Take 1pt off (simplify)

Name $\underline{\text{Key}}$ Mr. Konichek

UWSP Math 111 Section 4

Exam 2 (over sections 3.6-4.4) (show all your work, partial credit possible)

Do any 3 problems 1a – 4a. Cross out the 1 problem that I will not grade. (8 pts. Each)

1a. Find dy/dx by implicit differentiation if $2x^2 + y^2 = 16$

$$4x + 2y\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} = -\frac{4x}{2y} = \boxed{-\frac{2x}{y}}$$

Answer: dy/dx = $-\frac{2x}{y}$

2a. Find dy/dx by implicit differentiation if $2x^3 + 4y^3 + 5y - 13 = 0$

$$6x^2 + 12y^2\left(\frac{dy}{dx}\right) + 5\left(\frac{dy}{dx}\right) - 0 = 0$$

$$\frac{dy}{dx}\left(12y^2+5\right) = -6x^2$$

$$\boxed{\frac{dy}{dx} = -\frac{6x^2}{(12y^2+5)}}$$

Answer: dy/dx = $\frac{-6x^2}{(12y^2+5)}$

3a. Find an equation of the tangent line to the graph of the function f defined by the equation at the indicated point. $-4x^2 + y^2 = 25$ at the point (1 , 2)

(Hint: dy/dx = slope = m) (Hint: $(y - y_1) = m(x - x_1)$ where (x_1, y_1) is a point on the line and m is the slope.)

$$-8x + 2y\left(\frac{dy}{dx}\right) = 0$$

$$m = \left.\frac{dy}{dx} = \frac{8x}{2y} = \frac{4x}{y}\right]_{(1,2)} = \frac{4(1)}{(2)} = \frac{4}{2} = \boxed{2}$$

$$(y-2) = m(x-1)$$

$$(y-2) = 2(x-1)$$

$$y - 2 = 2x - 2$$

$$y = 2x - 2 + 2$$

$$\boxed{y = 2x}$$

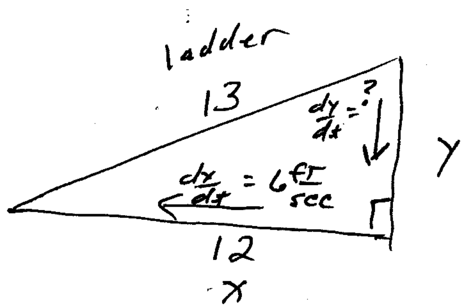
Answer: y = $2x$

Key

4a. A Sliding Ladder: **The base of a 13 ft ladder leaning against a wall begins to slide away from the wall. At the instant of time when the base is 12 ft from the wall, the base is moving at the rate (dx/dt) of 6 ft/sec. <u>How fast (dy/dt)</u> is the top of the ladder sliding down the wall at that instant of time?**

Hint: Make a diagram of the ladder leaning against the wall and note $x^2 + y^2 = 13^2$ (where x is the base from the bottom of the ladder to the wall and y is the height). Find y knowing x.

Hint: Differentiate the equation above implicitly with respect to t (meaning you will end up with both dx/dt's and dy/dt's). Now simply find dy/dt when you know x, y, and dx/dt.



$$12^2 + y^2 = 13^2$$
$$144 + y^2 = 169$$
$$y^2 = 25$$
$$\boxed{y = 5}$$

$\cancel{2x + 2y}$

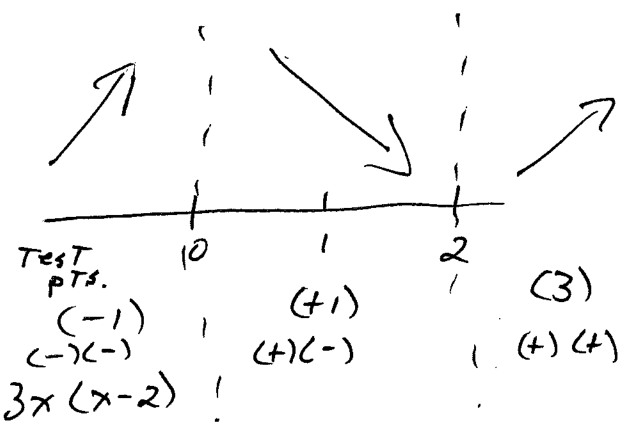
$$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 0$$
$$2(12)(6) + 2(5)\frac{dy}{dt} = 0$$
$$\frac{dy}{dt} = \frac{-\cancel{2}(12)(6)}{\cancel{10}\ 5} = \frac{-72}{5}$$
$$\frac{dy}{dt} = \frac{-72}{5} \text{ OR } -14.4\frac{ft}{sec}.$$

