PROBLEM SOLUTIONS

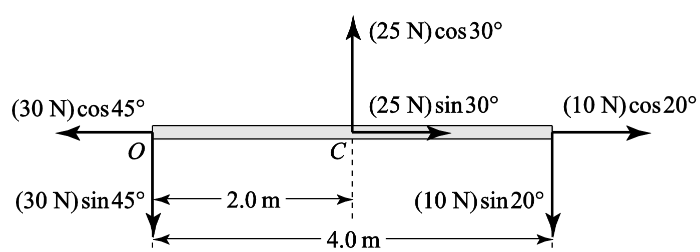
8.1 Since the friction force is tangential to a point on the rim of the wheel, it is perpendicular to the radius line connecting this point with the center of the wheel. The torque of this force about the axis through the center of the wheel is then $\tau = rf \sin 90.0^\circ = rf$, and the friction force is

$$f = \frac{\tau}{r} = \frac{76.0\ \text{N}\cdot\text{m}}{0.350\ \text{m}} = \boxed{217\ \text{N}}$$

8.2 The torque of the applied force is $\tau = rF\sin\theta$. Thus, if $r = 0.330$ m, $\theta = 75.0^\circ$, and the torque has the maximum allowed value of $\tau_{\max} = 65.0\ \text{N}\cdot\text{m}$, the applied force is

$$F = \frac{\tau_{\max}}{r\sin\theta} = \frac{65.0\ \text{N}\cdot\text{m}}{(0.330\ \text{m})\sin 75.0^\circ} = \boxed{204\ \text{N}}$$

8.3 First resolve all of the forces shown in Figure P8.3 into components parallel to and perpendicular to the beam as shown in the sketch below.



(a) $\tau_O = +\left[(25\ \text{N})\cos 30^\circ\right](2.0\ \text{m}) - \left[(10\ \text{N})\sin 20^\circ\right](4.0\ \text{m}) = \boxed{+30\ \text{N}\cdot\text{m}}$

or $\tau_0 = \boxed{30\ \text{N}\cdot\text{m counterclockwise}}$

(b) $\tau_C = +\left[(30\ \text{N})\sin 45^\circ\right](2.0\ \text{m}) - \left[(10\ \text{N})\sin 20^\circ\right](2.0\ \text{m}) = \boxed{+36\ \text{N}\cdot\text{m}}$

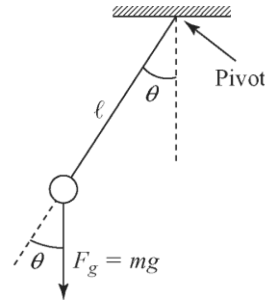
or $\tau_c = \boxed{30\ \text{N}\cdot\text{m counterclockwise}}$

8.4 The lever arm is $d = (1.20 \times 10^{-2}\ \text{m})\cos 48.0^\circ = 8.03 \times 10^{-3}\ \text{m}$, and the torque is

$$\tau = Fd = (80.0\text{ N})(8.03 \times 10^{-3}\text{ m}) = \boxed{0.642\text{ N}\cdot\text{m counterclockwise}}$$

8.5 (a) $|\tau| = F_g \cdot (\textit{lever arm}) = (mg)\cdot[\ell \sin\theta]$

$$= (3.0\text{ kg})(9.8\text{ m/s}^2)\cdot[(2.0\text{ m})\sin 5.0°] = \boxed{5.1\text{ N}\cdot\text{m}}$$



(b) The magnitude of the torque is proportional to the sin θ, where θ is the angle between the direction of the force and the line from the pivot to the point where the force acts. Note from the sketch that this is the same as the an gle the pendulum string makes with the vertical.

$\boxed{\text{Since sin } \theta \text{ increases as } \theta \text{ increases, the torque also increases with the angle.}}$

8.6 The object is in both translational and rotational equilibrium. Thus, we may write:

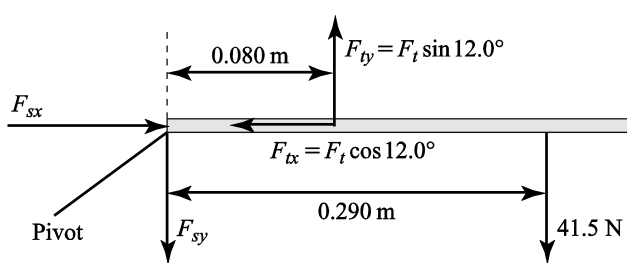
$$\Sigma F_x = 0 \Rightarrow \boxed{F_x - R_x = 0}$$

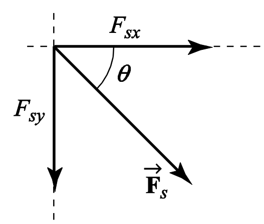
$$\Sigma F_y = 0 \Rightarrow \boxed{F_y + R_y - F_g = 0}$$

and

$$\Sigma\tau_O = 0 \Rightarrow \boxed{F_y(\ell\cos\theta) - F_x(\ell\sin\theta) - F_g\left(\frac{\ell}{2}\cos\theta\right) = 0}$$

8.7





Requiring that $\sum\tau = 0$, using the shoulder joint at point O as a pivot, gives

$$\Sigma\tau = (F_t \sin 12.0°)(0.080\text{ m}) - (41.5\text{ N})(0.290\text{ m}) = 0 \text{ or } F_t = \boxed{724\text{ N}}$$

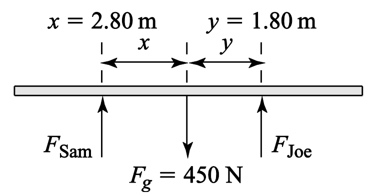
Then $\Sigma F_y = 0 \quad \Rightarrow \quad -F_{sy} + (724\text{ N})\sin 12.0° - 41.5\text{ N} = 0$, yielding $F_{sy} = 109\text{ N}$

$\Sigma F_y = 0$ gives $F_{sx} - (724\text{ N})\cos 12.0° = 0$, or $F_{sx} = 708\text{ N}$

Therefore,

$$F_s = \sqrt{F_{sx}^2 + F_{sy}^2} = \sqrt{(708\text{ N})^2 + (109\text{ N})^2} = \boxed{716\text{ N}}$$

8.8 (a) Since the beam is in equilibrium, we choose the center as our pivot point and require that



$$\Sigma\tau)_{\text{center}} = -F_{\text{Sam}}(2.80\text{ m}) + F_{\text{Joe}}(1.80\text{ m}) = 0$$

or

$$F_{\text{Joe}} = 1.56F_{\text{Sam}} \qquad [1]$$

Also,

$$\Sigma F_y = 0 \quad \Rightarrow \quad F_{\text{Sam}} + F_{\text{Joe}} = 450\text{ N} \qquad [2]$$

Substitute Equation [1] into [2] to get the following:

$$F_{\text{Sam}} + 1.56F_{\text{Sam}} = 450\text{ N} \quad \text{or} \quad F_{\text{Sam}} = \frac{450\text{ N}}{2.56} = \boxed{176\text{ N}}$$

Then, Equation [1] yields $F_{\text{Joe}} = 1.56(176\text{ N}) = \boxed{274\text{ N}}$.

(b) If Sam moves closer to the center of the beam, his lever arm about the beam center decreases, so the force $\boxed{F_{\text{Sam}}\text{ must increase}}$ to continue applying a clockwise torque capable of offsetting Joe's counterclockwise torque. At the same time, the force $\boxed{F_{\text{Joe}}\text{ would decrease}}$ since the sum of the two upward forces equal the magnitude of the downward gravitational force.

(c) If Sam moves to the right of the center of the beam, his torque about the midpoint would then be counterclockwise. $\boxed{\text{Joe would have to hold down on the beam}}$ in order to exert an offsetting clockwise torque.

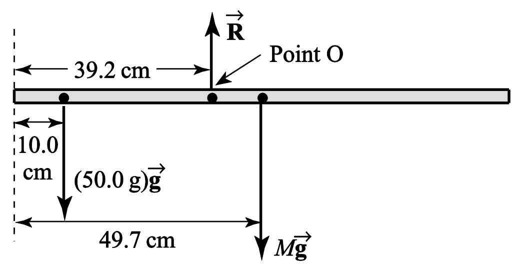
8.9 Require that $\sum\tau = 0$ about an axis through the elbow and perpendicular to the page. This gives

$$\Sigma\tau = +\left[(2.00 \text{ kg})\left(9.80 \text{ m/s}^2\right)\right](25.0 \text{ cm}+8.00 \text{ cm}) - (F_B \cos 75.0°)(8.00 \text{ cm}) = 0$$

or

$$F_B = \frac{(19.6 \text{ N})(33.0 \text{ cm})}{(8.00 \text{ cm})\cos 75.0°} = \boxed{312 \text{ N}}$$

8.10 Since the bare meter stick balances at the 49.7 cm mark when placed on the fulcrum, the center of gravity of the meter stick is located 49.7 cm from the zero end. Thus, the entire weight of the meter stick may be considered to be concentrated at this point. The free-body diagram of the stick when it is balanced with the 50.0-g mass attached at the 10.0 cm mark is as given at the right.



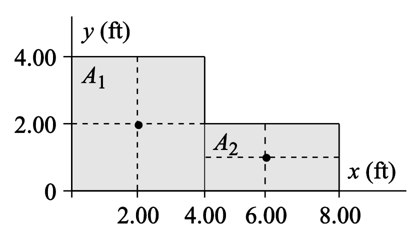
Requiring that the sum of the torques about point O be zero yields

$$+\left[(50.0 \text{ g})\cancel{g}\right](39.2 \text{ cm} - 10.0 \text{ cm}) - M\cancel{g}(49.7 \text{ cm} - 39.2 \text{ cm}) = 0$$

or

$$M = (50.0 \text{ g})\left(\frac{39.2 \text{ cm} - 10.0 \text{ cm}}{49.7 \text{ cm} - 39.2 \text{ cm}}\right) = \boxed{139 \text{ g}}$$

8. 11 Consider the remaining plywood to consist of two parts: A_1 is a 4.00-ft-by-4.00-ft section with center of gravity located at (2.00 ft, 2.00 ft), while A_2 is a 2.00-ft-by-4.00-ft section with center of gravity at (6.00 ft, 1.00 ft). Since the plywood is uniform, its mass per area σ is constant and the mass of a section having area A is $m = \sigma A$. The center of gravity of the remaining plywood has coordinates given by

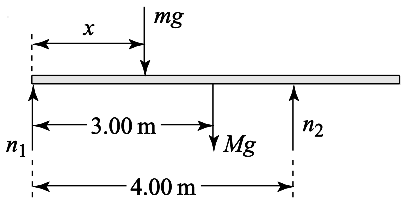


$$x_{cg} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{\cancel{\sigma}A_1x_1 + \cancel{\sigma}A_2x_2}{\cancel{\sigma}A_1 + \cancel{\sigma}A_2} = \frac{\left(16.0 \text{ ft}^2\right)(2.00 \text{ ft}) + \left(8.00 \text{ ft}^2\right)(6.00 \text{ ft})}{\left(16.0 \text{ ft}^2\right) + \left(8.00 \text{ ft}^2\right)} = \boxed{3.33 \text{ ft}}$$

and

$$y_{cg} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{\cancel{\sigma} A_1 y_1 + \cancel{\sigma} A_2 y_2}{\cancel{\sigma} A_1 + \cancel{\sigma} A_2} = \frac{\left(16.0\text{ ft}^2\right)\left(2.00\text{ ft}\right) + \left(8.00\text{ ft}^2\right)\left(1.00\text{ ft}\right)}{\left(16.0\text{ ft}^2\right) + \left(8.00\text{ ft}^2\right)} = \boxed{1.67\text{ ft}}$$

8.12 (a)



$$Mg = \left(90.0\text{ kg}\right)\left(9.80\text{ m/s}^2\right) = 882\text{ N} \qquad mg = \left(55.0\text{ kg}\right)\left(9.80\text{ m/s}^2\right) = 539\text{ N}$$

(b) $\boxed{\text{The woman is at } x = 0 \text{ when } n_1 \text{ is greatest}}$. With this location of the woman, the counterclockwise torque about the center of the beam is a maximum. Thus, n_1 must be exerting its maximum clockwise torque about the center to hold the beam in rotational equilibrium.

(c) $\boxed{n_1 = 0}$ As the woman walks to the right along the beam, she will eventually reach a point where the beam will start to rotate clockwise about the rightmost pivot. At this point, the beam is starting to lift up off of the left most pivot and the normal force exerted by that pivot will have diminished to zero.

(d) When the beam is about to tip, $n_1 = 0$ and $\sum F_y = 0$, gives $0 + n_2 - Mg - mg = 0$, or

$$n_2 = Mg + mg = 882\text{ N} + 539\text{ N} = \boxed{1.42 \times 10^3\text{ N}}$$

(e) Requiring that $\Sigma\tau\big)_{\substack{\text{rightmost}\\ \text{pivot}}} = 0$ when the beam is about to tip ($n_1 = 0$) gives

$$+\left(4.00\text{ m} - x\right)mg + \left(4.00\text{ m} - 3.00\text{ m}\right)Mg = 0$$

or $\left(mg\right)x = \left(1.00\text{ m}\right)Mg + \left(4.00\text{ m}\right)mg$, and

$$x = \left(1.00\text{ m}\right)\frac{M}{m} + 4.00\text{ m}$$

Thus,

$$x = (1.00\text{ m})\frac{(90.0\text{ kg})}{(55.0\text{ kg})} + 4.00\text{ m} = \boxed{5.64\text{ m}}$$

(f) When $n_1 = 0$ and $n_2 = 1.42 \times 10^3$ N, requiring that $\Sigma\tau)_{\substack{\text{left}\\\text{end}}} = 0$ gives

$$0 - (539\text{ N})x - (882\text{ N})(3.00\text{ m}) + (1.42 \times 10^3\text{ N})(4.00\text{ m}) = 0$$

or

$$x = \frac{-3.03 \times 10^3\text{ N}\cdot\text{m}}{-539\text{ N}} = \boxed{5.62\text{ N}}$$

which, within limits of rounding errors, is $\boxed{\text{the same as the answer to part (e)}}$.

8.13 Requiring that $x_{\text{cg}} = \Sigma m_i x_i/\Sigma m_i = 0$ gives

$$\frac{(5.0\text{ kg})(0) + (3.0\text{ kg})(0) + (4.0\text{ kg})(3.0\text{ m}) + (8.0\text{ kg})x}{(5.0 + 3.0 + 4.0 + 8.0)\text{ kg}} = 0$$

or $8.0\,x + 12$ m = 0 which yields $x = -1.5$ m

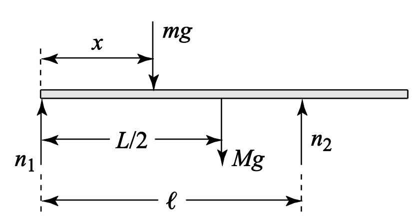
Also, requiring that $y_{\text{cg}} = \Sigma m_i y_i/\Sigma m_i = 0$ gives

$$\frac{(5.0\text{ kg})(0) + (3.0\text{ kg})(4.0\text{ m}) + (4.0\text{ kg})(0) + (8.0\text{ kg})y}{(5.0 + 3.0 + 4.0 + 8.0)\text{ kg}} = 0$$

or $8.0y + 12$ m = 0 yielding $y = -1.5$ m

Thus, the 8.0-kg object should be placed at coordinates $\boxed{(-1.5\text{ m}, -1.5\text{m})}$.

8 .14 (a) As the woman walks to the right along the beam, she will eventually reach a point where the beam will start to rotate clockwise about the rightmost pivot. At this point, the beam is starting to lift up off of the leftmost pivot and the normal force, n_1, exerted by that pivot will have diminished to



zero.

Then, $\Sigma F_y = 0 \quad \Rightarrow \quad 0 - mg - Mg + n_2 = 0$, or

$$\boxed{n_2 = (m + M)g}$$

(b) When $n_1 = 0$ and $n_2 = (m + M)g$, requiring that $\Sigma\tau)_{\substack{\text{left}\\\text{end}}} = 0$ gives

$$0 - (mg)x - (Mg)\frac{L}{2} + (mg + Mg)\ell = 0 \qquad \text{or} \qquad x = \left(1 + \frac{M}{m}\right)\ell - \left(\frac{M}{2m}\right)L$$

(c) If the woman is to just reach the right end of the beam ($x = L$) when $n_1 = 0$ and $n_2 = (m+M)g$ (i.e., the beam is ready to tip), then the result from Part (b) requires that

$$L = \left(1 + \frac{M}{m}\right)\ell - \left(\frac{M}{2m}\right)L \quad \text{or} \qquad \ell = \frac{(1 + M/2m)}{(1 + M/m)} = \boxed{\left(\frac{m + M/2}{m + M}\right)L}$$

8.15 In each case, the distance from the bar to the center of mass of the body is given by

$$x_{cg} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{m_{\text{arms}}x_{\text{arms}} + m_{\text{torso}}x_{\text{torso}} + m_{\text{thighs}}x_{\text{thighs}} + m_{\text{legs}}x_{\text{legs}}}{m_{\text{arms}} + m_{\text{torso}} + m_{\text{thighs}} + m_{\text{legs}}}$$

where the distance x for any body part is the distance from the bar to the center of gravity of that body part. <u>In each case, we shall take the positive direction for distances to run from the bar toward the location of the head.</u>

Note that $\Sigma m_i = (6.87 + 33.57 + 14.07 + 7.54)\text{ kg} = 62.05\text{ kg}$.

