as if it was in a modified gravitational field where the effective free-fall acceleration is

$$\text{“}g\text{”} = \frac{F}{m} = \frac{0.210\text{ N}}{1.00 \times 10^{-3}\text{ kg}} = 210\text{ m/s}^2$$

The period of the pendulum will be

$$T = 2\pi\sqrt{\frac{L}{\text{“}g\text{”}}} = 2\pi\sqrt{\frac{0.500\text{ m}}{210\text{ m/s}^2}} = \boxed{0.307\text{ s}}$$

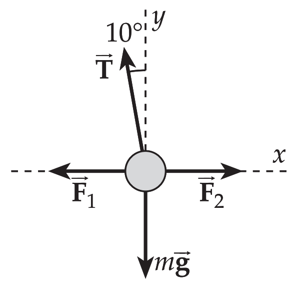
(b) $\boxed{\text{Yes}}$. The force of gravity is a significant portion of the total downward force acting on the ball. Without gravity, the effective acceleration would be

$$\text{“}g\text{”} = \frac{F_e}{m} = \frac{0.200\text{ N}}{1.00 \times 10^{-3}\text{ kg}} = 200\text{ m/s}^2$$

giving $T = 2\pi\sqrt{\frac{0.500\text{ m}}{200\text{ m/s}^2}} = 0.314\text{ s}$

a 2.28% difference from the correct value with gravity included.

15.57 The sketch at the right gives a free-body diagram of the positively charged sphere. Here, $F_1 = k_e |q|^2 / r^2$ is the attractive force exerted by the negatively charged sphere and $F_2 = qE$ is exerted by the electric field.



$$\Sigma F_y = 0 \;\Rightarrow\; T\cos 10° = mg \quad \text{or} \quad T = \frac{mg}{\cos 10°}$$

$$\Sigma F_x = 0 \;\Rightarrow\; F_2 = F_1 + T\sin 10° \quad \text{or} \quad qE = \frac{k_e |q|^2}{r^2} + mg\tan 10°$$

At equilibrium, the distance between the two spheres is $r = 2(L\sin 10°)$. Thus,

$$E = \frac{k_e |q|}{4(L\sin 10°)^2} + \frac{mg\tan 10°}{q}$$

$$= \frac{(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.0\times 10^{-8}\ \text{C})}{4[(0.100\ \text{m})\sin 10°]^2} + \frac{(2.0\times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)\tan 10°}{(5.0\times 10^{-8}\ \text{C})}$$

or the needed electric field strength is $E = \boxed{4.4\times 10^5\ \text{N/C}}$

15.58 (a) At any point on the x-axis in the range $0 < x < 1.00$ m, the contributions made to the resultant electric field by the two charges are both in the positive x direction. Thus, it is not possible for these contributions to cancel each other and yield a zero field.

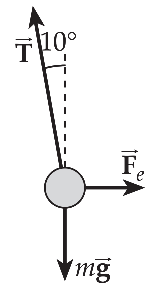
(b) Any point on the x-axis in the range $x < 0$ is located closer to the larger magnitude charge $(q = 5.00\ \mu\text{C})$ than the smaller magnitude charge $(|q| = 4.00\ \mu\text{C})$. Thus, the contribution to the resultant electric field by the larger charge will always have a greater magnitude than the contribution made by the smaller charge. It is not possible for these contributions to cancel to give a zero resultant field.

(c) If a point is on the x-axis in the region $x > 1.00$ m, the contributions made by the two charges are in opposite directions. Also, a point in this region is closer to the smaller magnitude charge than it is to the larger charge. Thus, there is a location in this region where the contributions of these charges to the total field will have equal magnitudes and cancel each other.

(d) When the contributions by the two charges cancel each other, their magnitudes must be equal. That is,

$$k_e \frac{(5.00\ \mu\text{C})}{x^2} = k_e \frac{(4.00\ \mu C)}{(x - 1.00\ \text{m})^2} \qquad \text{or} \qquad x - 1.00\ \text{m} = +\sqrt{\frac{4}{5}}\,x$$

Thus, the resultant field is zero at $x = \dfrac{1.00\ \text{m}}{1 - \sqrt{4/5}} = \boxed{+9.47\ \text{m}}$



15.59 We assume that the two spheres have equal charges, so the repulsive force that one exerts on the other has magnitude $F_e = k_e q^2/r^2$.

