

Name: Mr. Konichek EXAM 1: Ch 2.1 - 3.5

Draw a box around your final answers. Partial credit will be given.

130 min

1. a. Find the domain of  $f(x) = \frac{1}{x^2 - 3x - 4}$ .  $= \frac{1}{(x-4)(x+1)}$

$x: x \neq 4; x \neq -1$

b. What is  $f(-2)$ ?  $= \frac{1}{(-2)^2 - 3(-2)} - 4 = \frac{1}{4+6-4} = \boxed{\frac{1}{6}}$

c. What is  $f(-1)$ ?  $= \frac{1}{(-1)^2 - 3(-1)} - 4 = \frac{1}{1+3-4} = \frac{1}{0}$  undefined

d. What is  $f(0)$ ?  $= \frac{1}{(-4)(1)} = \boxed{-\frac{1}{4}}$

e. What is  $f(4)$ ? undefined

2. Evaluate  $k(-3)$  where  $k = f \circ g$  and  $f(x) = \frac{1}{2x+4}$ ,  $g(x) = x^2 - 4x - 5$

$$\begin{aligned} k &= f(g(x)) = f(x^2 - 4x - 5) \\ &= \frac{1}{2(x^2 - 4x - 5) + 4} = \frac{1}{2x^2 - 8x - 10 + 4} \end{aligned}$$

$$= \frac{1}{2x^2 - 8x - 6} = \frac{1}{2(x^2 - 4x - 3)}$$

$$= \frac{1}{2(x-3)(x-1)}$$

$$k(-3) = \frac{1}{2(-3-3)(-3-1)} = \frac{1}{2(-6)(-4)} = \boxed{\frac{1}{48}}$$

3. A manufacturer has a production cost of \$9 for each unit produced and a fixed cost of \$27,000. The product sells for \$13.50 per unit.

a. When does the manufacturer break even?

$$P(x) = 4.5x - 27,000$$

$$0 = 4.5x - 27,000$$

$$\begin{array}{l} 4.5x = 27,000 \\ \boxed{x = 6,000 \text{ units}} \end{array}$$

b. How many units must be sold in order to make a profit of \$9,000?

$$9,000 = 4.5x - 27,000$$

$$\begin{array}{l} 4.5x = 36,000 \\ \boxed{x = 8,000 \text{ units}} \end{array}$$

4. Find the limit. **(Be careful!)**

$$\lim_{x \rightarrow \infty} \frac{(2x^2 - 1)(3x + 1)(x - 4)(x + 2)}{(x - 7)(4 - 3x)(2x^2 + 3)(x^2 - 5)} = \lim_{x \rightarrow \infty} \frac{6x^5}{6x^6} = \lim_{x \rightarrow \infty} \frac{1}{x}$$

$$\cancel{=} \frac{1}{\infty} = \boxed{0}$$

5. Find the limit.

$$\lim_{x \rightarrow 0} \frac{3 - \sqrt{x+9}}{x} \cdot \frac{(3 + \sqrt{x+9})}{(3 + \sqrt{x+9})} = \lim_{x \rightarrow 0} \frac{9 - (x+9)}{3x + x\sqrt{x+9}}$$

$$= \lim_{x \rightarrow 0} \frac{-x}{x(3 + \sqrt{x+9})}$$

$$= \lim_{x \rightarrow 0} -\frac{1}{3 + \sqrt{x+9}}$$

$$= -\frac{1}{3 + \sqrt{9}} = \frac{1}{3+3} = \boxed{\frac{1}{6}}$$

6. Find the derivative and express your answer with positive exponents:

$$f(x) = \frac{8}{x^6} - \frac{7}{x^5} + \frac{6}{x^4} - \frac{5}{x^3} + \frac{4}{x^2} - \frac{3}{x} + 2x - 1 = 8x^{-6} - 7x^{-5} + 6x^{-4} - 5x^{-3} + 4x^{-2} - 3x^{-1} + 2x - 1$$

$$f'(x) = -\frac{48}{x^7} + \frac{35}{x^6} - \frac{24}{x^5} + \frac{15}{x^4} - \frac{8}{x^3} + \frac{3}{x^2} + 2$$

7. Use the limit definition to find  $f'(x)$  if  $f(x) = 3x - 2x^2$

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{3(x+h) - 2(x+h)^2 - (3x - 2x^2)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{3x + 3h - 2(x^2 + 2xh + h^2) - 3x + 2x^2}{h} \\
 &= \lim_{h \rightarrow 0} \frac{3h - 2x^2 - 4xh - 2h^2 + 2x^2}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{2x^2} + h(-4x - 2h + 3)}{h} \\
 &= \cancel{\frac{\cancel{2x^2} + \cancel{h}}{h}} + \lim_{h \rightarrow 0} \frac{-4x - 2h + 3}{h} = \boxed{3 - 4x}
 \end{aligned}$$

8. Find the derivative of:  $f(x) = (x^2 + 2)(x^3 + 2x + 1)$  (3 lines)

$$\begin{aligned}
 f'(x) &= (x^3 + 2x + 1)(2x) + (x^2 + 2)(3x^2 + 2) \\
 &= \cancel{2x^4 + 4x^2 + 2x} + 3x^4 + 2x^3 + 6x^2 + 4 \\
 f'(x) &= \boxed{5x^4 + 12x^2 + 2x + 4}
 \end{aligned}$$

9. Find the derivative of:  $f(x) = \frac{x^2 + 2}{x^2 + x + 1}$

$$f'(x) = \frac{(x^2 + x + 1)(2x) - (x^2 + 2)(2x + 1)}{(x^2 + x + 1)^2}$$

$$(2 \text{ lines})$$

$$\cancel{2x^3} - \cancel{x^2} - 4x - 2$$

$$\cancel{(2x^3 + x^2 + 4x + 2)}$$

$$f'(x) = \frac{(x^2 - 2x - 2)}{(x^2 + x + 1)^2}$$

10. Find the derivative of:  $f(x) = \sqrt[3]{x^3 + 4}$  (2 lines)

$$f(x) = (x^3 + 4)^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3} (x^3 + 4)^{-\frac{2}{3}} (3x^2)$$

$$f'(x) = \frac{2x^2}{3(x^3 + 4)^{\frac{2}{3}}}$$

$$= \frac{x^2}{\sqrt[3]{(x^3 + 4)^2}}$$

**Bonus Question:** (if you are in the mood for more torture!)

Find  $f^{(4)}(x)$  for:

[ 3 pts]

$$f(x) = 9x^9 + 8x^8 + 7x^7 + 6x^6 + 5x^5 + 4x^4 + 3x^3 + 2x^2 + x$$

$$f'(x) = 81x^8 + 64x^7 + 49x^6 + 36x^5 + 25x^4 + 16x^3 + 9x^2 + 4x + 1$$

$$f''(x) = 648x^7 + 448x^6 + 294x^5 + 180x^4 + 100x^3 + 48x^2 + 18x + 4$$

$$f'''(x) = 4536x^6 + 2688x^5 + 1470x^4 + 720x^3 + 300x^2 + 96x + 18$$

$$f''''(x) = 27216x^5 + 13440x^4 + 5880x^3 + 2160x^2 + 600x + 96$$

$$25 \frac{\text{min}}{\text{sec}}$$