Example 8.3 A vertical column made of cement has a base area of 0.5 m^2. If its height is 2 m, and the specific gravity of cement is 3, how much pressure does this column exert on the ground?

Solution. The force the column exerts on the ground is equal to its weight, mg, so we'll find the pressure it exerts by dividing this by the base area, A. The mass of the column is equal to ρV, which we calculate as follows:

$$\rho = sp.gr. \times \rho_{\text{water}} = 3 \times 1000 \frac{kg}{m^3} = 3000 \frac{kg}{m^3}$$

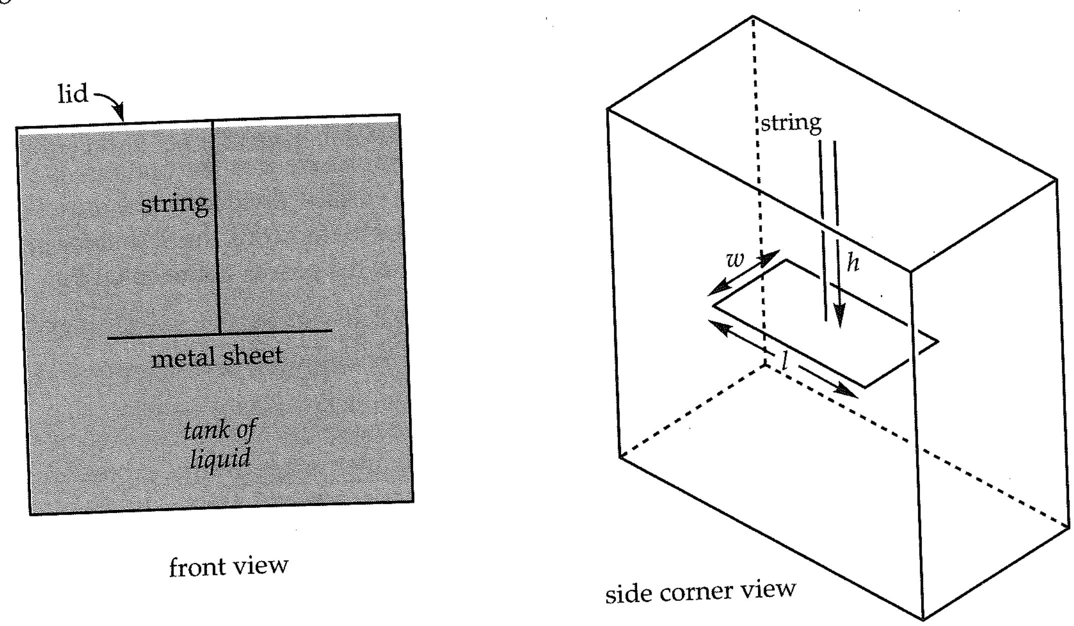
$$m = \rho V = \rho A h = (3 \times 10^3 \tfrac{\text{kg}}{\text{m}^3})(0.5 \text{ m}^2)(2 \text{ m}) = 3 \times 10^3 \text{ kg}$$

Therefore,

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{(3 \times 10^3 \text{ kg})(10 \tfrac{\text{N}}{\text{kg}})}{0.5 \text{ m}^2} = 6 \times 10^4 \text{ Pa} = 60 \text{ kPa}$$

HYDROSTATIC PRESSURE

Imagine that we have a tank with a lid on top, filled with some liquid. Suspended from this lid is a string, attached to a thin sheet of metal. The figures below show two views of this:



front view

side corner view

The weight of the liquid produces a force that pushes down on the metal sheet. If the sheet has length l and width w, and is at depth h below the surface of the liquid, then the weight of the liquid on top of the sheet is