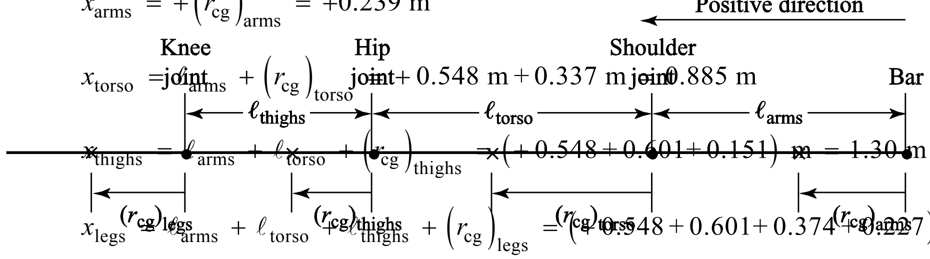
With the body positioned as shown in Figure P8.15b, the distances x for each body part is computed using the sketch given below:

$$x_{\text{arms}} = +\left(r_{\text{cg}}\right)_{\text{arms}} = +0.239\ \text{m}$$

$$x_{\text{torso}} = \ell_{\text{arms}} + \left(r_{\text{cg}}\right)_{\text{torso}} = +0.548\ \text{m} + 0.337\ \text{m} = +0.885\ \text{m}$$

$$x_{\text{thighs}} = \ell_{\text{arms}} + \ell_{\text{torso}} + \left(r_{\text{cg}}\right)_{\text{thighs}} = \left(+0.548 + 0.601 + 0.151\right)\ \text{m} = 1.30\ \text{m}$$

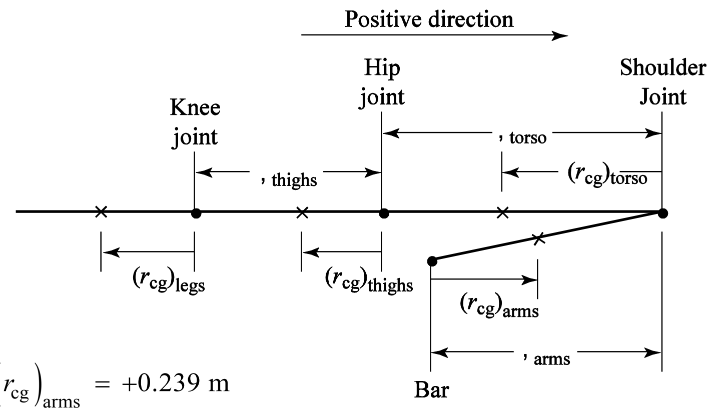
$$x_{\text{legs}} = \ell_{\text{arms}} + \ell_{\text{torso}} + \ell_{\text{thighs}} + \left(r_{\text{cg}}\right)_{\text{legs}} = \left(+0.548 + 0.601 + 0.374 + 0.227\right)\ \text{m} = 1.75\ \text{m}$$



With these distances and the given masses we find

$$x_{\text{cg}} = \frac{+62.8\ \text{kg}\cdot\text{m}}{62.05\ \text{kg}} = \boxed{+1.01\ \text{m}}$$

With the body positioned as shown in Figure P8.15c, we use the following sketch to determine the distance x for each body part:



$$x_{\text{arms}} = +\left(r_{\text{cg}}\right)_{\text{arms}} = +0.239\ \text{m}$$

$$x_{\text{torso}} = \ell_{\text{arms}} - \left(r_{\text{cg}}\right)_{\text{torso}} = +0.548\ \text{m} - 0.337\ \text{m} = +0.211\ \text{m}$$

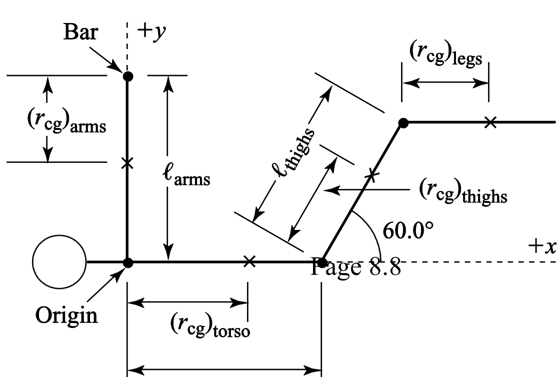
$$x_{\text{thighs}} = \ell_{\text{arms}} - \ell_{\text{torso}} - \left(r_{\text{cg}}\right)_{\text{thighs}} = \left(+0.548 - 0.601 - 0.151\right)\ \text{m} = -0.204\ \text{m}$$

$$x_{\text{legs}} = \ell_{\text{arms}} - \ell_{\text{torso}} - \ell_{\text{thighs}} - \left(r_{\text{cg}}\right)_{\text{legs}} = \left(+0.548 - 0.601 - 0.374 - 0.227\right)\ \text{m} = -0.654\ \text{m}$$

With these distances, the location (relative to the bar) of the center of gravity of the body is

$$x_{\text{cg}} = \frac{+0.924\ \text{kg}\cdot\text{m}}{62.05\ \text{kg}} = \boxed{+0.015\ \text{m}} = \boxed{0.015\ \text{m towards the head}}$$

8.16 With the coordinate system shown below, the coordinates of the center of gravity of each body part may be computed:



$$x_{\text{cg,arms}} = 0 \qquad y_{\text{cg,arms}} = \ell_{\text{arms}} - \left(r_{\text{cg}}\right)_{\text{arms}} = 0.309 \text{ m}$$

$$x_{\text{cg,torso}} = \left(r_{\text{cg}}\right)_{\text{torso}} = 0.337 \text{ m} \qquad y_{\text{cg,torso}} = 0$$

$$x_{\text{cg,thighs}} = \ell_{\text{torso}} + \left(r_{\text{cg}}\right)_{\text{thighs}} \cos 60.0° = 0.676 \text{ m} \qquad y_{\text{cg,thighs}} = \left(r_{\text{cg}}\right)_{\text{thighs}} \sin 60.0° = 0.131 \text{ m}$$

$$x_{\text{cg,legs}} = \ell_{\text{torso}} + \ell_{\text{thighs}} \cos 60.0° + \left(r_{\text{cg}}\right)_{\text{legs}} = 1.02 \text{ m} \qquad y_{\text{cg,legs}} = \ell_{\text{thighs}} \sin 60.0° = 0.324 \text{ m}$$

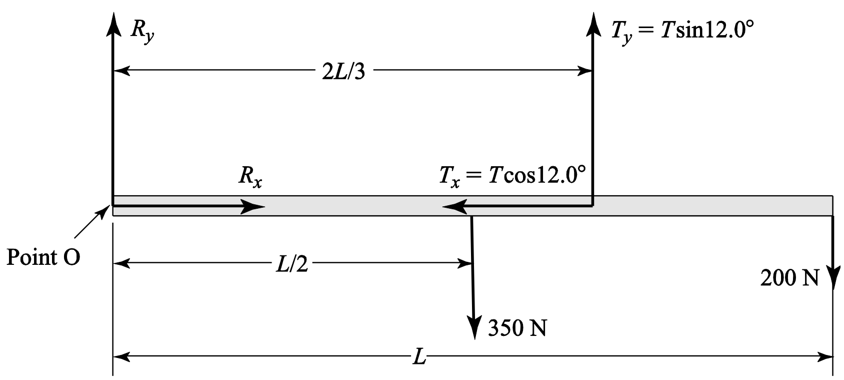
With these coordinates for individual body parts and the masses given in Problem 8.15, the coordinates of the center of mass for the entire body are found to be

$$x_{\text{cg}} = \frac{m_{\text{arms}} x_{\text{cg,arms}} + m_{\text{torso}} x_{\text{cg,torso}} + m_{\text{thighs}} x_{\text{cg,thighs}} + m_{\text{legs}} x_{\text{cg,legs}}}{m_{\text{arms}} + m_{\text{torso}} + m_{\text{thighs}} + m_{\text{legs}}} = \frac{28.5 \text{ kg} \cdot \text{m}}{62.05 \text{ kg}} = \boxed{0.459 \text{ m}}$$

and

$$y_{\text{cg}} = \frac{m_{\text{arms}} y_{\text{cg,arms}} + m_{\text{torso}} y_{\text{cg,torso}} + m_{\text{thighs}} y_{\text{cg,thighs}} + m_{\text{legs}} y_{\text{cg,legs}}}{m_{\text{arms}} + m_{\text{torso}} + m_{\text{thighs}} + m_{\text{legs}}} = \frac{6.41 \text{ kg} \cdot \text{m}}{62.05 \text{ kg}} = \boxed{0.103 \text{ m}}$$

8.17 The free-body diagram for the spine is shown below.



When the spine is in rotational equilibrium, the sum of the torques about the left end (point O) must be zero. Thus,

$$+T_y\left(\frac{2L}{3}\right) - (350 \text{ N})\left(\frac{L}{2}\right) - (200 \text{ N})(L) = 0$$

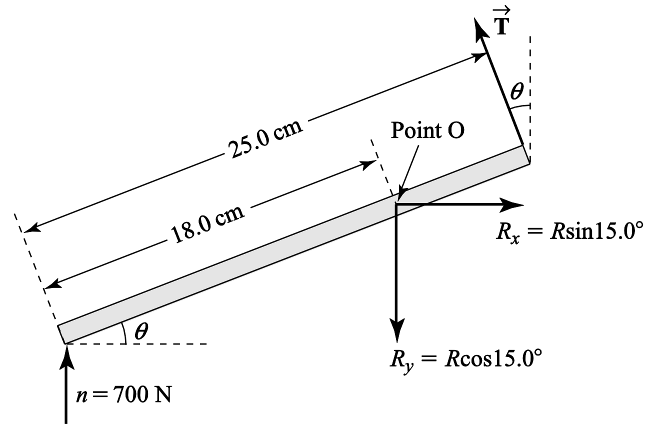
Yielding $T_y = T\sin 12.0° = 562 \text{ N}$.

The tension in the back muscle is then $T = \dfrac{562 \text{ N}}{\sin 12.0°} = 2.71 \times 10^3 \text{ N} = \boxed{2.71 \text{ kN}}$.

The spine is also in translational equilibrium, so $\Sigma F_x = 0 \Rightarrow R_x - T_x = 0$ and the compression force in the spine is

$$R_x = T_x = T\cos 12.0° = (2.71 \text{ kN})\cos 12.0° = \boxed{2.65 \text{ kN}}$$

8.18 In the free-body diagram of the foot given at the right, note that the force $\vec{\mathbf{R}}$ (exerted on the foot by the tibia) has been replaced by its horizontal and vertical components. Employing both conditions of equilibrium (using point O as the pivot point) gives the following three equations:



$$\Sigma F_x = 0 \Rightarrow R\sin 15.0° - T\sin\theta = 0$$

or

$$R = \frac{T\sin\theta}{\sin 15.0°} \qquad [1]$$

$$\Sigma F_y = 0 \Rightarrow 700 \text{ N} - R\cos 15.0° + T\cos\theta = 0 \qquad [2]$$

$$\Sigma\tau_O = 0 \Rightarrow -(700 \text{ N})\left[(18.0 \text{ cm})\cos\theta\right] + T(25.0 \text{ cm} - 18.0 \text{ cm}) = 0$$

or

$$T = (1\,800 \text{ N})\cos\theta \qquad [3]$$

Substituting Equation [3] into Equation [1] gives

$$R = \left(\frac{1\,800 \text{ N}}{\sin 15.0°}\right)\sin\theta\cos\theta \qquad [4]$$

Substituting Equations [3] and [4] into Equation [2] yields

$$\left(\frac{1\,800 \text{ N}}{\tan 15.0°}\right)\sin\theta\cos\theta - (1\,800 \text{ N})\cos^2\theta = 700 \text{ N}$$

which reduces to: $\sin\theta\cos\theta = (\tan 15.0°)\cos^2\theta + 0.104\,2$.

Squaring this result and using the identity $\sin^2\theta = 1 - \cos^2\theta$ gives

$$\left[\tan^2(15.0°) + 1\right]\cos^4\theta + \left[(2\tan 15.0°)(0.104\ 2) - 1\right]\cos^2 + (0.104\ 2)^2 = 0$$

In this last result, let $u = \cos^2\theta$ and evaluate the constants to obtain the quadratic equation

$$(1.071\ 8)u^2 - (0.944\ 2)u + (0.010\ 9) = 0$$

The quadratic formula yields the solutions $u = 0.869\ 3$ and $u = 1.011\ 7$.

Thus,

$$\theta = \cos^{-1}\left(\sqrt{0.869\ 3}\right) = 21.2° \text{ or } \theta = \cos^{-1}\left(\sqrt{0.011\ 7}\right) = 83.8°$$

We ignore the second solution since it is physically impossible for the human foot to stand with the sole inclined at 83.8° to the floor. We are the left with: $\theta = \boxed{21.2°}$.

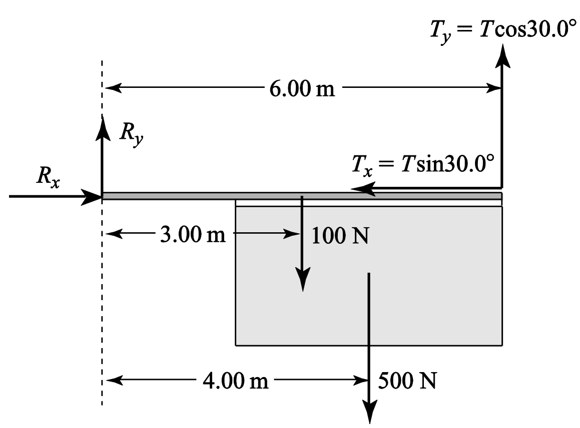
Equation [3] then yields

$$T = (1\ 800\ \text{N})\cos 21.2° = \boxed{1.68 \times 10^3\ \text{N}}$$

and Equation [1] gives

$$R = \frac{(1.68 \times 10^3\ \text{N})\sin 21.2°}{\sin 15.0°} = \boxed{2.34 \times 10^3\ \text{N}}$$

8.19 Consider the torques about an axis perpendicular to the page through the left end of the rod.



$$\Sigma\tau = 0 \Rightarrow T = \frac{(100\ \text{N})(3.00\ \text{m}) + (500\ \text{N})(4.00\ \text{m})}{(6.00\ \text{m})\cos 30.0°}$$

$$T = \boxed{443 \text{ N}}$$

$$\Sigma F_x = 0 \Rightarrow R_x = T\sin 30.0° = (443 \text{ N})\sin 30.0°$$

$$R_x = \boxed{221 \text{ N toward the right}}$$

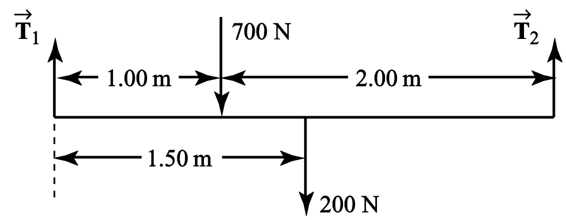
$$\Sigma F_y = 0 \Rightarrow R_y + T\cos 30.0° - 100 \text{ N} - 500 \text{ N} = 0$$

$$R_y = 600 \text{ N} - (443 \text{ N})\cos 30.0° = \boxed{217 \text{ N upward}}$$

8.20 Consider the torques about an axis perpendicular to the page through the left end of the scaffold.

$$\Sigma\tau = 0 \Rightarrow T_1(0) - (700 \text{ N})(1.00 \text{ m}) - (200 \text{ N})(1.50 \text{ m}) + T_2(3.00 \text{ m}) = 0$$

From which, $T_2 = \boxed{333 \text{ N}}$.



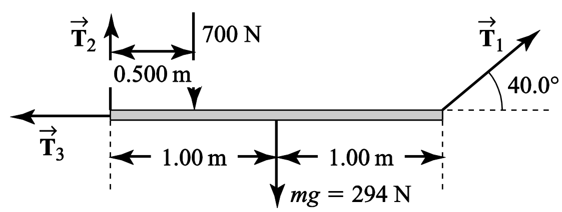
Then, from $\sum F_y = 0$, we have

$$T_1 + T_2 - 700 \text{ N} - 200 \text{ N} = 0$$

or

$$T_1 = 900 \text{ N} - T_2 = 900 \text{ N} - 333 \text{ N} = \boxed{567 \text{ N}}$$

8.21 Consider the torques about an axis perpendicular to the page and through the left end of the plank.



$\sum\tau = 0$ gives

$$-(700 \text{ N})(0.500 \text{ m}) - (294 \text{ N})(1.00 \text{ m}) + (T_1 \sin 40.0°)(2.00 \text{ m}) = 0$$

or T1 = $\boxed{501 \text{ N}}$.