Answer: dy/dt = $-14.4 \frac{ft}{second}$

Do any 3 problems 5b – 8b. Cross out the 1 problem that I will not grade. (8 pts. Each)

5b. Find the interval(s) where the <u>function is increasing</u> and the interval(s) where it is <u>decreasing</u>. $\mathbf{f(x) = x^3 - 3x^2}$

$$f'(x) = 3x^2 - 6x$$
$$3x(x-2) = 0$$
$$x = 0 \ \& \ x = 2$$



Increasing interval(s) $(-\infty, 0) \cup (2, \infty)$ Decreasing interval(s) $(0, 2)$

Key

6b. Find the relative maxima and relative minima, if any, of $f(x) = (1/3)x^3 - x^2 - 3x + 4$

(Step 1)

$f'(x) = x^2 - 2x - 3$

$(x-3)(x+1) = 0$

$x = 3 \quad x = -1$

(Step 2)

To find if max or min

$f''(x) = 2x - 2 = 2(x-1)$

$f''(+3) > 0$ ∪ so MIN

$f''(-1) < 0$ ∩ so MAX

(Step 3)

$f(3) = \frac{1}{3}(3)^3 - (3)^2 - 3(3) + 4 = 9 - 9 - 9 + 4 = -5$

$f(-1)\ \frac{1}{3}(-1)^3 - (-1)^2 - 3(-1) + 4 = \frac{1}{3}(-1) - 1 + 3 + 4 = -\frac{1}{3} + 6 = 5\frac{2}{3}$

NOTE: The y values alone also show MAX & MIN

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Relative maxima (-1 , $5\frac{2}{3}$)   Relative minima (3 , -5)

7b. Determine where the graph of the function is concave upward and where it is concave downward $f(x) = 3x^3 - 9x^2 - 24x + 14$

$f'(x) = 9x^2 - 18x - 24$

$f''(x) = 18x - 18 = 18(x-1)$

$18(x-1) = 0$

$x = 1$

(0) +1 (2)

$-$

$18(x-1)$

Concave upward $(1, \infty)$ Concave downward $(-\infty, 1)$

8b. Find the inflection point(s), if any, of $f(x) = 2x^3 - 36x^2 + 18x + 576$

(Step 1)

$f'(x) = 6x^2 - 72x + 18$

~~Step~~

$f''(x) = 12x - 72$

$12x - 72 = 0$

$6x - 36 = 0$

$x - 6 = 0$

$x = 6$

Step 2

$f(6) = 2(6)^3 - 36(6)^2 + 18(6) + 576 = 432 - \cancel{306}\ {-1296} + 108 + 576$

216, 36

126
108
576
810

Inflection point(s): (6 , ~~810~~ -180) ~~(,)~~ (there may be not be any)

Do any 6 problems 9c – 15c. Cross out the 1 problem that I will not grade. **(8 pts. Each)**

9c. Find the <u>relative extrema</u>, if any, of $f(x) = 2x^2 - 8x + 7$

Use the Second Derivative Test if applicable for determining if it is a <u>relative maximum</u> or <u>relative minimum</u>.