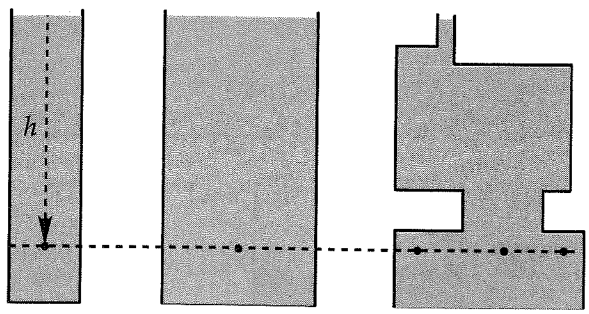
$$F_g = mg = \rho V g = \rho(lwh)g$$

where ρ is the liquid's density. If we divide this weight by the area of the sheet ($A = lw$), we get the pressure due to the liquid:

$$P_{\text{liquid}} = \frac{\text{force}}{\text{area}} = \frac{F_{g\,\text{liquid}}}{A} = \frac{\rho(lwh)g}{lw} = \rho g h$$

Since the liquid is at rest, this is known as **hydrostatic pressure**.

Note that the hydrostatic pressure due to the liquid, $P_{\text{liquid}} = \rho g h$, depends only on the density of the liquid and the depth below the surface; in fact, it's proportional to both of these quantities. One important consequence of this is that the shape of the container doesn't matter. For example, if all the containers in the figure below are filled with the same liquid, then the pressure is the same at every point along the horizontal dashed line (and within a container), simply because every point on this line is at the same depth, h, below the surface of the liquid.



If the liquid in the tank were open to the atmosphere, then the total (or absolute) pressure at depth h would be equal to the pressure pushing down on the surface—the atmospheric pressure, P_{atm}—plus the pressure due to the liquid alone:

$$\text{total (absolute) pressure: } P_{\text{total}} = P_{\text{atm}} + P_{\text{liquid}} = P_{\text{atm}} + \rho g h$$

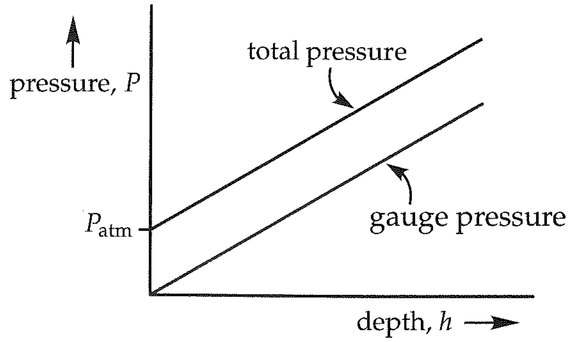
The difference between total pressure and atmospheric pressure is known as the **gauge pressure:**

$$P_{\text{gauge}} = P - P_{\text{atm}}$$

So, another way of writing the equation above is to say:

$$P_{\text{gauge}} = \rho g h$$

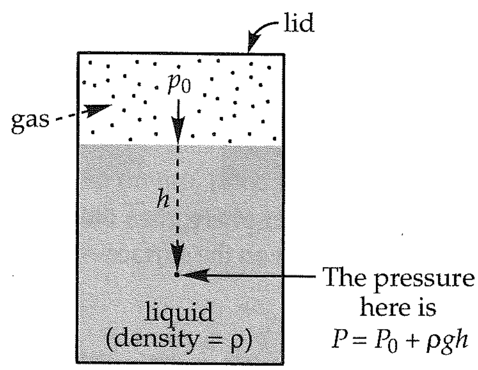
The following graphs show how gauge pressure and total pressure vary with depth. Note that although both graphs are straight lines, only gauge pressure is proportional to depth. In order for a graph to represent a direct proportion, it must be a straight line *through the origin*.



The lines will be straight as long as the density of the liquid remains constant as the depth increases. Actually, ρ increases as the depth increases, but the effect is small enough that we generally consider liquids to be incompressible—that is, that the density of a liquid remains constant and does not increase with depth.

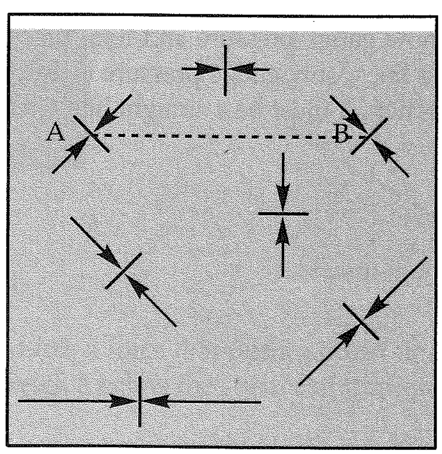
If we have a container of liquid where the pressure above the surface of the liquid is p_0, then the total pressure at depth h below the surface would be p_0 plus the pressure due just to the liquid:

$$P = P_0 + \rho gh$$



If there's no lid, then $P_0 = P_{atm}$, and we get the equation $P = P_{atm} + \rho gh$ again.