From Figure P15.59 in the textbook, observe that the distance separating the two spheres is

$$r = 3.0 \text{ cm} + 2\left[(5.0 \text{ cm})\sin 10°\right] = 4.7 \text{ cm} = 0.047 \text{ m}$$

From the free-body diagram of one sphere given above, observe that

$$\Sigma F_y = 0 \;\Rightarrow\; T\cos 10° = mg \text{ or } T = mg/\cos 10°$$

and $\quad \Sigma F_x = 0 \;\Rightarrow\; F_e = T\sin 10° = \left(\dfrac{mg}{\cos 10°}\right)\sin 10° = mg\tan 10°$

Thus, $k_e q^2/r^2 = mg\tan 10°$

or $\quad q = \sqrt{\dfrac{mgr^2\tan 10°}{k_e}} = \sqrt{\dfrac{(0.015 \text{ kg})(9.8 \text{ m/s}^2)(0.047 \text{ m})^2\tan 10°}{8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}}$

giving $\quad q = 8.0\times 10^{-8}$ C or $\boxed{q \sim 10^{-7} \text{ C}}$

15.60 The charges on the spheres will be equal in magnitude and opposite in sign. From $F = k_e q^2/r^2$, this charge must be

$$q = \sqrt{\frac{F\cdot r^2}{k_e}} = \sqrt{\frac{(1.00\times 10^4 \text{ N})(1.00 \text{ m})^2}{8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}} = 1.05\times 10^{-3} \text{ C}$$

The number of electrons transferred is

$$n = \frac{q}{e} = \frac{1.05\times 10^{-3} \text{ C}}{1.60\times 10^{-19} \text{ C}} = 6.59\times 10^{15}$$

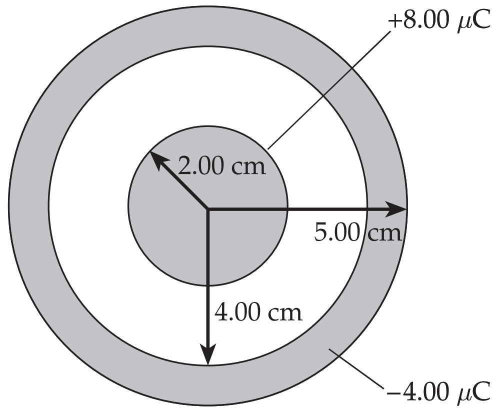
The total number of electrons in 100 g of silver is

$$N = \left(47 \ \frac{\text{electrons}}{\text{atom}}\right)\left(6.02\times 10^{23} \ \frac{\text{atoms}}{\text{mole}}\right)\left(\frac{1 \text{ mole}}{107.87 \text{ g}}\right)(100 \text{ g}) = 2.62\times 10^{25}$$

Thus, the fraction transferred is

$$\frac{n}{N} = \frac{6.59\times 10^{15}}{2.62\times 10^{25}} = \boxed{2.51\times 10^{-10}} \text{ (that is, 2.51 out of every 10 billion).}$$

15.61 Because of the spherical symmetry of the charge distribution, any electric field present will be radial in direction. If a field does exist at distance R from the center, it is the same as if the net charge located within $r \leq R$ were concentrated as a point charge at the center of the inner sphere. Charge located at $r > R$ does not contribute to the field at $r = R$.



(a) At $r = 1.00$ cm, $\boxed{E = 0}$ since static electric fields cannot exist within conducting materials.

(b) The net charge located at $r \leq 3.00$ cm is $Q = +8.00\ \mu\text{C}$.

Thus, at $r = 3.00$ cm,

$$E = \frac{k_e Q}{r^2}$$

$$= \frac{\left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(8.00 \times 10^{-6}\ \text{C}\right)}{\left(3.00 \times 10^{-2}\ \text{m}\right)^2} = \boxed{7.99 \times 10^7\ \text{N/C}\ \text{(outward)}}$$

(c) At $r = 4.50$ cm, $\boxed{E = 0}$ since this is located within conducting materials.

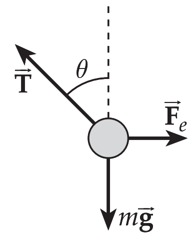
(d) The net charge located at $r \leq 7.00$ cm is $Q = +4.00\ \mu\text{C}$.

Thus, at $r = 7.00$ cm,

$$E = \frac{k_e Q}{r^2}$$

$$= \frac{\left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(4.00 \times 10^{-6}\ \text{C}\right)}{\left(7.00 \times 10^{-2}\ \text{m}\right)^2} = \boxed{7.34 \times 10^6\ \text{N/C}\ \text{(outward)}}$$

15.62 Consider the free-body diagram of the rightmost charge given below.



$$\Sigma F_y = 0 \ \Rightarrow\ T\cos\theta = mg \quad \text{or} \quad T = mg/\cos\theta$$

and $\Sigma F_x = 0 \ \Rightarrow\ F_e = T\sin\theta = \left(mg/\cos\theta\right)\sin\theta = mg\tan\theta$

But, $F_e = \dfrac{k_e q^2}{r_1^2} + \dfrac{k_e q^2}{r_2^2} = \dfrac{k_e q^2}{\left(L\sin\theta\right)^2} + \dfrac{k_e q^2}{\left(2L\sin\theta\right)^2} = \dfrac{5k_e q^2}{4L^2\sin^2\theta}$