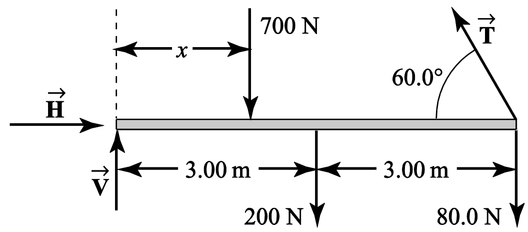
Then, $\Sigma F_x = 0$ gives $-T_3 + T_1\cos 40.0° = 0$, or

$$T_3 = (501 \text{ N})\cos 40.0° = \boxed{384 \text{ N}}$$

From $\Sigma F_y = 0$, $T_2 - 994 \text{ N} + T_1 \sin 40.0° = 0$,

or $T_2 = 994 \text{ N} - (501 \text{ N})\sin 40.0° = \boxed{672 \text{ N}}$.

8.22 (a) See the diagram below



(b) If $x = 1.00$ m, then

$$\Sigma\tau)_{\text{left end}} = 0 \Rightarrow -(700\text{ N})(1.00\text{ m}) - (200\text{ N})(3.00\text{ m}) - (80.0\text{ N})(6.00\text{ m}) + (T\sin 60.0°)(6.00\text{ m}) = 0$$

giving $T = \boxed{434\text{ N}}$.

Then, $\Sigma F_x = 0 \Rightarrow H - T\cos 60.0° = 0$, or $H = (343\text{ N})\cos 60.0° = \boxed{172\text{ N}}$

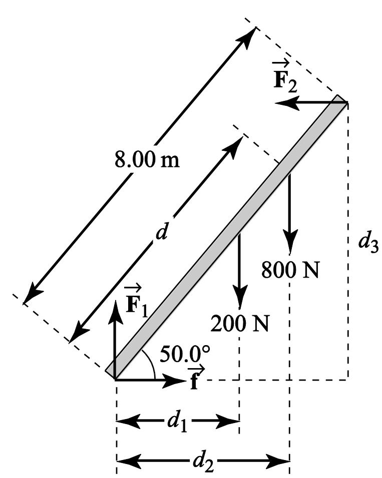
and $\Sigma F_y = 0 \Rightarrow V - 980\text{ N} + (343\text{ N})\sin 60.0° = 0$, or $V = \boxed{683\text{ N.}}$

(c) When the wire is on the verge of breaking, $T = 900$ N and

$$\Sigma\tau)_{\text{left end}} = -(700\text{ N})x_{\text{max}} - (200\text{ N})(3.00\text{ m}) - (80.0\text{ N})(6.00\text{ m}) + \left[(900\text{ N})\sin 60.0°\right](6.00\text{ m}) = 0$$

which gives $x_{\text{max}} = \boxed{5.14\text{ m}}$

8.23 The required dimensions are as follows:



$$d_1 = (4.00\text{ m})\cos 50.0° = 2.57\text{ m}$$

$$d_2 = d\cos 50.0° = (0.643)\ d$$

$$d_3 = (8.00\text{ m})\sin 50.0° = 6.13\text{ m}$$

$\Sigma F_y = 0$ yields $F_1 - 200\text{ N} - 800\text{ N} = 0$, or $F_1 = 1.00 \times 10^3$ N.

When the ladder is on the verge of slipping,

$$f = (f_s)_{max} = \mu_s n = \mu_s F_1 \quad \text{or} \quad f = (0.600)(1.00 \times 10^3 \text{ N}) = 600 \text{ N}$$

Then, $\sum F_x = 0$ gives $F_2 = 600$ N to the left.

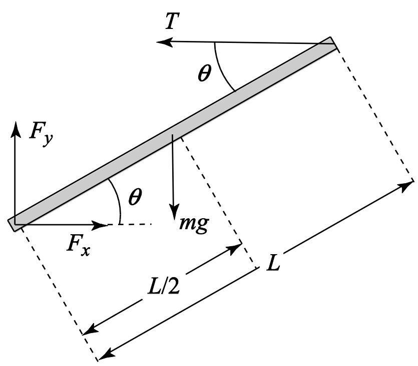
Finally, using an axis perpendicular to the page and through the lower end of the ladder $\sum \tau = 0$, gives

$$-(200 \text{ N})(2.57 \text{ m}) - (800 \text{ N})(0.643)d + (600 \text{ N})(6.13 \text{ m}) = 0$$

or

$$d = \frac{(3.68 \times 10^3 - 550) \text{ N} \cdot \text{m}}{0.643(800 \text{ N})} = \boxed{6.15 \text{ m}} \text{ when the ladder is ready to slip}$$

8.24 (a)



(b) The point of intersection of two unknown forces is always a good choice as the pivot point in a torque calculation. Doing this eliminates these two unknowns from the calculation (since they have zero lever arms about the chosen pivot) and makes it $\boxed{\text{easier to solve the resulting equilibrium equation}}$.

(c) $$\Sigma\tau)_{hinge} = 0 \quad \Rightarrow \quad \boxed{0 + 0 - mg\left(\frac{L}{2}\cos\theta\right) + T(L\sin\theta) = 0}$$

(d) Solving the above result for the tension in the cable gives

$$T = \frac{(mg/2)L\cos\theta}{L\sin\theta} = \frac{mg}{2\tan\theta}$$

or

$$T = \frac{(16.0 \text{ kg})(9.80 \text{ m/s}^2)}{2 \tan 30.0°} = \boxed{136 \text{ N}}$$

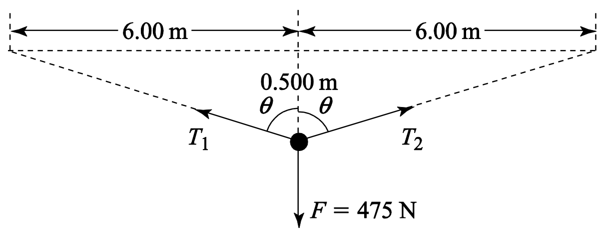
(e) $\Sigma F_x = 0 \Rightarrow \boxed{F_x - T = 0}$ and $\Sigma F_y = 0 \Rightarrow \boxed{F_y - mg = 0}$

(f) Solving the above results for the components of the hinge force gives

$F_x = T = \boxed{136 \text{ N}}$ and $F_y = mg = (16.0 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{157 \text{ N}}$

(g) Attaching the cable higher up would allow the cable to bear some of the weight, thereby reducing the stress on the hinge. It would also reduce the tension in the cable.

8.25 Consider the free-body diagram of the material making up the center point in the rope given at the right. Since this material is in equilibrium, it is necessary to have $\Sigma F_x = 0$ and $\Sigma F_y = 0$, giving



$$\Sigma F_x = 0: \quad +T_2 \sin\theta - T_1 \sin\theta = 0$$

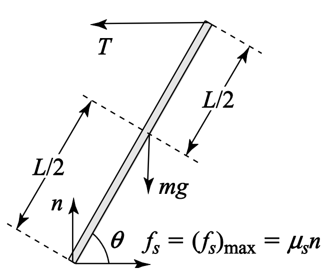
or $T_2 = T_1$, meaning that the rope has a uniform tension T throughout its length.

$$\Sigma F_y = 0: \quad T\cos\theta + T\cos\theta - 475 \text{ N} = 0 \text{ where } \cos\theta = \frac{0.500 \text{ m}}{\sqrt{(6.00 \text{ m})^2 + (0.500 \text{ m})^2}}$$

and the tension in the rope (force applied to the car) is

$$T = \frac{475 \text{ N}}{2\cos\theta} = \frac{(475 \text{ N})\sqrt{(6.00 \text{ m})^2 + (0.500 \text{ m})^2}}{2(0.500 \text{ m})} = 2.86 \times 10^3 \text{ N} = \boxed{2.86 \text{ kN}}$$

8.26 (a)



(b) $\Sigma\tau)_{\substack{\text{lower}\\ \text{end}}} = 0 \quad\Rightarrow\quad 0 + 0 - mg\left(\frac{L}{2}\cos\theta\right) + T\left(L\sin\theta\right) = 0$

or $\quad T = \frac{mg}{2}\left(\frac{\cos\theta}{\sin\theta}\right) = \boxed{\frac{mg}{2}\cot\theta}$

(c) $\Sigma F_x = 0 \quad\Rightarrow\quad -T + \mu_s n = 0 \qquad$ or $\qquad T = \mu_s n \qquad [1]$

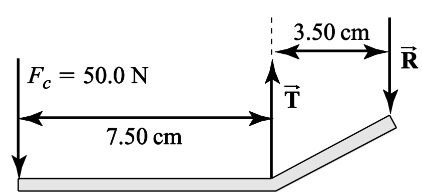
$\Sigma F_y = 0 \quad\Rightarrow\quad n - mg = 0 \qquad\qquad n = mg \qquad [2]$

Substitute Equation [2] into [1] to obtain $\boxed{T = \mu_s mg}$.

(d) Equate the results of parts (b) and (c) to obtain $\boxed{\mu_s = \cot\theta/2}$
This result is valid only at the critical angle θ where the beam is on the verge of slipping (i.e., where $f_s = (f_s)_{\max}$ is valid.)

(e) At angles below the critical angle (where $f_s = \left(f_s\right)_{\max}$ is valid), the beam will slip. At larger angles, the static friction force is reduced below the maximum value, and it is no longer appropriate to use μ_s in the calculation.

8.27 Consider the torques about an axis perpendicular to the page and through the point where the force $\vec{\mathbf{T}}$ acts on the jawbone.



$\Sigma\tau = 0 \Rightarrow (50.0\text{ N})(7.50\text{ cm}) - R(3.50\text{ cm}) = 0$, which yields $R = \boxed{107\text{ N}}$.

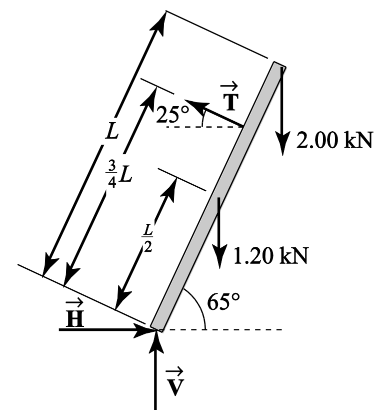
Then, $\Sigma F_y = 0 \Rightarrow -\left(50.0\text{ N}\right) + T - 107\text{ N} = 0$, or $T = \boxed{157\text{ N}}$.

8.28 Observe that the cable is perpendicular to the boom. Then, using $\Sigma\tau = 0$ for an axis perpendicular to the page and through the lower end of the boom gives

$$-\left(1.20\text{ kN}\right)\left(\frac{L}{2}\cos 65°\right) + T\left(\frac{3}{4}L\right) - \left(2.00\text{ kN}\right)\left(L\cos 65°\right) = 0$$

or .$T = \boxed{1.47\text{ kN}}$

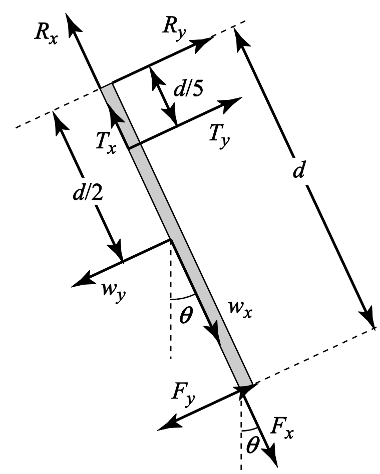
From $\Sigma F_x = 0$, $H = T\cos 25° = \boxed{1.33 \text{ kN to the right}}$



and $\Sigma F_y = 0$ gives

$$V = 3.20 \text{ kN} - T\sin 25° = \boxed{2.58 \text{ kN upward}}$$

8.29 First, we resolve all forces into components parallel to and perpendicular to the tibia, as shown. Note that $\theta = 40.0°$ and



$$w_y = (30.0 \text{ N})\sin 40.0° = 19.3 \text{ N}$$

$$F_y = (12.5 \text{ N})\sin 40.0° = 8.03 \text{ N}$$

and

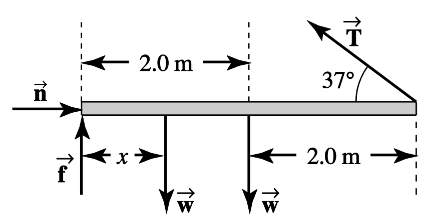
$$T_y = T\sin 25.0°$$

Using $\Sigma\tau = 0$ for an axis perpendicular to the page and through the upper end of the tibia gives

$$\left(T\sin 25.0°\right)\frac{d}{5} - \left(19.3 \text{ N}\right)\frac{d}{2} - \left(8.03 \text{ N}\right)d = 0$$

or $T = \boxed{209 \text{ N}}$.

8.30 When $x = x_{\min}$, the rod is on the verge of slipping, so $f = \left(f_s\right)_{\max} = \mu_s n = 0.50\,n$



From $\Sigma F_x = 0$, $n - T\cos 37° = 0$, or $n = 0.80\,T$. Thus, $f = 0.50\left(0.80\,T\right) = 0.40\,T$

From $\Sigma F_y = 0$, $f + T\sin 37° - 2w = 0$, or $0.40\,T + 0.60\,T - 2w = 0$, giving $T = 2w$

Using $\Sigma\tau = 0$ for an axis perpendicular to the page and through the left end of the beam gives $-w \cdot x_{\text{min}} - w(2.0\text{ m}) + \left[(2w)\sin 37°\right](4.0\text{ m}) = 0$, which reduces to $x_{\text{min}} = \boxed{2.8\text{ m}}$.

8.31 The moment of inertia for rotations about an axis is $I = \Sigma m_i r_i^2$, where r_i is the distance mass m_i is from that axis.

(a) For rotation about the x-axis,

$$I_x = (3.00\text{ kg})(3.00\text{ m})^2 + (2.00\text{ kg})(3.00\text{ m})^2 + (2.00\text{ kg})(3.00\text{ m})^2 + (4.00\text{ kg})(3.00\text{ m})^2 = \boxed{99.0\text{ kg}\cdot\text{m}^2}$$

(b) When rotating about the y-axis,

$$I_y = (3.00\text{ kg})(2.00\text{ m})^2 + (2.00\text{ kg})(2.00\text{ m})^2 + (2.00\text{ kg})(2.00\text{ m})^2 + (4.00\text{ kg})(2.00\text{ m})^2 = \boxed{44.0\text{ kg}\cdot\text{m}^2}$$

(c) For rotations about an axis perpendicular to the page through point O, the distance r_i for each mass is

$$r_i = \sqrt{(2.00\text{ m})^2 + (3.00\text{ m})^2} = \sqrt{13.0}\text{ m}$$

Thus,

$$I_O = \left[(3.00 + 2.00 + 2.00 + 4.00)\text{ kg}\right](13.0\text{ m}^2) = \boxed{143\text{ kg}\cdot\text{m}^2}$$

8.32 The required torque in each case is $\tau = I\alpha$. Thus,

$$\tau_x = I_x\,\alpha = (99.0\text{ kg}\cdot\text{m}^2)(1.50\text{ rad/s}^2) = \boxed{149\text{ N}\cdot\text{m}}$$

$$\tau_y = I_y\,\alpha = (44.0\text{ kg}\cdot\text{m}^2)(1.50\text{ rad/s}^2) = \boxed{66.0\text{ N}\cdot\text{m}}$$

and

$$\tau_O = I_O\,\alpha = (143\text{ kg}\cdot\text{m}^2)(1.50\text{ rad/s}^2) = \boxed{215\text{ N}\cdot\text{m}}$$

8.33 (a) $\tau_{net} = I\alpha \quad \Rightarrow \quad I = \dfrac{\tau_{net}}{\alpha} = \dfrac{rF\sin 90°}{\alpha} = \dfrac{(0.330\ \text{m})(250\ \text{N})}{0.940\ \text{rad/s}^2} = \boxed{87.8\ \text{kg}\cdot\text{m}^2}$

(b) For a solid cylinder, $I = Mr^2/2$, so

$$M = \frac{2I}{r^2} = \frac{2\left(87.8\ \text{kg}\cdot\text{m}^2\right)}{(0.330\ \text{m})^2} = \boxed{1.61\times 10^3\ \text{kg}}$$

(c) $\omega = \omega_0 + \alpha t = 0 + \left(0.940\ \text{rad/s}^2\right)(5.00\ \text{s}) = \boxed{4.70\ \text{rad/s}}$

8.34 (a) $I = 2I_{disk} + I_{cylinder} = 2\left(MR^2/2\right) + mr^2/2$ or $\boxed{I = MR^2 + mr^2/2}$

(b) $\boxed{\tau_g = 0}$ Since the line of action of the gravitational force passes through the rotation axis, it has zero lever arm about this axis and zero torque.

