PROBLEM SOLUTIONS

7.1 (a) Earth rotates 2π radians (360°) on its axis in 1 day. Thus,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{1 \cancel{\text{ day}}}\left(\frac{1 \cancel{\text{ day}}}{8.64\times10^4 \text{ s}}\right) = \boxed{7.27\times10^{-5} \text{ rad/s}}$$

(b) Because of its rotation about its axis, $\boxed{\text{Earth bulges at the equator.}}$

7.2 The distance traveled is $s = r\,\theta$, where θ is in radians.

For 30°,

$$s = r\theta = (4.1 \text{ m})\left[30°\left(\frac{\pi \text{ rad}}{180°}\right)\right] = \boxed{2.1 \text{ m}}$$

For 30 radians,

$$s = r\theta = (4.1 \text{ m})(30 \text{ rad}) = \boxed{1.2\times10^2 \text{ m}}$$

For 30 revolutions,

$$s = r\theta = (4.1 \text{ m})\left[30 \text{ rev}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\right] = \boxed{7.7\times10^2 \text{ m}}$$

7.3 (a) $\theta = \dfrac{s}{r} = \dfrac{60\,000 \text{ mi}}{1.0 \text{ ft}}\left(\dfrac{5280 \text{ ft}}{1 \text{ mi}}\right) = \boxed{3.2\times10^8 \text{ rad}}$

(b) $\theta = 3.2\times10^8 \text{ rad}\left(\dfrac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{5.0\times10^7 \text{ rev}}$

7.4 (a) $\alpha = \dfrac{\Delta\omega}{\Delta t} = \dfrac{1.00 \text{ rev/s} - 0}{30.0 \text{ s}} = \left(3.33\times10^{-2}\ \dfrac{\cancel{\text{rev}}}{\text{s}^2}\right)\left(\dfrac{2\pi \text{ rad}}{1 \cancel{\text{ rev}}}\right) = \boxed{0.209 \text{ rad/s}^2}$

(b) $\boxed{\text{Yes.}}$ When an object starts from rest, its angular speed is related to the angular acceleration and time by the equation $\omega = \alpha(\Delta t)$. Thus, the angular speed is directly proportional to both the angular acceleration and the time interval. It the time interval is held constant, doubling the angular acceleration will double the angular speed attained during the interval.

7.5 (a) $$\alpha = \frac{\left(2.51\times10^{4}\ \text{rev/min} - 0\right)}{3.20\ \text{s}}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60.0\ \text{s}}\right) = \boxed{821\ \text{rad/s}^2}$$

(b) $$\theta = \omega_i t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}\left(821\ \frac{\text{rad}}{\text{s}^2}\right)(3.20\ \text{s})^2 = \boxed{4.21\times10^3\ \text{rad}}$$

7.6 $$\omega_i = 3\,600\ \frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60.0\ \text{s}}\right) = 377\ \text{rad/s}$$

$$\Delta\theta = 50.0\ \text{rev}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 314\ \text{rad}$$

Thus,

$$\alpha = \frac{w^2 - \omega_i^2}{2\Delta\theta} = \frac{0-(377\ \text{rad/s})^2}{2(314\ \text{rad})} = \boxed{-226\ \text{rad/s}^2}$$

7.7 (a) From $\omega^2 = \omega_0^2 + 2\alpha(\Delta\theta)$, the angular displacement is

$$\Delta\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(2.2\ \text{rad/s})^2 - (0.06\ \text{rad/s})^2}{2(0.70\ \text{rad/s}^2)} = \boxed{3.5\ \text{rad}}$$

(b) From the equation given above for $\Delta\theta$, observe that when the angular acceleration is constant, the displacement is proportional to the difference in the *squares* of the final and initial angular speeds. Thus, the angular displacement would $\boxed{\text{increase by a factor of 4}}$ if both of these speeds were doubled.

7.8 (a) The maximum height h depends on the drop's vertical speed at the instant it leaves the tire and becomes a projectile. The vertical speed at this instant is the same as the tangential speed, $v_t = r\omega$, of points on the tire. Since the second drop rose to a lesser height, $\boxed{\text{the tangential speed decreased}}$ during the intervening rotation of the tire.

(b) From $v^2 = v_0^2 + 2a_y(\Delta y)$, with $v_0 = v_t$, $a_y = -g$, and $v = 0$ when $\Delta y = h$, the relation between the tangential speed of the tire and the maximum height h is found to be

$$0 = v_t^2 + 2(-g)h \qquad \text{or} \qquad v_t = \sqrt{2gh}$$

Thus, the angular speed of the tire when the first drop left was

$$\omega_1 = \frac{(v_t)_1}{r} = \frac{\sqrt{2gh_1}}{r}$$

and when the second drop left, the angular speed was

$$\omega_2 = \frac{(v_t)_2}{r} = \frac{\sqrt{2gh_2}}{r}$$

From $\omega^2 = \omega_0^2 + 2\alpha(\Delta\theta)$, with $\Delta\theta = 2\pi$, the angular acceleration is found to be

$$\alpha = \frac{\omega_2^2 - \omega_1^2}{2(\Delta\theta)} = \frac{2gh_2/r^2 - 2gh_1/r^2}{2(\Delta\theta)} = \frac{g}{r^2(\Delta\theta)}(h_2 - h_1)$$

or

$$\alpha = \frac{(9.80\ \text{m/s}^2)}{(0.381\ \text{m})^2(2\pi\ \text{rad})}(0.510\ \text{m} - 0.540\ \text{m}) = \boxed{-0.322\ \text{rad/s}^2}$$