Because pressure is the *magnitude* of the force per area, pressure is a scalar. It has no direction. The direction of the force due to the pressure on any small surface is perpendicular to that surface. For example, in the figure below, the pressure at Point A is the same as the pressure at Point B, because they're at the same depth.



$$P = \cancel{P} + (\rho g h) - \cancel{(1\times 10^{5}\text{ Pa})}$$

$1025\,\frac{kg}{m^3}(10\,\frac{m}{s^2})(10\,m)$

$102{,}500$ Pa

102.5 kPa

But, as you can see, the direction of the force due to the pressure varies depending on the orientation of the surface—and even which side of the surface—the force is pushing on.

> **Example 8.4** What is the hydrostatic gauge pressure at a point 10 m below the surface of the ocean? (Note: The specific gravity of seawater is 1.025.)

Solution. Using ρ_{seawater} = sp. gr. $\times\ \rho_{\text{water}} = 1025\ \text{kg/m}^3$, we find that

$$P_{\text{gauge}} = \rho g h = (1025\ \tfrac{\text{kg}}{\text{m}^3})(10\ \tfrac{\text{N}}{\text{kg}})(10\ \text{m}) = 1.025 \times 10^5\ \text{Pa}$$

Note that this is just about equal to one atmosphere (1 atm = 1.013×10^5 Pa). In fact, a rule of thumb used by scuba and deep-sea divers is that the pressure increases by about 1 atmosphere for every 10 m (or 30.5 feet) of depth.

> **Example 8.5** A swimming pool has a depth of 4 m. What is the hydrostatic gauge pressure at a point 1 m below the surface?

Solution. Using $\rho = \rho_{\text{water}} = 1000\ \text{kg/m}^3$ for the density of the pool water, we find that

$$P_{\text{gauge}} = \rho g h = (1000\ \tfrac{\text{kg}}{\text{m}^3})(10\ \tfrac{\text{N}}{\text{kg}})(1\ \text{m}) = 1 \times 10^4\ \text{Pa}$$

Note that the total depth of the swimming pool is irrelevant; all that matters is the depth below the surface.

> **Example 8.6** What happens to the gauge pressure if we double our depth below the surface of a liquid? What happens to the total pressure?

Solution. Since $P_{\text{gauge}} = \rho g h$, we see that $P_{\textit{gauge}}$ is proportional to the depth, h. Therefore, if we double h, then P_{gauge} will double. However, the total pressure, $P_{\text{atm}} + \rho g h$, is not proportional to h. The term $\rho g h$ will double, but the term P_{atm}, being a constant, will not. So, the total pressure will increase, but it will not double; in fact, it will increase by less than a factor of 2.

> **Example 8.7** A flat piece of wood, of area 0.5 m², is lying at the bottom of a lake. If the depth of the lake is 30 m, what is the force on the wood due to the pressure? (Use $P_{\text{atm}} = 1 \times 10^5$ Pa.)

Solution. Using $\rho = \rho_{\text{water}} = 1000\ \text{kg/m}^3$ for the density of the water in the lake, we find that

$$P_{\text{gauge}} = \rho g h = (1000\ \tfrac{\text{kg}}{\text{m}^3})(10\ \tfrac{\text{N}}{\text{kg}})(30\ \text{m}) = 3 \times 10^5\ \text{Pa}$$

Therefore, the total pressure on the wood is $P = P_{\text{atm}} + P_{\text{gauge}} = 4 \times 10^5$ Pa. Now, by definition, we have $P = F/A$, so $F = PA$; this gives

$$F = PA = (4 \times 10^5\ \text{Pa})(0.5\ \text{m}^2) = 2 \times 10^5\ \text{N}$$