(c) The torque due to the tension force is $\boxed{\text{positive.}}$ Imagine gripping the cylinder with your right hand so your fingers on the front side of the cylinder point upward in the direction of the tension force. The thumb of your right hand then points toward the left (positive direction) along the rotation axis. Because $\vec{\tau} = I\vec{\alpha}$, the torque and angular acceleration have the same direction. Thus, a positive torque produces a $\boxed{\text{positive angular acceleration}}$. When released, the center of mass of the yoyo drops downward, in the negative direction. $\boxed{\text{The translational acceleration is negative.}}$

(d) Since, with the chosen sign convention, the translational acceleration is negative when the angular acceleration is positive, we must include a negative sign in the proportionality between these two quantities. Thus, we write: $a = -r\alpha$ or $\boxed{\alpha = -a/r}$

(e) Translation:

$$\Sigma F_y = m_{total}a \quad \Rightarrow \quad \boxed{T - (2M + m)g = (2M + m)a} \qquad [1]$$

(f) Rotational:

$$\Sigma\tau = I\alpha \quad \Rightarrow \quad rT\sin 90° = I\alpha \qquad \text{or} \qquad \boxed{rT = I\alpha} \qquad [2]$$

(g) Substitute the results of (d) and (a) into Equation [2] to obtain

$$T = I\left(\frac{\alpha}{r}\right) = I\left(\frac{-a/r}{r}\right) = -\left(MR^2 + \frac{mr^2}{2}\right)\frac{a}{r^2} \qquad \text{or} \qquad T = -\left[M\left(\frac{R}{r}\right)^2 + \frac{m}{2}\right]as \qquad [3]$$

Substituting Equation [3] into [1] yields

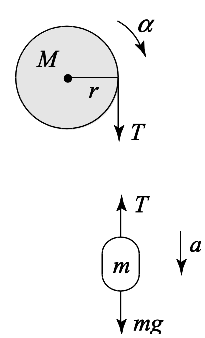
$$-\left[M\left(R/r\right)^2 + m/2\right]a - \left(2M + m\right)g = \left(2M + m\right)a \qquad \text{or} \qquad \boxed{a = \frac{-\left(2M + m\right)g}{2M + M\left(R/r\right)^2 + 3m/2}}$$

(h) $$a = \frac{-\left[2\left(2.00\text{ kg}\right) + 1.00\text{ kg}\right]\left(9.80\text{ m/s}^2\right)}{2\left(2.00\text{ kg}\right) + \left(2.00\text{ kg}\right)\left(10.0/4.00\right)^2 + 3\left(1.00\text{ kg}\right)/2} = \boxed{-2.72\text{ m/s}^2}$$

(i) From Equation [1], $T = \left(2M + m\right)\left(g + a\right) = \left(5.00\text{ kg}\right)\left(9.80\text{ m/s}^2 - 2.72\text{ m/s}^2\right) = \boxed{35.4\text{ N}}$.

(j) $$\Delta y = \left(0\right)t + at^2/2 \quad \Rightarrow \quad t = \sqrt{\frac{2\left(\Delta y\right)}{a}} = \sqrt{\frac{2\left(-1.00\text{ m}\right)}{-2.72\text{ m/s}^2}} = \boxed{0.857\text{ s}}$$

8.35 (a) Consider the free-body diagrams of the cylinder and man given at the right. Note that we shall adopt a sign convention with clockwise and downward as the positive directions. Thus, both a and α are positive in the indicated directions and $a = r\alpha$. We apply the appropriate form of Newton's second law to each diagram to obtain the following:



Rotation of Cylinder: $\tau = I\alpha \quad \Rightarrow \quad rT\sin 90° = I\alpha$, or $T = I\alpha/r$,

so

$$T = \frac{1}{r}\left(\frac{1}{2}Mr^2\right)\left(\frac{a}{r}\right) \qquad \text{and} \qquad T = \frac{1}{2}Ma \qquad [1]$$

Translation of man:

$$\Sigma F_y = ma \quad \Rightarrow \quad mg - T = ma \qquad \text{or} \qquad T = m\left(g - a\right) \qquad [2]$$

Equating Equations [1] and [2] gives $\frac{1}{2} Ma = m(g - a)$, or

$$a = \frac{mg}{m + M/2} = \frac{(75.0 \text{ kg})(9.80 \text{ m/s}^2)}{75.0 \text{ kg} + (225 \text{ kg}/2)} = \boxed{3.92 \text{ m/s}^2}$$

(b) From $a = r\alpha$, we have

$$\alpha = \frac{a}{r} = \frac{3.92 \text{ m/s}^2}{0.400 \text{ m}} = \boxed{9.80 \text{ rad/s}^2}$$

(c) As the rope leaves the cylinder, the mass of the cylinder decreases, thereby decreasing the moment of inertia. At the same time, the weight of the rope leaving the cylinder would increase the downward force acting tangential to the cylinder, and hence increase the torque exerted on the cylinder. Both of these effects will cause the acceleration of the system to increase with time. (The increase would be slight in this case, given the large mass of the cylinder.)

8.36 The angular acceleration is $\alpha = (\omega_f - \omega_i)/\Delta t = -(\omega_i/\Delta t)$ since $\omega_f = 0$

Thus, the torque is $\tau = I\alpha = -(I\omega_i/\Delta t)$. But, the torque is also $\tau = -fr$, so the magnitude of the required friction force is

$$f = \frac{I\omega_i}{r(\Delta t)} = \frac{(12 \text{ kg}\cdot\text{m}^2)(50 \text{ rev/min})}{(0.50 \text{ m})(6.0 \text{ s})}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 21 \text{ N}$$

Therefore, the coefficient of friction is

$$\mu_k = \frac{f}{n} = \frac{21 \text{ N}}{70 \text{ N}} = \boxed{0.30}$$

8.37 (a) $\tau = F \cdot r = (0.800 \text{ N})(30.0 \text{ m}) = \boxed{24.0 \text{ N}\cdot\text{m}}$

(b) $\alpha = \frac{\tau}{I} = \frac{\tau}{mr^2} = \frac{24.0 \text{ N}\cdot\text{m}}{(0.750 \text{ kg})(30.0 \text{ m})^2} = \boxed{0.0356 \text{ rad/s}^2}$

(c) $a_t = r\alpha = (30.0\ \text{m})(0.0356\ \text{rad/s}^2) = \boxed{1.07\ \text{m/s}^2}$

8.38 $I = MR^2 = (1.80\ \text{kg})(0.320\ \text{m})^2 = 0.184\ \text{kg}\cdot\text{m}^2$

$\tau_{\text{net}} = \tau_{\text{applied}} - \tau_{\text{resistive}} = I\alpha$, or $F\cdot r - f\cdot R = I\alpha$

yielding

$$F = \frac{I\alpha + f\cdot R}{r}$$

(a) $$F = \frac{(0.184\ \text{kg}\cdot\text{m}^2)(4.50\ \text{rad/s}^2) + (120\ \text{N})(0.320\ \text{m})}{4.50\times10^{-2}\ \text{m}} = \boxed{872\ \text{N}}$$

(b) $$F = \frac{(0.184\ \text{kg}\cdot\text{m}^2)(4.50\ \text{rad/s}^2) + (120\ \text{N})(0.320\ \text{m})}{2.80\times10^{-2}\ \text{m}} = \boxed{1.40\ \text{kN}}$$

8.39 $I = \frac{1}{2}MR^2 = \frac{1}{2}(150\ \text{kg})(1.50\ \text{m})^2 = 169\ \text{kg}\cdot\text{m}^2$

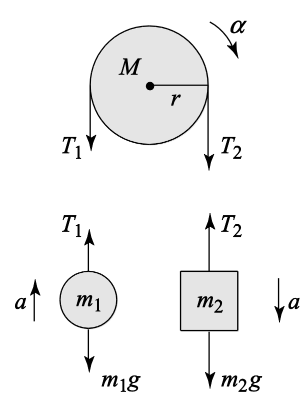
and

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{(0.500\ \text{rev/s} - 0)}{2.00\ \text{s}}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = \frac{\pi}{2}\ \text{rad/s}^2$$

Thus, $\tau = F\cdot r = I\alpha$ gives

$$F = \frac{I\alpha}{r} = \frac{(169\ \text{kg}\cdot\text{m}^2)\left(\frac{\pi}{2}\ \text{rad/s}^2\right)}{1.50\ \text{m}} = \boxed{177\ \text{N}}$$

8.40 (a) It is necessary that the tensions T_1 and T_2 be different in order $\boxed{\text{to provide a net torque}}$ about the axis of the pulley $\boxed{\text{and produce and angular acceleration}}$ of the pulley. Since intuition tells us that the system will accelerate in the directions shown in the diagrams at the right when $m_2 > m_1$, it is necessary that $T_2 > T_1$.



(b) We adopt a sign convention for each object with the positive direction being the indicated direction of the acceleration of that object in the diagrams at the right. Then, apply Newton's second law to each object:

For m_1: $\Sigma F_y = m_1 a \quad \Rightarrow \quad T_1 - m_1 g = m_1 a \qquad$ or $\qquad T_1 = m_1 \left(g + a\right) \qquad$ [1]

For m_2: $\Sigma F_y = m_2 a \quad \Rightarrow \quad m_2 g - T_2 = m_2 a\, m_2 \qquad$ or $\qquad T_2 = m_2 \left(g - a\right) \qquad$ [2]

For M: $\Sigma \tau = I\alpha \quad \Rightarrow \quad rT_2 - rT_1 = I\alpha \qquad$ or $\qquad T_2 - T_1 = I\alpha/r \qquad$ [3]

Substitute Equations [1] and [2], along with the relations $I = Mr^2/2$ and $\alpha = a/r$, into Equation [3] to obtain

$$m_2\left(g - a\right) - m_1\left(g + a\right) = \frac{Mr^2}{2r}\left(\frac{a}{r}\right) = \frac{Ma}{2} \qquad \text{or} \qquad \left(m_1 + m_2 + \frac{M}{2}\right)a = \left(m_2 - m_1\right)g$$

and

$$a = \frac{\left(m_2 - m_1\right)g}{m_1 + m_2 + M/2} = \frac{\left(20.0\text{ kg} - 10.0\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{20.0\text{ kg} + 10.0\text{ kg} + \left(8.00\text{ kg}\right)/2} = \boxed{2.88\text{ m/s}^2}$$

(c) From Equation [1]: $T_1 = \left(10.0\text{ kg}\right)\left(9.80\text{ m/s}^2 + 2.88\text{ m/s}^2\right) = \boxed{127\text{ N}}$.

From Equation [2]: $T_2 = \left(20.0\text{ kg}\right)\left(9.80\text{ m/s}^2 - 2.88\text{ m/s}^2\right) = \boxed{138\text{ N}}$.

8.41 The initial angular velocity of the wheel is zero, and the final angular velocity is

$$\omega_f = \frac{v}{r} = \frac{50.0\text{ m/s}}{1.25\text{ m}} = 40.0\text{ rad/s}$$

Hence, the angular acceleration is

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{40.0\text{ rad/s} - 0}{0.480\text{ s}} = 83.3\text{ rad/s}^2$$

The torque acting on the wheel is $\tau = f_k \cdot r$, so $\tau = I\alpha$ gives

$$f_k = \frac{I\alpha}{r} = \frac{\left(110 \text{ kg} \cdot \text{m}^2\right)\left(83.3 \text{ rad/s}^2\right)}{1.25 \text{ m}} = 7.33 \times 10^3 \text{ N}$$

Thus, the coefficient of friction is

$$\mu_k = \frac{f_k}{n} = \frac{7.33 \times 10^3 \text{ N}}{1.40 \times 10^4 \text{ N}} = \boxed{0.524}$$

8.42 (a) The moment of inertia of the flywheel is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(500 \text{ kg})(2.00 \text{ m})^2 = 1.00 \times 10^3 \text{ kg} \cdot \text{m}^2$$

and the angular velocity is

$$\omega = \left(5000 \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 524 \text{ rad/s}$$

Therefore, the stored kinetic energy is

$$KE_{\text{stored}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(1.00 \times 10^3 \text{ kg} \cdot \text{m}^2\right)(524 \text{ rad/s})^2 = \boxed{1.37 \times 10^8 \text{ J}}$$

(b) A 10.0-hp motor supplies energy at the rate of

$$\mathcal{P} = (10.0 \text{ hp})\left(\frac{746 \text{ W}}{1 \text{ hp}}\right) = 7.46 \times 10^3 \text{ J/s}$$

The time the flywheel could supply energy at this rate is

$$t = \frac{KE_{\text{stored}}}{\mathcal{P}} = \frac{1.37 \times 10^8 \text{ J}}{7.46 \times 10^3 \text{ J/s}} = 1.84 \times 10^4 \text{ s} = \boxed{5.10 \text{ h}}$$

8.43 The moment of inertia of the cylinder is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}\left(\frac{w}{g}\right)R^2 = \frac{1}{2}\left(\frac{800\text{ N}}{9.80\text{ m/s}^2}\right)(1.50\text{ m})^2 = 91.8\text{ kg}\cdot\text{m}^2$$

The angular acceleration is given by

$$\alpha = \frac{\tau}{I} = \frac{F\cdot R}{I} = \frac{(50.0\text{ N})(1.50\text{ m})}{91.8\text{ kg}\cdot\text{m}^2} = 0.817\text{ rad/s}^2$$

At $t = 3.00$ s, the angular velocity is

$$\omega = \omega_i + \alpha t = 0 + (0.817\text{ rad/s}^2)(3.00\text{ s}) = 2.45\text{ rad/s}$$

and the kinetic energy is

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(91.8\text{ kg}\cdot\text{m}^2)(2.45\text{ rad/s})^2 = \boxed{276\text{ J}}$$

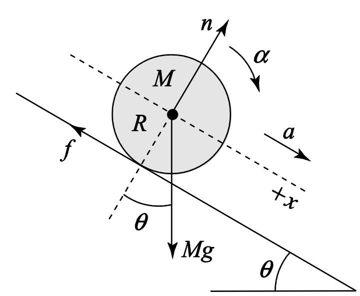
8.44 (a) Hoop: $I = MR^2 = (4.80\text{ kg})(0.230\text{ m})^2 = \boxed{0.254\text{ kg}\cdot\text{m}^2}$

Solid Cylinder: $I = \frac{1}{2}MR^2 = \frac{1}{2}(4.80\text{ kg})(0.230\text{ m})^2 = \boxed{0.127\text{ kg}\cdot\text{m}^2}$

Solid Sphere: $I = \frac{2}{5}MR^2 = \frac{2}{5}(4.80\text{ kg})(0.230\text{ m})^2 = \boxed{0.102\text{ kg}\cdot\text{m}^2}$

Thin, Spherical, Shell: $I = \frac{2}{3}MR^2 = \frac{2}{3}(4.80\text{ kg})(0.230\text{ m})^2 = \boxed{0.169\text{ kg}\cdot\text{m}^2}$

(b) When different objects of mass M and radius R roll without slipping $(\Rightarrow\ a = R\alpha)$ down a ramp, the one with the largest translational acceleration a will have the highest translational speed at the bottom. To determine the translational acceleration for the various objects, consider the free-body diagram at the right:



$$\Sigma F_x = Ma \quad \Rightarrow \quad Mg\sin\theta - f = Ma \qquad [1]$$

$$\tau = I\alpha \quad \Rightarrow \quad f\,R = I\left(a/R\right) \quad \text{or} \quad f = Ia/R^2 \qquad [2]$$

Substitute Equation [2] into [1] to obtain

$$Mg\sin\theta - Ia/R^2 = Ma \qquad \text{or} \qquad a = \frac{Mg\sin\theta}{M + I/R^2}$$

Since M, R, g are θ the same for all of the objects, we see that the translational acceleration (and hence the translational speed) increases as the moment of inertia decreases. Thus, the proper rankings from highest to lowest by translational speed will be:

$\boxed{\text{Solid sphere; solid cylinder; thin, spherical, shell; and hoop}}$

(c) When an object rolls down the ramp without slipping, the friction force does no work and mechanical energy is conserved. Then, the total kinetic energy gained equals the gravitational potential energy given up: $KE_r + KE_t = -\Delta PE_g = Mgh$ and $KE_r = Mgh - \frac{1}{2}Mv^2$, where h is the vertical drop of the ramp and v is the translational speed at the bottom. Since M, g, and h are the same for all of the objects, the rotational kinetic energy decreases as the translational speed increases. Using this fact, along with the result of Part (b), we rank the object's final rotational kinetic energies, from highest to lowest, as:

$\boxed{\text{hoop; thin, spherical, shell; solid cylinder; and solid sphere}}$

8.45 (a) Treating the particles on the ends of the rod as point masses, the total moment of inertia of the rotating system is $I = I_{\text{rod}} + I_3 + I_4 = m_{\text{rod}}L^2/12 + m_3(L/2)^2 + m_4(L/2)^2$. If the mass of the rod can be ignored, this reduces to

$$I = 0 + \left(m_3 + m_4\right)\left(\frac{L}{2}\right)^2 = \left(3.00\text{ kg} + 4.00\text{ kg}\right)\left(0.500\text{ m}\right)^2 = 1.75\text{ kg}\cdot\text{m}^2$$

and the rotational kinetic energy is

$$KE_r = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(1.75\text{ kg}\cdot\text{m}^2\right)\left(2.50\text{ rad/s}\right)^2 = \boxed{5.47\text{ J}}$$

(b) If the rod has mass $m_{\text{rod}} = 2.00\text{ kg}$

$$I = \frac{1}{12}\left(2.00\text{ kg}\right)\left(1.00\text{ m}\right)^2 + 1.75\text{ kg}\cdot\text{m}^2 = 1.92\text{ kg}\cdot\text{m}^2$$

and

$$KE_r = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(1.92\text{ kg}\cdot\text{m}^2\right)\left(2.50\text{ rad/s}\right)^2 = \boxed{6.00\text{ J}}$$

8.46 Using conservation of mechanical energy,

$$\left(KE_{\text{trans}} + KE_{\text{rot}} + PE_g\right)_f = \left(KE_{\text{trans}} + KE_{\text{rot}} + PE_g\right)_i$$

or

$$\frac{1}{2} M v_t^2 + \frac{1}{2} I\omega^2 + 0 = 0 + 0 + Mg\left(L\sin\theta\right)$$