Step 1 $\quad f'(x) = 4x - 8 \Rightarrow 4x - 8 = 0$

$x - 2 = 0$

$x = 2$

Step 2 $\quad f''(x) = 4$

$f''(2) > 0$ ∪ ← so min

Step 3 $\quad f(2) = 2(2)^2 - 8(2) + 7$

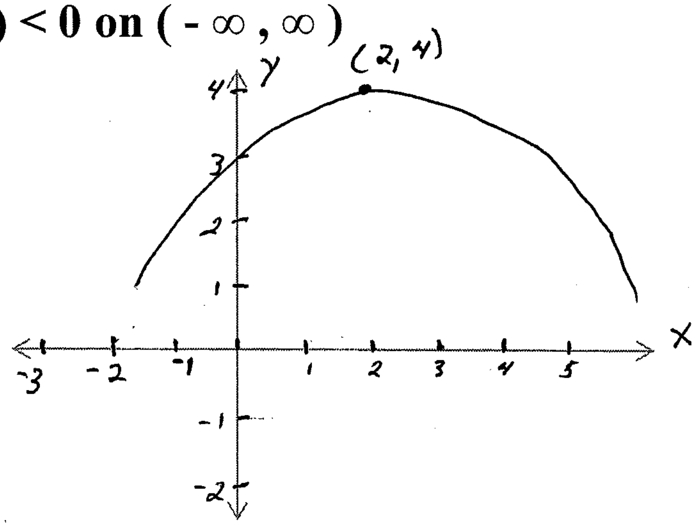
$f(2) = 8 - 16 + 7$

$f(2) = -1$

Answer: the relative extrema is (2 , −1) (4 pts.) and it is a relative minimum (4 pts.)

10c. Sketch the graph of a function having the given properties.

$f(2) = 4$, $f'(2) = 0$, $f''(x) < 0$ on $(-\infty, \infty)$



11c. & 12c. Find the horizontal and vertical asymptotes of the graph of the function.

(You need NOT sketch the graph)

11c. $\quad f(x) = 1 / (x + 2)$

$x + 2 = 0$

$x = -2$ vertical asymptote

$\lim_{x \to \infty} \frac{1}{x+2} = \frac{\frac{1}{x}}{\frac{x}{x} + \frac{2}{x}} = 0$ so $y = 0$ horizontal asymptote

horizontal asymptote(s) ~~[illegible]~~ $y = 0$

~~horizontal~~ vertical asymptote(s) $x = -2$

12c. $\quad f(x) = 2x^3 + x^2 + 1$

Polynomial so NO ASYMPTOTES

horizontal asymptote(s) NONE

vertical asymptote(s)

13c. **Sketch the graph of the function** $\quad$ **f(x) = (x² – 9) / (x² – 4)**

$x^2 - 4 = 0$

$x^2 = 4$

$\boxed{x = \pm 2}$ vertical asymptotes

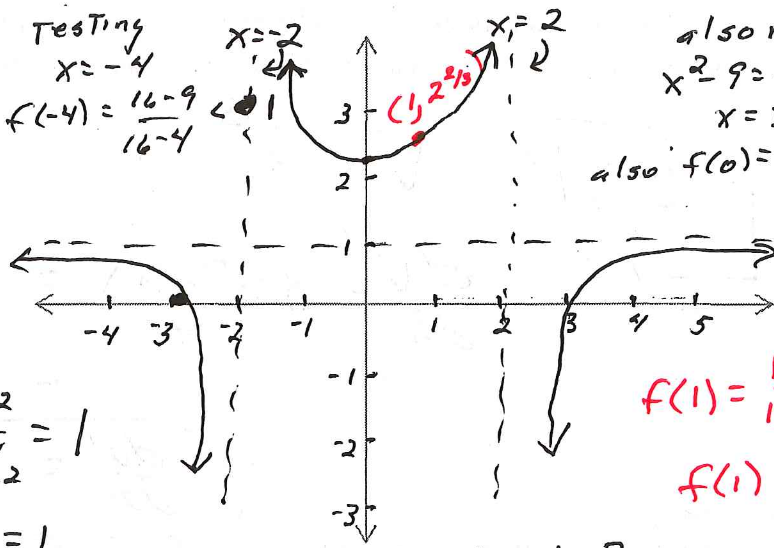
Testing $x = -4$

$f(-4) = \frac{16-9}{16-4} < 1$

also note $x^2 - 9 = 0$ (x int.)