Thus, $\dfrac{5k_e q^2}{4L^2\sin^2\theta} = mg\tan\theta$ or $q = \sqrt{\dfrac{4L^2 mg\sin^2\theta\tan\theta}{5k_e}}$

If $\theta = 45°$, $m = 0.10$ kg, and $L = 0.300$ m then

$$q = \sqrt{\frac{4\left(0.300\ \text{m}\right)^2\left(0.10\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\sin^2\left(45°\right)\tan\left(45°\right)}{5\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}}$$

or $\quad q = 2.0\times10^{-6}\ \text{C} = \boxed{2.0\ \mu\text{C}}$

15.63 (a) When an electron (negative charge) moves distance Δx in the direction of an electric field, the work done on it is

$$W = F_e\left(\Delta x\right)\cos\theta = eE\left(\Delta x\right)\cos 180° = -eE\left(\Delta x\right)$$

From the work-energy theorem $\left(W_{\text{net}} = KE_f - KE_i\right)$ with $KE_f = 0$, we have

$$-eE\left(\Delta x\right) = -KE_i, \text{ or } E = \frac{KE_i}{e\left(\Delta x\right)} = \frac{1.60\times10^{-17}\ \text{J}}{\left(1.60\times10^{-19}\ \text{C}\right)\left(0.100\ \text{m}\right)} = \boxed{1.00\times10^3\ \text{N/C}}$$

(b) The magnitude of the retarding force acting on the electron is $F_e = eE$, and Newton's second law gives the acceleration as $a = -F_e/m = -eE/m$. Thus, the time required to bring the electron to rest is

$$t = \frac{v - v_0}{a} = \frac{0 - \sqrt{2\left(KE_i\right)/m}}{-eE/m} = \frac{\sqrt{2m\left(KE_i\right)}}{eE}$$

or

$$t = \frac{\sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.60\times10^{-17}\ \text{J}\right)}}{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.00\times10^3\ \text{N/C}\right)} = 3.37\times10^{-8}\ \text{s} = \boxed{33.7\ \text{ns}}$$

(c) After bringing the electron to rest, the electric force continues to act on it causing the electron to $\boxed{\text{accelerate in the direction opposite to the field}}$ at a rate of

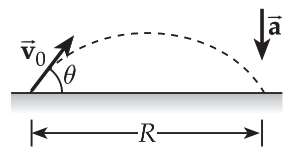
$$|a| = \frac{eE}{m} = \frac{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.00\times10^3\ \text{N/C}\right)}{9.11\times10^{-31}\ \text{kg}} = \boxed{1.76\times10^{14}\ \text{m/s}^2}$$

15.64 (a) The acceleration of the protons is downward (in the direction of the field) and

$$\left|a_y\right| = \frac{F_e}{m} = \frac{eE}{m} = \frac{\left(1.60\times10^{-19}\ \mathrm{C}\right)\left(720\ \mathrm{N/C}\right)}{1.67\times10^{-27}\ \mathrm{kg}} = 6.90\times10^{10}\ \mathrm{m/s^2}$$

The time of flight for the proton is twice the time required to reach the peak of the arc, or

$$t = 2t_{\text{peak}} = 2\left(\frac{v_{0y}}{\left|a_y\right|}\right) = \frac{2v_0\sin\theta}{\left|a_y\right|}$$



The horizontal distance traveled in this time is

$$R = v_{0x}t = \left(v_0\cos\theta\right)\left(\frac{2v_0\sin\theta}{\left|a_y\right|}\right) = \frac{v_0^2\sin 2\theta}{\left|a_y\right|}$$

Thus, if $R = 1.27\times10^{-3}$ m, we must have

$$\sin 2\theta = \frac{\left|a_y\right|R}{v_0^2} = \frac{\left(6.90\times10^{10}\ \mathrm{m/s^2}\right)\left(1.27\times10^{-3}\ \mathrm{m}\right)}{\left(9\ 550\ \mathrm{m/s}\right)^2} = 0.961$$

giving $2\theta = 73.9°$ or $2\theta = 180° - 73.9° = 106.1°$.

Hence, $\theta = \boxed{37.0° \text{ or } 53.0°}$.

(b) The time of flight for each possible angle of projection is:

For $\theta = 37.0°$: $t = \dfrac{2v_0\sin\theta}{\left|a_y\right|} = \dfrac{2\left(9\ 550\ \mathrm{m/s}\right)\sin 37.0°}{6.90\times10^{10}\ \mathrm{m/s^2}} = \boxed{1.66\times10^{-7}\ \mathrm{s}}$

For $\theta = 53.0°$: $t = \dfrac{2v_0\sin\theta}{\left|a_y\right|} = \dfrac{2\left(9\ 550\ \mathrm{m/s}\right)\sin 53.0°}{6.90\times10^{10}\ \mathrm{m/s^2}} = \boxed{2.21\times10^{-7}\ \mathrm{s}}$