Since $I = 2/5\,MR^2$ for a solid sphere and $v_t = R\omega$ when rolling without slipping, this becomes

$$\frac{1}{2} MR^2\omega^2 + \frac{1}{5} MR^2\omega^2 = Mg\left(L\sin\theta\right)$$

and reduces to

$$\omega = \sqrt{\frac{10gL\sin\theta}{7R^2}} = \sqrt{\frac{10\left(9.8\text{ m/s}^2\right)\left(6.0\text{ m}\right)\sin 37^\circ}{7\left(0.20\text{ m}\right)^2}} = \boxed{36\text{ rad/s}}$$

8.47 (a) Assuming the disk rolls without slipping, the friction force between the disk and the ramp does no work. In this case, the total mechanical energy of the disk is constant with the value $E = KE_i + \left(PE_g\right)_i = 0 + Mgh = MgL\sin\theta$. When the disk gets to the bottom of the ramp, $PE_g = 0$ and $KE_f = KE_t + KE_r = E = MgL\sin\theta$. Also, since the disk does not slip, $\omega = v/R$ and

$$KE_r = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}\left(\frac{1}{2}Mv^2\right) = \frac{1}{2}KE_t$$

Then,

$$KE_{\text{total}} = KE_t + \frac{1}{2}KE_t = E = MgL\sin\theta \quad\text{or}\quad \frac{3}{2}\left(\frac{1}{2}\cancel{M}v^2\right) = \cancel{M}gL\sin\theta$$

and

$$v = \sqrt{\frac{4gL\sin\theta}{3}} = \sqrt{\frac{4\left(9.80\ \text{m/s}^2\right)\left(4.50\ \text{m}\right)\sin 15.0^\circ}{3}} = \boxed{3.90\ \text{m/s}}$$

(b) The angular speed of the disk at the bottom is

$$\omega = \frac{v}{R} = \frac{3.90\ \text{m/s}}{0.250\ \text{m}} = \boxed{15.6\ \text{rad/s}}$$

8.48 (a) Assuming the solid sphere starts from rest, and taking y =0 at the level of the bottom of the incline, the total mechanical energy will be split among three distinct forms of energy $E = \left(PE_g\right)_i = mgh$ as the sphere rolls down the incline. These are

$$\boxed{\text{rotational kinetic energy, } \frac{1}{2}I\omega^2}$$

$$\boxed{\text{translational kinetic energy, } \frac{1}{2}mv^2}$$

and

$$\boxed{\text{gravitational potential energy, } mgy}$$

where y is the current height of the center of mass of the sphere above the level of the bottom of the incline.

(b) The $\boxed{\text{force of static friction, exerted on the sphere by the incline}}$ and directed up the incline, exerts a torque about the center of mass giving the sphere an angular acceleration.

(c) $KE_t = \frac{1}{2}Mv^2$ and $KE_r = \frac{1}{2}I\omega^2$ where $v = R\omega$ (since the sphere rolls without slipping) and $I = \frac{2}{5}MR^2$ for a solid sphere. Therefore,

$$\frac{KE_r}{KE_t + KE_r} = \frac{I\omega^2/2}{Mv^2/2 + I\omega^2/2} = \frac{\left(2MR^2/5\right)\omega^2}{M\left(R\omega\right)^2 + \left(2MR^2/5\right)\omega^2} = \frac{2\cancel{MR^2\omega^2}}{5\cancel{MR^2\omega^2} + 2\cancel{MR^2\omega^2}} = \boxed{\frac{2}{7}}$$

8.49 Using $W_{net} = KE_f - KE_i = \frac{1}{2}I\omega_f^2 - 0$, we have

$$\omega_f = \sqrt{\frac{2\,W_{net}}{I}} = \sqrt{\frac{2\,F\cdot s}{I}} = \sqrt{\frac{2(5.57\text{ N})(0.800\text{ m})}{4.00\times10^{-4}\text{ kg}\cdot\text{m}^2}} = \boxed{149\text{ rad/s}}$$

8.50 The work done on the grindstone is $W_{net} = F\cdot s = F\cdot(r\,\theta) = (F\cdot r)\,\theta = \tau\cdot\theta$.

Thus, $W_{net} = \tau\cdot\theta = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$, or

$$(25.0\text{ N}\cdot\text{m})(15.0\text{ rev})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = \frac{1}{2}(0.130\text{ kg}\cdot\text{m}^2)\omega_f^2 - 0$$

This yields

$$\omega_f = \left(190\ \frac{\text{rad}}{\text{s}}\right)\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = \boxed{30.3\text{ rev/s}}$$

8.51 (a) $KE_{trans} = \frac{1}{2}mv_t^2 = \frac{1}{2}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{500\text{ J}}$

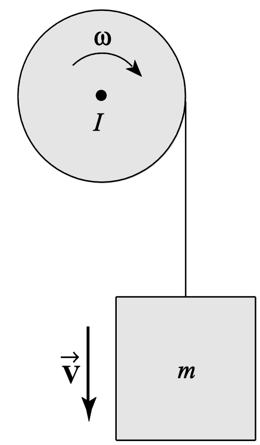
(b)

$$KE_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v_t^2}{R^2}\right)$$

$$= \frac{1}{4}mv_t^2 = \frac{1}{4}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{250\text{ J}}$$

(c) $KE_{total} = KE_{trans} + KE_{rot} = \boxed{750\text{ J}}$

8.52 As the bucket drops, it loses gravitational potential energy. The spool gains rotational kinetic energy and the bucket gains translational kinetic energy. Since the string does not slip on the spool, $\upsilon = r\omega$ where r is the radius of the spool. The moment of inertia of the spool is $I = \frac{1}{2}Mr^2$, where M is the mass of the spool. Conservation of energy gives



$$\left(KE_t + KE_r + PE_g\right)_f = \left(KE_t + KE_r + PE_g\right)_i$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgy_f = 0 + 0 + mgy_i$$

or

$$\frac{1}{2}m\left(r\omega\right)^2 + \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\omega^2 = mg\left(y_i - y_f\right)$$

This gives

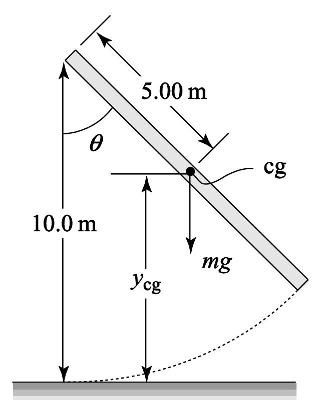
$$\omega = \sqrt{\frac{2mg\left(y_i - y_f\right)}{\left(m + \frac{1}{2}M\right)r^2}} = \sqrt{\frac{2\left(3.00\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\left(4.00\text{ m}\right)}{\left[3.00\text{ kg}+\frac{1}{2}\left(5.00\text{ kg}\right)\right]\left(0.600\text{ m}\right)^2}} = \boxed{10.9\text{ rad/s}}$$

8.53 (a) The arm consists of a uniform rod of 10.0 m length and the mass of the seats at the lower end is negligible. The center of gravity of this system is then located at the $\boxed{\text{geometric center of the arm, located 5.00 m from the upper end}}$.

From the sketch at the right, the height of the center of gravity above the zero level is $y_{cg} = 10.0\text{ m} - \left(5.00\text{ m}\right)\cos\theta$.

(b) When $\theta = 45.0°$, $y_{cg} = 10.0\text{ m} - \left(5.00\text{ m}\right)\cos 45.0° = 6.46\text{ m}$

and



$$PE_g = mgy_{cg} = \left(365\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\left(6.46\text{ m}\right) = \boxed{2.31 \times 10^4\text{ J}}$$

(c) In the vertical orientation, $\theta = 0°$ and $\cos\theta = 1$, giving $y_{cg} = 10.0\text{ m} - 5.00\text{ m} = 5.00\text{ m}$. Then,

$$PE_g = mgy_{cg} = \left(365\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\left(5.00\text{ m}\right) = \boxed{1.79 \times 10^4\text{ J}}$$

(d) Using conservation of mechanical energy as the arm starts from rest in the 45° orientation and rotates about the upper end to the vertical orientation gives

$$\frac{1}{2} I_{end}\omega_f^2 + mg\left(y_{cg}\right)_f = 0 + mg\left(y_{cg}\right)_i \quad \text{or} \qquad \omega_f = \sqrt{\frac{2mg\left[\left(y_{cg}\right)_i - \left(y_{cg}\right)_f\right]}{I_{end}}} \qquad [1]$$

For a long, thin rod: $I_{end} = mL^2/3$. Equation [1] then becomes

$$\omega_f = \sqrt{\frac{2\cancel{m}g\left[\left(y_{cg}\right)_i - \left(y_{cg}\right)_f\right]}{\cancel{m}L^2/3}} = \sqrt{\frac{6g\left[\left(y_{cg}\right)_i - \left(y_{cg}\right)_f\right]}{L^2}}$$

$$= \sqrt{\frac{6\left(9.80\ \text{m/s}^2\right)\left(6.46\ \text{m} - 5.00\ \text{m}\right)}{\left(10.0\ \text{m}\right)^2}} = 0.927\ \text{rad/s}$$

Then, from $\upsilon = r\omega$, the translational speed of the seats at the lower end of the rod is

$$\upsilon = \left(10.0\ \text{m}\right)\left(0.927\ \text{rad/s}\right) = \boxed{9.27\ \text{m/s}}$$

8.54 (a) $L = I\omega = \left(MR^2\right)\omega = \left(2.40\ \text{kg}\right)\left(0.180\ \text{m}\right)^2\left(35.0\ \text{rad/s}\right) = \boxed{2.72\ \text{kg}\cdot\text{m}^2/\text{s}}$

(b) $L = I\omega = \left(\frac{1}{2}MR^2\right)\omega = \frac{1}{2}\left(2.40\ \text{kg}\right)\left(0.180\ \text{m}\right)^2\left(35.0\ \text{rad/s}\right) = \boxed{1.36\ \text{kg}\cdot\text{m}^2/\text{s}}$

(c) $L = I\omega = \left(\frac{2}{5}MR^2\right)\omega = \frac{2}{5}\left(2.40\ \text{kg}\right)\left(0.180\ \text{m}\right)^2\left(35.0\ \text{rad/s}\right) = \boxed{1.09\ \text{kg}\cdot\text{m}^2/\text{s}}$

(d) $L = I\omega = \left(\frac{2}{3}MR^2\right)\omega = \frac{2}{3}\left(2.40\ \text{kg}\right)\left(0.180\ \text{m}\right)^2\left(35.0\ \text{rad/s}\right) = \boxed{1.81\ \text{kg}\cdot\text{m}^2/\text{s}}$

8.55 (a) The rotational speed of Earth is

$$\omega_E = \frac{2\pi \text{ rad}}{1 \text{ d}}\left(\frac{1 \text{ d}}{8.64 \times 10^4 \text{ s}}\right) = 7.27 \times 10^{-5} \text{ rad/s}$$

$$L_{spin} = I_{sphere}\omega_E = \left(\frac{2}{5}M_E R_E^2\right)\omega_E$$
$$= \left[\frac{2}{5}\left(5.98 \times 10^{24} \text{ kg}\right)\left(6.38 \times 10^6 \text{ m}\right)^2\right]\left(7.27 \times 10^{-5} \text{ rad/s}\right) = \boxed{7.08 \times 10^{33} \text{ J} \cdot \text{s}}$$

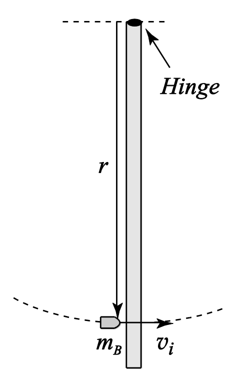
(b) For Earth's orbital motion,

$$\omega_{orbit} = \frac{2\pi \text{ rad}}{1 \text{ y}}\left(\frac{1 \text{ y}}{3.156 \times 10^7 \text{ s}}\right) = 1.99 \times 10^{-7} \text{ rad/s}$$

and using data from Table 7.3, we find

$$L_{orbit} = I_{point}\omega_{orbit} = \left(M_E R_{orbit}^2\right)\omega_{orbit}$$
$$= \left(5.98 \times 10^{24} \text{ kg}\right)\left(1.496 \times 10^{11} \text{ m}\right)^2\left(1.99 \times 10^{-7} \text{ rad/s}\right) = \boxed{2.67 \times 10^{40} \text{ J} \cdot \text{s}}$$

8.56 (a) $\boxed{\text{Yes}}$, the bullet has angular momentum about an axis through the hinges of the door before the collision. Consider the sketch at the right, showing the bullet the instant before it hits the door. The physical situation is identical to that of a point mass m_g moving in a circular path of radius r with tangential speed $v_t = v_i$. For that situation the angular momentum is



$$L_i = I_i\omega_i = \left(m_B r^2\right)\left(\frac{v_i}{r}\right) = m_B r v_i$$

and this is also the angular momentum of the bullet about the axis through the hinge at the instant just before impact.

(b) $\boxed{\text{No, mechanical energy is not conserved}}$ in the collision. The bullet embeds itself in the door with the two moving as a unit after impact. This is a perfectly inelastic collision in which a significant amount of mechanic cal energy is converted to other forms, notably thermal energy.

(c) Apply conservation of angular momentum with $L_i = m_B r \upsilon_i$ as discussed in part (a). After impact,

$L_f = I_f \omega_f = \left(I_{door} + I_{bullet}\right)\omega_f = \left(\frac{1}{2} M_{door} L^2 + m_B r^2\right)\omega_f$ where L = 1.00 m = the width of the door and $r = L$ – 10.0 cm = 0.900 m. Then,

$$L_f = L_i \quad \Rightarrow \quad \omega_f = \frac{m_B r \upsilon_i}{\frac{1}{3}\left(M_{door} L^2\right) + m_B r^2} = \frac{(0.005\text{ kg})(0.900\text{ m})\left(1.00 \times 10^3\text{ m/s}\right)}{\frac{1}{3}(18.0\text{ kg})(1.00\text{ m})^2 + (0.005\text{ kg})(0.900\text{ m})^2}$$

yielding $\boxed{\omega_f = 0.749\text{ rad/s}}$.

(d) The kinetic energy of the door-bullet system immediately after impact is

$$KE_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{2}\left[\frac{1}{3}(18.0\text{ kg})(1.00\text{ m})^2 + (0.005\text{ kg})(0.900\text{ m})^2\right](0.749\text{ rad/s})^2$$

or $\boxed{KE_f = 1.68\text{ J}}$.

The kinetic energy (of the bullet) just before impact was

$$KE_i = \frac{1}{2} m_B \upsilon_i^2 = \frac{1}{2}(0.005\text{ kg})\left(1.00 \times 10^3\text{ m/s}\right)^2 = \boxed{2.50 \times 10^3\text{ J}}$$

8.57 Each mass moves in a circular path of radius r = 0.500 m/s about the center of the connecting rod. Their angular speed is

$$\omega = \frac{\upsilon}{r} = \frac{5.00\text{ m/s}}{0.500\text{ m}} = 10.0\text{ m/s}$$

Neglecting the moment of inertia of the light connecting rod, the angular momentum of this rotating system is

$$L = I\omega = \left[m_1 r^2 + m_2 r^2\right]\omega = (4.00\text{ kg} + 3.00\text{ kg})(0.500\text{ m})^2(10.0\text{ rad/s}) = \boxed{17.5\text{ J}\cdot\text{s}}$$

8.58 Using conservation of angular momentum, $L_{aphelion} = L_{perihelion}$.

Thus, $\left(m r_a^2\right)\omega_a = \left(m r_p^2\right)\omega_p$. Since $\omega = \upsilon_{\text{t}}/r$ at both aphelion and perihelion, this is equivalent to $\left(m r_a^2\right)\upsilon_a/r_a = \left(m r_p^2\right)\upsilon_p/r_p$, giving

$$\upsilon_a = \left(\frac{r_p}{r_a}\right)\upsilon_p = \left(\frac{0.59\text{ A.U.}}{35\text{ A.U.}}\right)(54\text{ km/s}) = \boxed{0.91\text{ km/s}}$$

8.59 The initial moment of inertia of the system is

$$I_i = \Sigma m_i r_i^2 = 4\left[M(1.0\text{ m})^2\right] = M\left(4.0\text{ m}^2\right)$$

The moment of inertia of the system after the spokes are shortened is

$$I_f = \Sigma m_f r_f^2 = 4\left[M(0.50\text{ m})^2\right] = M\left(1.0\text{ m}^2\right)$$

From conservation of angular momentum, $I_f\,\omega_f = I_i\,\omega_i$, or

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = (4)(2.0\text{ rev/s}) = \boxed{8.0\text{ rev/s}}$$

8.60 From conservation of angular momentum: $\left(I_{child} + I_{m\text{-}g\text{-}r}\right)_f \omega_f = \left(I_{child} + I_{m\text{-}g\text{-}r}\right)_i \omega_i$

where $I_{m\text{-}g\text{-}r} = 275\text{ kg}\cdot\text{m}^2$ is the constant moment of inertia of the merry-go-round.