$x = \pm 3$

also $f(0) = \frac{-9}{-4} = \frac{9}{4}$



$f(1) = \frac{1-9}{1-4} = \frac{-8}{-3}$

$f(1) = 2\frac{2}{3}$

$\lim_{x \to \infty} \frac{x^2 - 9}{x^2 - 4} = \lim_{x \to \infty} \frac{\frac{x^2}{x^2} - \frac{9}{x^2}}{\frac{x^2}{x^2} - \frac{4}{x^2}} = 1$

$y = 1$ horizontal asymptote

horizontal asymptote(s) ~~x = ±2~~ $y = 1$ $\quad$ vertical asymptote(s) $x = \pm 2$

14c. & 15c. Find the absolute maximum value and the absolute minimum value, if any, of each function. (optimization I)

14c. f(x) = x² - 2x – 3 on [0 , 4]

$f'(x) = 2x - 2$

$2(x-1) = 0$

$x = 1$

$f(0) = -3$

$f(1) = 1 - 2 - 3 = -4 \Leftarrow$ smallest

$f(4) = 16 - 8 - 3 = 5 \Leftarrow$ largest

absolute maximum value ~~3~~ 5

absolute minimum value ~~3~~ -4

15c. f(x) = 9x + 1 / x on [1 , 3] (+ x^{-1})

$f'(x) = 9 - \frac{1}{x^2} = \frac{9x^2 - 1}{x^2}$

$f'(x) = 0$ when $9x^2 - 1 = 0$

$9x^2 = 1$

$x^2 = \frac{1}{9}$

$x = \pm\frac{1}{3}$ outside [1, 3]

~~$f(-\frac{1}{3}) = 9(-\frac{1}{3}) + \frac{1}{(-\frac{1}{3})}$~~ $= -3 - 3 = -6$ smallest

~~$f(+\frac{1}{3}) = +3 + 3 = +6$~~

$f(1) = 9 + 1 = 10$ largest

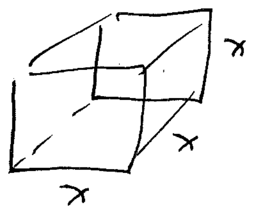
$f(3) = 27 + \frac{1}{3} = 27\frac{1}{3}$

absolute maximum value $27\frac{1}{3}$

absolute minimum value ~~-6~~ ✓ 10

Everyone does problem 16d.

16d. (4 pts.) The volume V of a cube with sides of length x in. is changing with respect to time. At a certain instant of time, the sides of the cube are 5 in. long and and increasing at the rate (dx/dt) of 0.2 in./sec. How fast (dV'/dt) is the volume of the cube changing at that instant of time?



$V = x^3$

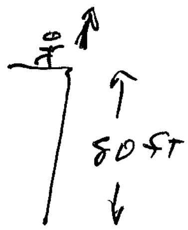
$\frac{dV'}{dt} = 3x^2\left(\frac{dx}{dt}\right) = 3(5\,in.)^2\left(0.2\,\frac{in}{sec}\right)$

$= 3(\cancel{25}^{5}\,in^2)\left(\frac{1}{\cancel{5}}\,\frac{in}{sec}\right)$

$\frac{dv'}{dt} = 15\,\frac{in^3}{sec}$

Extra Credit

1e. (5 points) A stone is thrown straight up from the roof of an 80 ft. building. The height (in feet) of the stone at any time t (in seconds), measured from the ground, is given by h(t) = - 16 t^2 + 64 t + 80



$h'(t) = -32t + 64$

$-32t + 64 = 0$

$-16t + 32 = 0$

$-t + 2 = 0$

$t = 2$

$h(2) = -16(2)^2 + 64(2) + 80$

$h(2) = -\cancel{64} + \cancel{64} + 64 + 80$

$h(2) = 144\,ft$

What is the maximum height [h(t) where you find t first] the stone reaches? 144 ft

2e. (5 points) Maximizing Profits Acrosonic's total profit (in dollars) from manufacturing and selling x units of their model F loudspeaker systems is given by

P(x) = - 0.02 x^2 + 300 x – 200,000 (0 ≤ x ≤ 20,000) (hint: optimization I)

(Step 1)

$P'(x) = -.04x + 300$

$-.04x + 300 = 0$

$-4x + 30000 = 0$

$-2x + 15000 = 0$

$-x + 7500 = 0$

$x = +7500$

(Step 2)

$P(7500) = -.02(7500)^2 + 300(7500) - 200{,}000$

$P(7500) = 925{,}000$ make almost a million

$P(0) = -200{,}000$

$P(20{,}000) =$ can see even a much larger loss in the millions

How many units of the loudspeaker system must Acrosonic produce to maximize its profits? 7,500