Treating the child as a point object, $I_{child} = mr^2$ where r is the distance the child is from the rotation axis. Conservation of angular momentum then gives

$$\omega_f = \left(\frac{mr_i^2 + I_{m\text{-}g\text{-}r}}{mr_f^2 + I_{m\text{-}g\text{-}r}}\right)\omega_i = \left[\frac{(25.0\text{ kg})(1.00\text{ m})^2 + 275\text{ kg}\cdot\text{m}^2}{(25.0\text{ kg})(2.00\text{ m})^2 + 275\text{ kg}\cdot\text{m}^2}\right](14.0\text{ rev/min})$$

or

$$\omega_f = 11.2\ \frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = \boxed{1.17\text{ rad/s}}$$

8.61 The moment of inertia of the cylinder before the putty arrives is

$$I_i = \frac{1}{2}MR^2 = \frac{1}{2}(10.0\text{ kg})(1.00\text{ m})^2 = 5.00\text{ kg}\cdot\text{m}^2$$

After the putty sticks to the cylinder, the moment of inertia is

$$I_f = I_i + mr^2 = 5.00\ \text{kg}\cdot\text{m}^2 + (0.250\ \text{kg})(0.900\ \text{m})^2 = 5.20\ \text{kg}\cdot\text{m}^2$$

Conservation of angular momentum gives $I_f\omega_f = I_i\omega_i$, or

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{5.00\ \text{kg}\cdot\text{m}^2}{5.20\ \text{kg}\cdot\text{m}^2}\right)(7.00\ \text{rad/s}) = \boxed{6.73\ \text{rad/s}}$$

8.62 The total moment of inertia of the system is

$$I_{total} = I_{masses} + I_{student} = 2\left(mr^2\right) + 3.0\ \text{kg}\cdot\text{m}^2$$

Initially, $r = 1.0$ m, and $I_i = 2\left[(3.0\ \text{kg})(1.0\ \text{m})^2\right] + 3.0\ \text{kg}\cdot\text{m}^2 = 9.0\ \text{kg}\cdot\text{m}^2$.

Afterward, $r = 0.30$ m, so

$$I_f = 2\left[(3.0\ \text{kg})(0.30\ \text{m})^2\right] + 3.0\ \text{kg}\cdot\text{m}^2 = 3.5\ \text{kg}\cdot\text{m}^2$$

(a) From conservation of angular momentum, $I_f\,\omega_f = I_i\,\omega_i$, or

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{9.0\ \text{kg}\cdot\text{m}^2}{3.5\ \text{kg}\cdot\text{m}^2}\right)(0.75\ \text{rad/s}) = \boxed{1.9\ \text{rad/s}}$$

(b) $$KE_i = \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}\left(9.0\ \text{kg}\cdot\text{m}^2\right)(0.75\ \text{rad/s})^2 = \boxed{2.5\ \text{J}}$$

$$KE_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}\left(3.5\ \text{kg}\cdot\text{m}^2\right)(1.9\ \text{rad/s})^2 = \boxed{6.3\ \text{J}}$$

8.63 The initial angular velocity of the puck is

$$\omega_i = \frac{(v_t)_i}{r_i} = \frac{0.800\ \text{m/s}}{0.400\ \text{m}} = 2.00\ \frac{\text{rad}}{\text{s}}$$

Since the tension in the string does not exert a torque about the axis of revolution, the angular momentum of the puck is conserved, or $I_f\,\omega_f = I_i\,\omega_i$.

Thus,

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{mr_i^2}{mr_f^2}\right)\omega_i = \left(\frac{0.400\text{ m}}{0.250\text{ m}}\right)^2\left(2.00\ \text{rad/s}\right) = 5.12\ \text{rad/s}$$

The net work done on the puck is

$$W_{net} = KE_f - KE_i = \frac{1}{2}I_f\,\omega_f^2 - \frac{1}{2}I_i\,\omega_i^2 = \frac{1}{2}\left[\left(mr_f^2\right)\omega_f^2 - \left(mr_i^2\right)\omega_i^2\right] = \frac{m}{2}\left[r_f^2\omega_f^2 - r_i^2\omega_i^2\right]$$

or

$$W_{net} = \frac{(0.120\text{ kg})}{2}\left[(0.250\text{ m})^2(5.12\ \text{rad/s})^2 - (0.400\text{ m})^2(2.00\ \text{rad/s})^2\right]$$

This yields $W_{net} = \boxed{5.99\times10^{-2}\text{ J}}$.

8.64 For one of the crew, $\Sigma F_c = m\,a_c$ becomes $n = m\left(v_t^2/r\right) = mr\,\omega_i^2$. We require $n = mg$, so the initial angular velocity must be $\omega_i = \sqrt{g/r}$. From conservation of angular momentum, $I_f\,\omega_f = I_i\,\omega_i$ or $\omega_f = (I_i/I_f)\omega_i$. Thus, the angular velocity of the station during the union meeting is

$$\omega_f = \left(\frac{I_i}{I_f}\right)\sqrt{\frac{g}{r}} = \left[\frac{5.00\times10^8\text{ kg}\cdot\text{m}^2 + 150(65.0\text{ kg})(100\text{ m})^2}{5.00\times10^8\text{ kg}\cdot\text{m}^2 + 50(65.0\text{ kg})(100\text{ m})^2}\right]\sqrt{\frac{g}{r}} = 1.12\sqrt{\frac{g}{r}}$$

The centripetal acceleration experienced by the managers still on the rim is

$$a_c = r\omega_f^2 = r\,12.3\ \text{m/s}^2\,(1.12)^2\frac{g}{r} = (1.12)^2\left(9.80\ \text{m/s}^2\right) = \boxed{12.3\ \text{m/s}^2}$$

8.65 (a) From conservation of angular momentum, $I_f\omega_f = I_i\omega_i$, so

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \boxed{\left(\frac{I_1}{I_1+I_2}\right)\omega_o}$$

(b) $$KE_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{2}\left(I_1 + I_2\right)\left(\frac{I_1}{I_1 + I_2}\right)^2 \omega_o^2 = \left(\frac{I_1}{I_1 + I_2}\right)\left[\frac{1}{2} I_1 \omega_o^2\right] = \left(\frac{I_1}{I_1 + I_2}\right) KE_i$$

or

$$\frac{KE_f}{KE_i} = \boxed{\frac{I_1}{I_1 + I_2}}$$

Since this is less than 1.0, kinetic energy was lost.

8.66 The initial angular velocity of the system is

$$\omega_i = \left(0.20 \ \frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 0.40\pi \ \text{rad/s}$$

The total moment of inertia is given by

$$I = I_{man} + I_{cylinder} = mr^2 + \frac{1}{2} MR^2 = \left(80 \text{ kg}\right) r^2 + \frac{1}{2}\left(25 \text{ kg}\right)\left(2.0 \text{ m}\right)^2$$

Initially, the man is at $r = 2.0 \text{ m}$ from the axis, and this gives $I_i = 3.7 \times 10^2 \ \text{kg} \cdot \text{m}^2$. At the end, when $r = 1.0$, the moment of inertia is $I_f = 1.3 \times 10^2 \ \text{kg} \cdot \text{m}^2$.

(a) From conservation of angular momentum, $I_f \omega_f = I_i \omega_i$, or

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{3.7 \times 10^2 \ \text{kg} \cdot \text{m}^2}{1.3 \times 10^2 \ \text{kg} \cdot \text{m}^2}\right)\left(0.40\pi \ \text{rad/s}\right) = 1.14\,\pi \ \text{rad/s} = \boxed{3.6 \ \text{rad/s}}$$

(b) The change in kinetic energy is $\Delta KE = \frac{1}{2} I_f \omega_f^2 - \frac{1}{2} I_f \omega_i^2$, or

$$\Delta KE = \frac{1}{2}\left(1.3 \times 10^2 \ \text{kg} \cdot \text{m}^2\right)\left(1.14\,\pi \ \frac{\text{rad}}{\text{s}}\right)^2 - \frac{1}{2}\left(3.7 \times 10^2 \ \text{kg} \cdot \text{m}^2\right)\left(0.40\,\pi \ \frac{\text{rad}}{\text{s}}\right)^2$$

or $\Delta KE = \boxed{5.4 \times 10^2 \ \text{J}}$. The difference is the work done by the man as he walks inward.

8.67 (a) The table turns counterclockwise, opposite to the way the woman walks. Its angular momentum cancels that of the woman so the total angular momentum maintains a constant value of $L_{total} = L_{woman} + L_{table} = 0$.

Since the final angular momentum is $L_{total} = I_w \omega_w + I_t \omega_t = 0$, we have

$$\omega_t = -\left(\frac{I_w}{I_t}\right)\omega_w = -\left(\frac{m_w r^2}{I_t}\right)\left(\frac{\upsilon_w}{r}\right) = -\left(\frac{m_w r}{I_t}\right)\upsilon_w$$

or

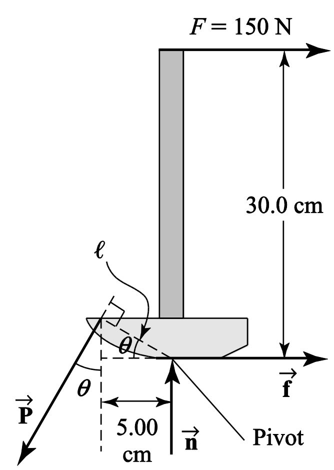
$$\omega_t = -\left[\frac{(60.0 \text{ kg})(2.00 \text{ m})}{500 \text{ kg}\cdot\text{m}^2}\right](-1.50 \text{ m/s}) = 0.360 \text{ rad/s}$$

Hence, $\omega_{table} = \boxed{0.360 \text{ rad/s counterclockwise}}$.

(b) $$W_{net} = \Delta KE = KE_f - 0 = \frac{1}{2}m\upsilon_w^2 + \frac{1}{2}I_t\omega_t^2$$

$$W_{net} = \frac{1}{2}(60.0 \text{ kg})(1.50 \text{ m/s})^2 + \frac{1}{2}(500 \text{ kg}\cdot\text{m}^2)(0.360 \text{ rad/s})^2 = \boxed{99.9 \text{ J}}$$

8.68 (a) In the sketch at the right, choose an axis perpendicular to the page and passing through the indicated pivot. Then, with $\theta = 30.0°$, the lever arm of the force $\vec{\mathbf{P}}$ is observed to be



$$\ell = \frac{5.00 \text{ cm}}{\cos\theta} = \frac{5.00 \text{ cm}}{\cos 30.0°} = 5.77 \text{ cm}$$

and $\Sigma\tau = 0$ gives

$$+P(5.77 \text{ cm}) - (150 \text{ N})(30.0 \text{ cm}) = 0$$

so

$$P = \frac{(150 \text{ N})(30.0 \text{ cm})}{5.77 \text{ cm}} = \boxed{780 \text{ N}}$$

(b) $\Sigma F_y = 0 \Rightarrow n - P\cos 30.0° = 0$, giving

$$n = P\cos 30.0° = (780 \text{ N})\cos 30.0° = 675 \text{ N}$$

$\Sigma F_x = 0 \Rightarrow f + F - P\sin 30.0° = 0$, or

$$f = P\sin 30.0° - F = (780 \text{ N})\sin 30.0° - 150 \text{ N} = 240 \text{ N}$$

The resultant force exerted on the hammer at the pivot is

$$R = \sqrt{f^2 + n^2} = \sqrt{(240 \text{ N})^2 + (675 \text{ N})^2} = 716 \text{ N}$$

at $\theta = \tan^{-1}(n/f) = \tan^{-1}(675 \text{ N}/240 \text{ N}) = 70.4°$, or

$$\vec{\mathbf{R}} = \boxed{716 \text{ N at } 70.4° \text{ above the horizontal to the right}}$$

8.69 (a) Since no horizontal force acts on the child-boat system, the center of gravity of this system will remain stationary, or

$$x_{\text{cg}} = \frac{m_{child}x_{child} + m_{boat}x_{boat}}{m_{child} + m_{boat}} = \text{constant}$$

The masses do not change, so this result becomes $m_{\text{child}}\, x_{\text{child}} + m_{\text{boat}} x_{\text{boat}} = \text{constant}$.

Thus, $\boxed{\text{as the child walks to the right, the boat will move to the left}}$.

(b) Measuring distances from the stationary pier, with away from the pier being positive, the child is initially at $(x_{\text{child}})_{\text{i}} = 3.00$ m and the center of gravity of the boat is at $(x_{\text{boat}})_{\text{i}} = 5.00$. At the end, the child is at the right end of the boat, so $(x_{\text{child}})_f = (x_{\text{boat}})_f + 2.00$ m. Since the center of gravity of the system does not move,

we have $\left(m_{child}x_{child} + m_{boat}x_{boat}\right)_f = \left(m_{child}x_{child} + m_{boat}x_{boat}\right)_i$, or

$$m_{child}\left(x_{child}\right)_f + m_{boat}\left[\left(x_{child}\right)_f - 2.00 \text{ m}\right] = m_{child}\left(3.00 \text{ m}\right) + m_{boat}\left(5.00 \text{ m}\right)$$

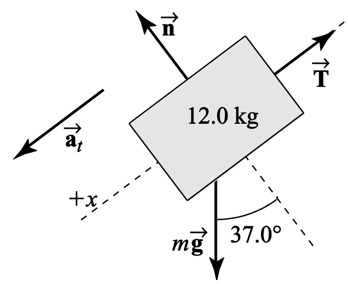
and

$$\left(x_{child}\right)_f = \frac{m_{child}\left(3.00 \text{ m}\right) + m_{boat}\left(5.00 \text{ m} + 2.00 \text{ m}\right)}{m_{child} + m_{boat}}$$

$$\left(x_{child}\right)_f = \frac{\left(40.0 \text{ kg}\right)\left(3.00 \text{ m}\right) + \left(70.0 \text{ kg}\right)\left(5.00 \text{ m} + 2.00 \text{ m}\right)}{40.0 \text{ kg} + 70.0 \text{ kg}} = \boxed{5.55 \text{ m}}$$

(c) When the child arrives at the right end of the boat, the greatest distance from the pier that he can reach is $x_{\max} = \left(x_{child}\right)_f + 1.00 \text{ m} = 5.55 \text{ m} + 1.00 \text{ m} = 6.55 \text{ m}$ This leaves him $\boxed{\text{0.45 m short of reaching the turtle}}$.

8.70 (a) Consider the free-body diagram of the block given at the right. If the $+x$-axis is directed down the incline, $\Sigma F_x = ma_x$ gives

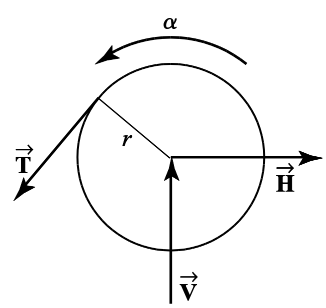


$$mg\sin 37.0° - T = m\,a_t\text{, or } T = m\left(g\sin 37.0° - a_t\right)$$

$$T = \left(12.0 \text{ kg}\right)\left[\left(9.80 \text{ m/s}^2\right)\sin 37.0° - 2.00 \text{ m/s}^2\right]$$

$$= \boxed{46.8 \text{ N}}$$

(b) Now, consider the free-body diagram of the pulley. Choose an axis perpendicular to the page and passing through the center of the pulley, $\Sigma\tau = I\,\alpha$ gives



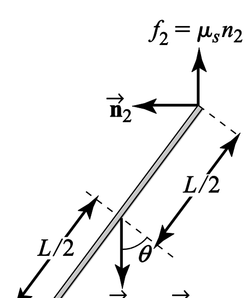
$$T \cdot r = I\left(\frac{a_t}{r}\right)$$

or

$$I = \frac{T \cdot r^2}{a_t} = \frac{\left(46.8 \text{ N}\right)\left(0.100 \text{ m}\right)^2}{2.00 \text{ m/s}^2} = \boxed{0.234 \text{ kg} \cdot \text{m}^2}$$

(c) $\omega = \omega_i + \alpha t = 0 + \left(\frac{a_t}{r}\right)t = \left(\frac{2.00 \text{ m/s}^2}{0.100 \text{ m}}\right)\left(2.00 \text{ s}\right) = \boxed{40.0 \text{ rad/s}}$

8.71 If the ladder is on the verge of slipping, $f = \left(f_s\right)_{\max} = \mu_s n$ at both the



floor and the wall.

From $\Sigma F_x = 0$, we find $f_1 - n_2 = 0$, or

$$n_2 = \mu_s n_1 \qquad [1]$$

Also, $\Sigma F_x = 0$ gives $n_1 - w + \mu_s n_2 .= 0$

Using Equation [1], this becomes

$$n_1 - w + \mu_s \left(\mu_s n_1\right) = 0$$

or

$$n_1 = \frac{w}{1+\mu_s^2} = \frac{w}{1.25} = 0.800\,w \qquad [2]$$

Thus, Equation [1] gives

$$n_2 = 0.500\left(0.800\,w\right) = 0.400\,w \qquad [3]$$

Choose an axis perpendicular to the page and passing through the lower end of the ladder. Then, $\Sigma\tau = 0$ yields

$$-w\left(\frac{L}{2}\cos\theta\right) + n_2\left(L\sin\theta\right) + f_2\left(L\cos\theta\right) = 0$$

Making the substitutions $n_2 = 0.400\,w$ and $f_2 = \mu_s n_2 = 0.200\,w$, this becomes

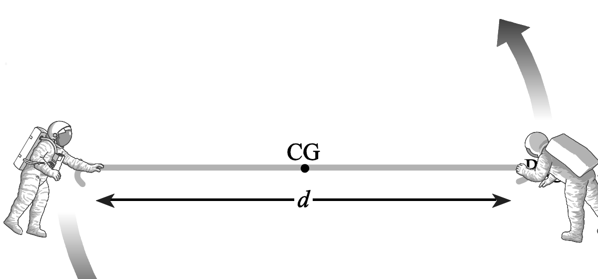
$$-w\left(\frac{L}{2}\cos\theta\right) + \left(0.400\,w\right)\left(L\sin\theta\right) + \left(0.200\,w\right)\left(L\cos\theta\right) = 0$$

and reduces to

$$\sin\theta = \left(\frac{0.500-0.200}{0.400}\right)\cos\theta$$

Hence, $\tan\theta = 0.750$ and $\theta = \boxed{36.9°}$.

8.72



We treat each astronaut as a point object, m, moving at speed v in a circle of radius $r = d/2$. Then the total angular momentum is

$$L = I_1\omega + I_2\omega = 2\left[\left(mr^2\right)\left(\frac{\upsilon}{r}\right)\right] = 2m\upsilon r$$

(a) $L_i = 2m\upsilon_i r_i = 2(75.0\text{ kg})(5.00\text{ m/s})(5.00\text{ m})$

$L_i = \boxed{3.75 \times 10^3\text{ kg}\cdot\text{m}^2/\text{s}}$

(b) $KE_i = \frac{1}{2}m_1\upsilon_{1i}^2 + \frac{1}{2}m_2\upsilon_{2i}^2 = 2\left(\frac{1}{2}m\upsilon_i^2\right)$

$KE_i = (75.0\text{ kg})(5.00\text{ m/s})^2 = 1.88 \times 10^3\text{ J} = \boxed{1.88\text{ kJ}}$

(c) Angular momentum is conserved: $L_f = L_i = \boxed{3.75 \times 10^3\text{ kg}\cdot\text{m}^2/\text{s}}$.

(d) $\upsilon_f = \dfrac{L_f}{2\left(mr_f\right)} = \dfrac{3.75 \times 10^3\text{ kg}\cdot\text{m}^2/\text{s}}{2(75.0\text{ kg})(2.50\text{ m})} = \boxed{10.0\text{ m/s}}$

(e) $KE_f = 2\left(\frac{1}{2}m\upsilon_f^2\right) = (75.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{7.50\text{ kJ}}$

(f) $W_{net} = KE_f - KE_i = \boxed{5.62\text{ kJ}}$

8.73 (a) $L_i = 2\left[M\upsilon\left(\frac{d}{2}\right)\right] = \boxed{M\upsilon d}$

(b) $KE_i = 2\left(\frac{1}{2}M\upsilon_i^2\right) = \boxed{M\upsilon^2}$

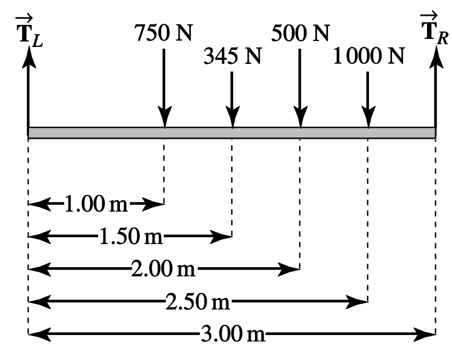
(c) $L_f = L_i = \boxed{M\upsilon d}$

(d) $$\upsilon_f = \frac{L_f}{2\left(M r_f\right)} = \frac{M\upsilon d}{2\,M\left(d/4\right)} = \boxed{2\,\upsilon}$$

(e) $$KE_f = 2\left(\frac{1}{2}M\upsilon_f^2\right) = M\left(2\,\upsilon\right)^2 = \boxed{4M\upsilon^2}$$

(f) $$W_{net} = KE_f - KE_i = \boxed{3M\upsilon^2}$$

8.74 Choose an axis that is perpendicular to the page and passing through the left end of the scaffold. Then $\Sigma\tau = 0$gives



$$\begin{aligned}&-\left(750\text{ N}\right)\left(1.00\text{ m}\right) - \left(345\text{ N}\right)\left(1.50\text{ m}\right)\\ &\quad - \left(500\text{ N}\right)\left(2.00\text{ m}\right) - \left(1000\text{ N}\right)\left(2.50\text{ m}\right)\\ &\qquad + T_R\left(3.00\text{ m}\right) = 0\end{aligned}$$

or

$$T_R = 1.59\times10^3\text{ N} = \boxed{1.59\text{ kN}}$$

Then,

$$\Sigma F_y = 0 \Rightarrow T_L = \left(750 + 345 + 500 + 1000\right)\text{ N} - 1.59\times10^3\text{ N} = \boxed{1.01\text{ kN}}$$

8.75 (a) From conservation of angular momentum, $L_f = I_f\omega_f = I_i\omega_i = L_i$, or

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{\frac{2}{5}MR_i^2}{\frac{2}{5}MR_f^2}\right)\omega_i = \left(\frac{R_i}{R_f}\right)^2\omega_i = \left(\frac{1.50\times10^9\text{ m}}{15.0\times10^3\text{ m}}\right)^2\left(0.010\,0\text{ rev/d}\right)$$

giving

$$\omega_f = 1.00\times10^8\ \frac{\cancel{\text{rev}}}{\cancel{\text{d}}}\left(\frac{2\pi\text{ rad}}{1\ \cancel{\text{rev}}}\right)\left(\frac{1\ \cancel{\text{d}}}{8.64\times10^4\text{ s}}\right) = \boxed{7.27\times10^3\text{ rad/s}}$$

(b) $\left(\upsilon_t\right)_f = R_f\omega_f = \left(15.0\times10^3\text{ m}\right)\left(7.27\times10^3\text{ rad/s}\right) = \boxed{1.09\times10^8\text{ m/s}}$ (which is about one-third the speed of light).

8.76 (a) Taking $PE_g = 0$ at the level of the horizontal axis passing through the center of the rod, the total energy of the rod in the vertical position is

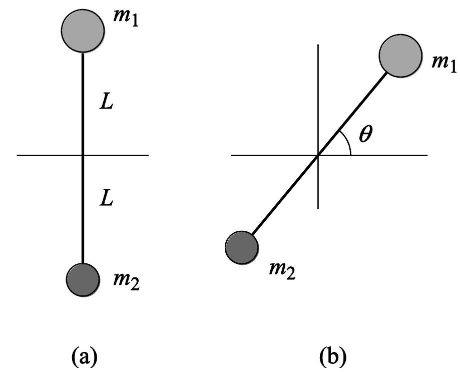


Figure P8.76

$$E = KE + PE_g$$
$$= 0 + m_1 g\left(+L\right) + m_2 g\left(-L\right) = \boxed{\left(m_1 - m_2\right) gL}$$

(b) In the rotated position of Figure P8.76b, the rod is in motion and the total energy is

$$E = KE_r + PE_g = \frac{1}{2} I_{total}\omega^2 + m_1 g y_1 + m_2 g y_2$$

$$= \frac{1}{2}\left(m_1 L^2 + m_2 L^2\right)\omega^2 + m_1 g\left(+L \sin\theta\right) + m_2 g\left(-L \sin\theta\right)$$

or

$$E = \boxed{\frac{\left(m_1 + m_2\right) L^2 \omega^2}{2} + \left(m_1 - m_2\right) gL \sin\theta}$$

(c) In the absence of any nonconservative forces that do work on the rotating system, the total mechanical energy of the system is constant. Thus, the results of parts (a) and (b) may be equated to yield an equation that can be solved for the angular speed, ω, of the system as a function of angle θ.

(d) In the vertical position, the net torque acting on the system is zero, $\boxed{\tau_{\text{net}} = 0}$. This is because the lines of action of both external gravitational forces ($m_1 g$ and $m_2 g$)pass through the pivot and hence have zero lever arms about the rotation axis. In the rotated position, the net torque (taking clockwise as positive) is

$$\tau_{net} = \Sigma\tau = m_1 g\left(L \cos\theta\right) - m_2 g\left(L \cos\right) = \boxed{\left(m_1 - m_2\right) gL \cos\theta}$$

Note that the net torque is not constant as the system rotates. Thus, the angular acceleration of the rotating system, given by $\alpha = \tau_{\text{net}}/I$, will vary as a function of θ. Since a net torque of varying magnitude acts on the system, $\boxed{\text{the angular momentum of the system will change}}$ at a nonuniform rate.

(e) In the rotated position, the angular acceleration is

$$\alpha = \frac{\tau_{net}}{I} = \frac{(m_1 - m_2)gL\cos\theta}{m_1L^2 + m_2L^2} = \boxed{\frac{(m_1 - m_2)g\cos\theta}{(m_1 + m_2)L}}$$

8.77 Let m_p be the mass of the pulley, m_1 be the mass of the sliding block, and m_2 be the mass of the counterweight.

(a) The moment of inertia of the pulley is $I = \frac{1}{2}m_pR_p^2$ and its angular velocity at any time is, $\omega = \upsilon/R_p$ where υ is the linear speed of the other objects. The friction force retarding the sliding block is $f_k = \mu_k n = \mu_k(m_1 g)$

Choose $PE_g = 0$ at the level of the counterweight when the sliding object reaches the second photogate. Then, from the work–energy theorem,

$$W_{nc} = \left(KE_{trans} + KE_{rot} + PE_g\right)_f - \left(KE_{trans} + KE_{rot} + PE_g\right)_i$$

$$-f_k \cdot s = \frac{1}{2}(m_1 + m_2)\upsilon_f^2 + \frac{1}{2}\left(\frac{1}{2}m_pR_p^2\right)\left(\frac{\upsilon_f^2}{R_p^2}\right) + 0$$
$$- \frac{1}{2}(m_1 + m_2)\upsilon_i^2 - \frac{1}{2}\left(\frac{1}{2}m_pR_p^2\right)\left(\frac{\upsilon_i^2}{R_p^2}\right) - m_2gs$$

or

$$\frac{1}{2}\left(m_1 + m_2 + \frac{1}{2}m_p\right)\upsilon_f^2 = \frac{1}{2}\left(m_1 + m_2 + \frac{1}{2}m_p\right)\upsilon_i^2 + m_2gs - \mu_k(m_1g)\cdot s$$

This reduces to

$$\upsilon_f = \sqrt{\upsilon_i^2 + \frac{2(m_2 - \mu_k m_1)gs}{m_1 + m_2 + \frac{1}{2}m_p}}$$

and yields

$$\upsilon_f = \sqrt{\left(0.820\ \frac{\text{m}}{\text{s}}\right)^2 + \frac{2(0.208\text{ kg})(9.80\text{ m/s}^2)(0.700\text{ m})}{1.45\text{ kg}}} = \boxed{1.63\text{ m/s}}$$

(b) $\omega_f = \dfrac{v_f}{R_p} = \dfrac{1.63 \text{ m/s}}{0.030\ 0 \text{ m}} = \boxed{54.2 \text{ rad/s}}$

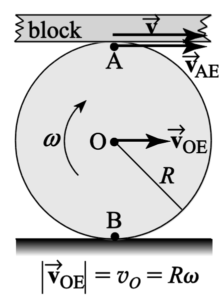
8.78 (a) The frame and the center of each wheel moves forward at $\upsilon = 3.35$ m/s and each wheel also turns at angular speed $\omega = \upsilon/R$. The total kinetic energy of the bicycle is $KE = KE_t + KE_r$, or

$$\begin{aligned} KE &= \frac{1}{2}\left(m_{frame} + 2\, m_{wheel}\right)\upsilon^2 + 2\left(\frac{1}{2} I_{wheel}\, \omega^2\right) \\ &= \frac{1}{2}\left(m_{frame} + 2\, m_{wheel}\right)\upsilon^2 + \frac{1}{2}\left(m_{wheel} R^2\right)\left(\frac{\upsilon^2}{R^2}\right) \end{aligned}$$

This yields

$$\begin{aligned} KE &= \frac{1}{2}\left(m_{frame} + 3\, m_{wheel}\right)v^2 \\ &= \frac{1}{2}\left[8.44 \text{ kg} + 3\left(0.820 \text{ kg}\right)\right]\left(3.35 \text{ m/s}\right)^2 = \boxed{61.2 \text{ J}} \end{aligned}$$

(b) Since the block does not slip on the roller, its forward speed must equal that of point A, the uppermost point on the rim of the roller. That is, $\upsilon = \left|\vec{\mathbf{v}}_{AE}\right|$ where $\vec{\mathbf{v}}_{AE}$ is the velocity of A relative to Earth.



Since the roller does not slip on the ground, the velocity of point O (the roller center) must have the same magnitude as the tangential speed of point B (the point on the roller rim in contact with the ground). That is, $\left|\vec{\mathbf{v}}_{OE}\right| = R\omega = \upsilon_O$. Also, note that the velocity of point A relative to the roller center has a magnitude equal to the tangential speed $R\omega$, or $\left|\vec{\mathbf{v}}_{AO}\right| = R\omega = \upsilon_O$.

From the discussion of relative velocities in Chapter 3, we know that $\vec{\mathbf{v}}_{AE} = \vec{\mathbf{v}}_{AO} + \vec{\mathbf{v}}_{OE}$. Since all of these velocities are in the same direction, we may add their magnitudes getting $\left|\vec{\mathbf{v}}_{AE}\right| = \left|\vec{\mathbf{v}}_{AO}\right| + \left|\vec{\mathbf{v}}_{OE}\right|$, or $\upsilon = \upsilon_O + \upsilon_O = 2\upsilon_O = 2R\omega$.

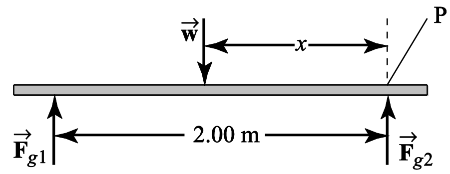
The total kinetic energy is $KE = KE_t + KE_r$, or

$$KE = \frac{1}{2}m_{stone}\upsilon^2 + 2\left[\frac{1}{2}m_{tree}\left(\frac{\upsilon}{2}\right)^2\right] + 2\left(\frac{1}{2}I_{tree}\,\omega^2\right)$$
$$= \left(\frac{1}{2}m_{stone} + \frac{1}{4}m_{tree}\right)\upsilon^2 + \frac{1}{2}m_{tree}R^2\left(\frac{\upsilon^2}{4R^2}\right)$$

This gives $KE = \frac{1}{2}\left(m_{stone} + \frac{3}{4}m_{tree}\right)\upsilon^2$, or

$$KE = \frac{1}{2}\left[844\text{ kg} + \frac{3}{4}(82.0\text{ kg})\right](0.335\text{ m/s})^2 = \boxed{50.8\text{ J}}$$

8.79 We neglect the weight of the board and assume that the woman's feet are directly above the point of support by the rightmost scale. Then, the free-body diagram for the situation is as shown at the right.



From $\Sigma F_y = 0$, we have $F_{g1} + F_{g2} - w = 0$, or $w = 380\text{ N} + 320\text{ N} = 700\text{ N}$.

Choose an axis perpendicular to the page and passing through point P.

Then $\Sigma\tau = 0$ gives $w \cdot x - F_{g1}(2.00\text{ m}) = 0$ or

$$x = \frac{F_{g1}(2.00\text{ m})}{w} = \frac{(380\text{ N})(2.00\text{ m})}{700\text{ N}} = \boxed{1.09\text{ m}}$$

8.80 Choose $PE_g = 0$ at the level of the base of the ramp. Then, conservation of mechanical energy gives

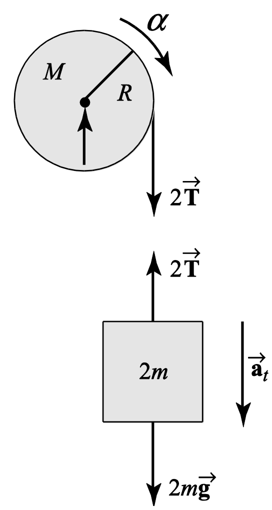
$$\left(KE_{trans} + KE_{rot} + PE_g\right)_f = \left(KE_{trans} + KE_{rot} + PE_g\right)_i$$

$$0 + 0 + (mg)(s \cdot \sin\theta) = \frac{1}{2}m\upsilon_i^2 + \frac{1}{2}\left(mR^2\right)\left(\frac{\upsilon_i}{R}\right)^2 + 0$$

or

$$s = \frac{\upsilon_i^2}{g\sin\theta} = \frac{R^2\omega_i^2}{g\sin\theta} = \frac{(3.0\text{ m})^2(3.0\text{ rad/s})^2}{(9.80\text{ m/s}^2)\sin 20^\circ} = \boxed{24\text{ m}}$$

8.81 Choose an axis perpendicular to the page and passing through the center of the cylinder. Then, applying $\Sigma\tau = I\alpha$ to the cylinder gives



$$(2T)\cdot R = \left(\frac{1}{2}MR^2\right)\alpha = \left(\frac{1}{2}MR^2\right)\left(\frac{a_t}{R}\right) \text{ or } T = \frac{1}{4}Ma_t \qquad [1]$$

Now $\Sigma F_y = ma_y$ apply to the falling objects to obtain

$$(2m)g - 2T = (2m)a_t \text{ or } a_t = g - \frac{T}{m} \qquad [2]$$

(a) Substituting Equation [2] into [1] yields

$$T = \frac{Mg}{4} - \left(\frac{M}{4m}\right)T$$

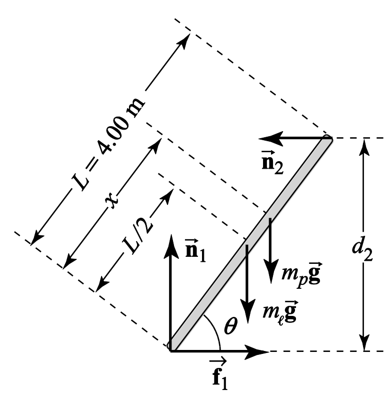
which reduces to $T = \boxed{\dfrac{Mmg}{M + 4m}}$

(b) From Equation [2] above,

$$a_t = g - \frac{1}{m}\left(\frac{Mmg}{M + 4m}\right) = g - \frac{Mg}{M + 4m} = \boxed{\frac{4mg}{M + 4m}}$$

8.82 (a) A smooth (that is, frictionless) wall cannot exert a force parallel to its surface. Thus, the only force the vertical wall can exert on the upper end of the ladder is a horizontal normal force.

(b) Consider the free-body diagram of the ladder given at the right. If the rotation axis is perpendicular to the page and passing through the lower end of the ladder, the lever arm of the normal force $\vec{\mathbf{n}}_2$ that the wall exerts on the upper end of the ladder is



$d2 = \boxed{L \sin\theta}$

(c) The lever arm of the force of gravity, $m_\ell \vec{\mathbf{g}}$, acting on the ladder is

$$d_\ell = (L/2)\cos\theta = \boxed{(L\cos\theta)/2}$$

(d) Refer to the free-body diagram given in part (b) of this solution and make use of the fact that the ladder is in both translational and rotational equilibrium.

$$\Sigma F_y = 0 \;\Rightarrow\; n_1 - m_\ell g - m_p g = 0 \text{ or } n_1 = (m_\ell + m_p)g$$

When the ladder is on the verge of slipping, $f_1 = (f_1)_{\max} = \mu_s n_1 = \mu_s (m_\ell + m_p) g$.

Then $\Sigma F_x = 0 \;\Rightarrow\; n_2 = f_1$, or $n_2 = \mu_s (m_\ell + m_p) g$.

Finally, $\Sigma\tau = 0 \;\Rightarrow\; n_2 (L\sin\theta) - m_\ell g((L/2)\cos\theta) - m_p g x\cos\theta = 0$ where x is the maximum distance the painter can go up the ladder before it will start to slip. Solving for x gives

$$x = \frac{n_2 (L\sin\theta) - m_\ell g\left(\dfrac{L}{2}\cos\theta\right)}{m_p g\cos\theta} = \mu_s\left(\frac{m_\ell}{m_p} + 1\right)L\tan\theta - \left(\frac{m_\ell}{2m_p}\right)L$$

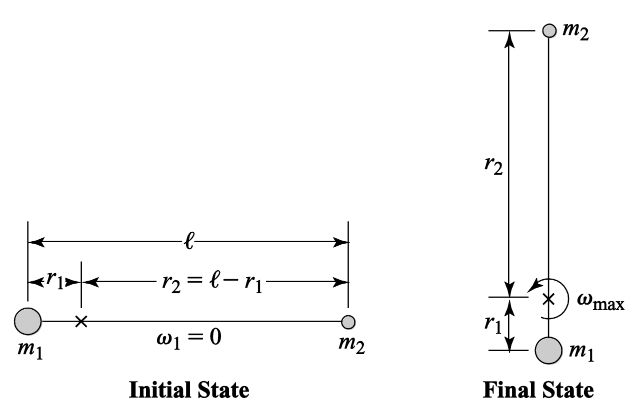
and using the given numerical data, we find

$$x = (0.45)\left(\frac{30\text{ kg}}{80\text{ kg}} + 1\right)(4.0\text{ m})\tan 53^\circ - \left[\frac{30\text{ kg}}{2(80\text{ kg})}\right](4.0\text{ m}) = \boxed{2.5\text{ m}}$$

8.83 The large mass ($m_1 = 60.0$ kg) moves in a circular path of radius $r_1 = 0.140$ m, while the radius of the path for the small mass ($m_2 = 0.120$ kg) is

$$r_2 = \ell - r_1$$
$$= 3.00\text{ m} - 0.140\text{ m} = 2.86\text{ m}$$

The system has maximum angular speed when the rod is in the vertical position as shown at the right.



We take $PE_g = 0$ at the level of the horizontal rotation axis and use conservation of energy to find:

$$KE_f + \left(PE_g\right)_f = KE_i + \left(PE_g\right)_i \quad \Rightarrow \quad \left(\frac{1}{2}I_1\omega_{\max}^2 + \frac{1}{2}I_2\omega_{\max}^2\right) + \left(m_2 g r_2 - m_1 g r_1\right) = 0 + 0$$

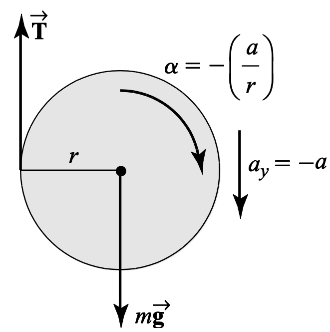
Approximating the two objects as point masses, we have $I_1 = m_1 r_1^2$ and $I_2 = m_2 r_2^2$. The energy conservation equation then becomes $\frac{1}{2}\left(m_1 r_1^2 + m_2 r_2^2\right)\omega_{\max}^2 = \left(m_1 r_1 - m_2 r_2\right) g$ and yields

$$\omega_{\max} = \sqrt{\frac{2\left(m_1 r_1 - m_2 r_2\right) g}{m_1 r_1^2 + m_2 r_2^2}} = \sqrt{\frac{2\left[(60.0\ \text{kg})(0.140\ \text{m}) - (0.120\ \text{kg})(2.86\ \text{m})\right]\left(9.80\ \text{m/s}^2\right)}{(60.0\ \text{kg})(0.140\ \text{m})^2 + (0.120\ \text{kg})(2.86\ \text{m})^2}}$$

or $\omega_{\max} = 8.56\ \text{rad/s}$. The maximum linear speed of the small mass object is then

$$\left(\upsilon_2\right)_{\max} = r_2\omega_{\max} = (2.86\ \text{m})(8.56\ \text{rad/s}) = \boxed{24.5\ \text{m/s}}$$

8.84 (a) Note that the cylinder has both translational and rotational motion. The center of gravity accelerates downward while the cylinder rotates around the center of gravity. Thus, we apply both the translational and the rotational forms of Newton's second law to the cylinder:



$$\Sigma F_y = ma_y \quad \Rightarrow \quad T - mg = m(-a)$$

or

$$T = m(g - a) \qquad [1]$$

$$\Sigma\tau = I\alpha \quad \Rightarrow \quad -Tr = I(-a/r)$$

For a uniform, solid cylinder, $I = \frac{1}{2}mr^2$ so our last result becomes

$$Tr = \left(\frac{mr^2}{2}\right)\left(\frac{a}{r}\right) \text{ or } a = \frac{2T}{m} \qquad [2]$$

Substituting Equation [2] into Equation [1] gives $T = mg - 2T$, and solving for T yields $T = \boxed{mg/3}$.

(b) From Equation [2] above,

$$a = \frac{2T}{m} = \frac{2}{m}\left(\frac{mg}{3}\right) = \boxed{2g/3}$$

(c) Considering the translational motion of the center of gravity, $\upsilon_y^2 = \upsilon_{0y}^2 + 2a_y\Delta y$ gives

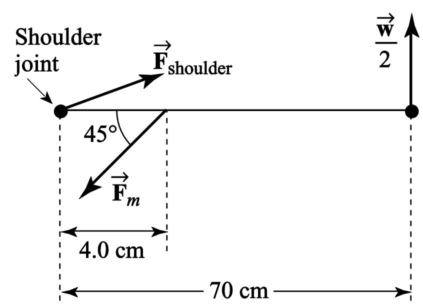
$$\upsilon_y = \sqrt{0 + 2\left(-\frac{2g}{3}\right)(-h)} = \boxed{\sqrt{4gh/3}}$$

Using conservation of energy with PE_g at the final level of the cylinder gives

$$\left(KE_t + KE_r + PE_g\right)_f = \left(KE_t + KE_r + PE_g\right)_i \text{ or } \tfrac{1}{2}m\upsilon_y^2 + \tfrac{1}{2}I\omega^2 + 0 = 0 + 0 + mgh$$

Since $\omega = \upsilon_y/r$ and $I = \frac{1}{2}mr^2$, this becomes $\frac{1}{2}m\upsilon_y^2 + \frac{1}{2}\left(\frac{1}{2}m\cancel{r^2}\right)\left(\frac{\upsilon_y^2}{\cancel{r^2}}\right) = mgh$, or $\frac{3}{4}m\upsilon_y^2 = mgh$

yielding $\upsilon_y = \boxed{\sqrt{4gh/3}}$.



8.85 Considering the shoulder joint as the pivot, the second condition of equilibrium gives

$$\Sigma\tau = 0 \Rightarrow \frac{w}{2}(70\text{ cm}) - (F_m \sin 45°)(4.0\text{ cm}) = 0$$

or

$$F_m = \frac{w(70\text{ cm})}{2(4.0\text{ cm})\sin 45°} = 12.4w$$

Recall that this is the total force exerted on the arm by a set of two muscles. If we approximate that the two muscles of this pair exert equal magnitude forces, the force exerted by each muscle is

$$F_{\substack{each\\muscle}} = \frac{F_m}{2} = \frac{12.4w}{2} = \boxed{6.2w} = 6.2(750\text{ N}) = 4.6 \times 10^3\text{ N} = \boxed{4.6\text{ kN}}$$

8.86 Observe that since the torque opposing the rotational motion of the gymnast is constant, the work done by nonconservative forces as the gymnast goes from position 1 to position 2 (an angular displacement of $\pi/2$ rad) will be the same as that done while the gymnast goes from position 2 to position 3 (another angular displacement of $\pi/2$ rad).

Choose $PE_g = 0$ at the level of the bar, and let the distance from the bar to the center of gravity of the outstretched body be r_{cg}. Applying the work–energy theorem, $W_{nc} = (KE + PE_g)_f - (KE + PE_g)_i$, to the rotation from position 1 to position 2 gives

$$(W_{nc})_{12} = \left(\tfrac{1}{2}I\omega_2^2 + 0\right) - \left(0 + mgr_{\text{cg}}\right) \text{ or } (W_{nc})_{12} = \tfrac{1}{2}I\omega_2^2 - mgr_{\text{cg}} \qquad [1]$$

Now, apply the work–energy theorem to the rotation from position 2 to position 3 to obtain

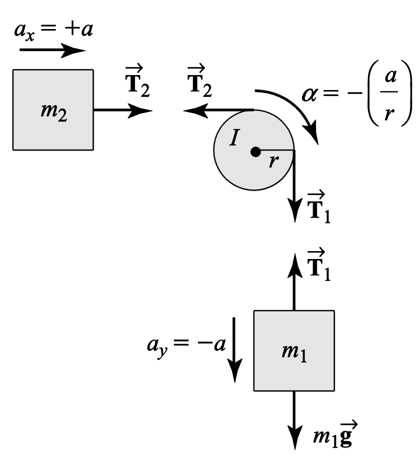
$$(W_{nc})_{23} = \left[\tfrac{1}{2}I\omega_3^2 + mg(-r_{\text{cg}})\right] - \left(\tfrac{1}{2}I\omega_2^2 + 0\right) \text{ or } (W_{nc})_{23} = \tfrac{1}{2}I\omega_3^2 - \tfrac{1}{2}I\omega_2^2 - mgr_{\text{cg}} \qquad [2]$$

Since the frictional torque is constant and these two segments of the motion involve equal angular displacements, $(W_{nc})_{23} = (W_{nc})_{12}$. Thus, equating Equation [2] to Equation [1] gives

$$\tfrac{1}{2}I\omega_3^2 - \tfrac{1}{2}I\omega_2^2 - \cancel{mgr_{\text{cg}}} = \tfrac{1}{2}I\omega_2^2 - \cancel{mgr_{\text{cg}}}$$

which yields $\omega_3^2 = 2\omega_2^2$, or $\omega_3 = \sqrt{2}\omega_2 = \sqrt{2}(4.0\text{ rad/s}) = \boxed{5.7\text{ rad/s}}$.

8.87 (a) Free-body diagrams for each block and the pulley are given at the right. Observe that the angular acceleration of the pulley will be clockwise in direction and has been given a negative sign. Since $\Sigma\tau = I\alpha$, the positive sense for torques and angular acceleration must be the same (counterclockwise).



For m_1: $\Sigma F_y = ma_y \Rightarrow T_1 - m_1 g = m_1(-a)$

$$T_1 = m_1(g - a) \quad [1]$$

For m_2:

$$\Sigma F_x = ma_x \quad \Rightarrow \quad T_2 = m_2 a \qquad [2]$$

For the pulley: $\Sigma\tau = I\alpha \;\Rightarrow\; T_2 r - T_1 r = I(-a/r)$ or

$$T_1 - T_2 = \left(\frac{I}{r^2}\right)a \qquad [3]$$

Substitute Equations [1] and [2] into Equation [3] and solve for a to obtain

$$a = \frac{m_1 g}{\left(I/r^2\right) + m_1 + m_2}$$

or

$$a = \frac{(4.00\ \text{kg})(9.80\ \text{m/s}^2)}{\left(0.500\ \text{kg}\cdot\text{m}^2\right)/(0.300\ \text{m})^2 + 4.00\ \text{kg} + 3.00\ \text{kg}} = \boxed{3.12\ \text{m/s}^2}$$

(b) Equation [1] above gives: $T_1 = (4.00\ \text{kg})\left(9.80\ \text{m/s}^2 - 3.12\ \text{m/s}^2\right) = \boxed{26.7\ \text{N}}$,

and Equation [2] yields: $T_2 = (3.00\ \text{kg})\left(3.12\ \text{m/s}^2\right) = \boxed{9.37\ \text{N}}$.

8.88 (a)

(b) $\Sigma F_y = 0 \quad \Rightarrow \quad n_F - 120\ \text{N} - m_{monkey}g = 0$

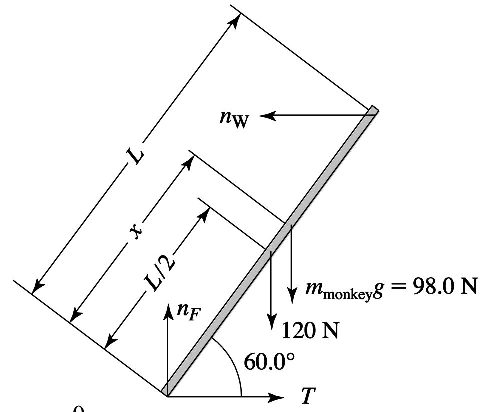
$$n_F = 120\ \text{N} + (10.0\ \text{kg})\left(9.80\ \text{m/s}^2\right) = \boxed{218\ \text{N}}$$

(c) When $x = 2L/3$, we consider the bottom end of the ladder as our pivot and obtain

$$\Sigma\tau)_{\substack{bottom\\ end}} = 0 \quad \Rightarrow \quad -(120\ \text{N})\left(\frac{\cancel{L}}{2}\cos 60.0^\circ\right) - (98.0\ \text{N})\left(\frac{2\cancel{L}}{3}\cos 60.0^\circ\right) + n_W\left(\cancel{L}\sin 60.0^\circ\right) = 0$$

or

$$n_W = \frac{\left[60.0\text{ N} + (196/3)\text{ N}\right]\cos 60.0^\circ}{\sin 60.0^\circ} = 72.4\text{ N}$$



Then,

$$\Sigma F_x = 0 \quad \Rightarrow \quad T - n_W = 0 \text{ or } T = n_W = \boxed{72.4\text{ N}}$$

(d) When the rope is ready to break, $T = n_W = 80.0\text{ N}$. Then $\Sigma\tau)_{\substack{bottom\\end}} = 0$ yields

$$-(120\text{ N})\left(\frac{L}{2}\cos 60.0^\circ\right) - (98.0\text{ N})\,x\cos 60.0^\circ + (80.0\text{ N})(L\sin 60.0^\circ) = 0$$

or

$$x = \frac{\left[(80.0\text{ N})\sin 60.0^\circ - (60.0\text{ N})\cos 60.0^\circ\right]L}{(98.0\text{ N})\cos 60.0^\circ} = 0.802\,L = 0.802(3.00\text{ m}) = \boxed{2.41\text{ m}}$$

(e) If the horizontal surface were rough and the rope removed, a horizontal static friction force directed toward the wall would act on the bottom end of the ladder. Otherwise, the analysis would be much as what is done above. The maximum distance the monkey could climb would correspond to the condition that the friction force have its maximum value, $\mu_s n_F$, so you would need to know the coefficient of static friction to solve part (d).