7.9 Main Rotor:

$$v = r\omega = (3.80\ \text{m})\left(450\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{179\ \text{m/s}}$$

$$v = \left(179\ \frac{\text{m}}{\text{s}}\right)\left(\frac{v_{\text{sound}}}{343\ \text{m/s}}\right) = \boxed{0.522\ v_{\text{sound}}}$$

Tail Rotor:

$$v = r\omega = (0.510\ \text{m})\left(4\,138\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{221\ \text{m/s}}$$

$$v = \left(221\ \frac{\text{m}}{\text{s}}\right)\left(\frac{v_{\text{sound}}}{343\ \text{m/s}}\right) = \boxed{0.644\ v_{\text{sound}}}$$

7.10 We will break the motion into two stages: (1) an acceleration period and (2) a deceleration period.

The angular displacement during the acceleration period is

$$\theta_1 = \omega_{\text{av}}t = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left[\frac{(5.0\ \text{rev/s})(2\pi\ \text{rad/1 rev}) + 0}{2}\right](8.0\ \text{s}) = 126\ \text{rad}$$

and while decelerating,

$$\theta_2 = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left[\frac{0 + (5.0 \text{ rev/s})(2\pi \text{ rad/1 rev})}{2}\right](12 \text{ s}) = 188 \text{ rad}$$

The total displacement is $\theta = \theta_1 + \theta_2 = \left[(126 + 188) \text{ rad}\right]\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{50 \text{ rev}}$.

7.11 (a) The linear distance the car travels in coming to rest is given by $v_f^2 = v_0^2 + 2a(\Delta x)$ as

$$\Delta x = \frac{v_f^2 - v_0^2}{2a} = \frac{0 - (29.0 \text{ m/s})^2}{2(-1.75 \text{ m/s}^2)} = 240 \text{ m}$$

Since the car does not skid, the linear displacement of the car and the angular displacement of the tires are related by $\Delta x = r(\Delta\theta)$. Thus, the angular displacement of the tires is

$$\Delta\theta = \frac{\Delta x}{r} = \frac{240 \text{ m}}{0.330 \text{ m}} = (728 \text{ rad})\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{116 \text{ rev}}$$

(b) When the car has traveled 120 m (one half of the total distance), the linear speed of the car is

$$v = \sqrt{v_0^2 + 2a(\Delta x)} = \sqrt{(29.0 \text{ m/s})^2 + 2(-1.75 \text{ m/s}^2)(120 \text{ m})} = 20.5 \text{ m/s}$$

and the angular speed of the tires is

$$\omega = \frac{v}{r} = \frac{20.5 \text{ m/s}}{0.330 \text{ m}} = \boxed{62.1 \text{ rad/s}}$$

7.12 (a) The angular speed is $\omega = \omega_0 + \alpha t = 0 + (2.50 \text{ rad/s}^2)(2.30 \text{ s}) = \boxed{5.75 \text{ rad/s}}$.

(b) Since the disk has a diameter of 45.0 cm, its radius is $r = (00.450 \text{ m})/2 = 0.225$ m. Thus,

$$v_t = r\omega = (0.225 \text{ m})(5.75 \text{ rad/s}) = \boxed{1.29 \text{ m/s}}$$

and

$$a_t = r\alpha = (0.225 \text{ m})(2.50 \text{ rad/s}^2) = \boxed{0.563 \text{ m/s}^2}$$

(c) The angular displacement of the disk is

$$\Delta\theta = \theta_f - \theta_0 = \frac{\omega_f^2 - \omega_0^2}{2\alpha} = \frac{(5.75 \text{ rad/s})^2 - 0}{2(2.50 \text{ rad/s}^2)} = (6.61 \text{ rad})\left(\frac{360°}{2\pi \text{ rad}}\right) = 379°$$

and the final angular position of the radius line through point P is

$$\theta_f = \theta_0 + \Delta\theta = 57.3° + 379° = 436°$$

or it is at $\boxed{76° \text{ counterclockwise from the } +x\text{-axis}}$ after turning 19° beyond one full revolution.

7.13 From $\Delta\theta = \omega_{av}t = \left(\frac{\omega + \omega_i}{2}\right)t$, we find the initial angular speed to be

$$\omega_i = \frac{2\,\Delta\theta}{t} - \omega = \frac{2(37.0 \text{ rev})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)}{3.00 \text{ s}} - 98.0 \text{ rad/s} = 57.0 \text{ rad/s}$$

The angular acceleration is then

$$\alpha = \frac{\omega - \omega_i}{t} = \frac{98.0 \text{ rad/s} - 57.0 \text{ rad/s}}{3.00 \text{ s}} = \boxed{13.7 \text{ rad/s}^2}$$

7.14 (a) The initial angular speed is

$$\omega_0 = 1.00 \times 10^2 \frac{\cancel{\text{rev}}}{\cancel{\text{min}}}\left(\frac{2\pi \text{ rad}}{1 \cancel{\text{rev}}}\right)\left(\frac{1 \cancel{\text{min}}}{60.0 \text{ s}}\right) = 10.5 \text{ rad/s}$$

The time to stop (i.e., reach a speed of $\omega = 0$) with $\alpha = -2.00 \text{ rad/s}^2$ is

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 10.5 \text{ rad/s}}{-2.00 \text{ rad/s}^2} = \boxed{5.25 \text{ s}}$$

(b) $\Delta\theta = \omega_{av}t = \left(\frac{\omega + \omega_0}{2}\right)t = \left(\frac{0 + 10.5 \text{ rad/s}}{2}\right)(5.25 \text{ s}) = \boxed{27.6 \text{ rad}}$

7.15 The centripetal acceleration is $a_c = v_t^2/r = r\omega^2$ where r radius of the circular path followed by the object in question. The angular speed of the rotating Earth is

$$\omega = 2\pi \frac{\text{rad}}{\text{day}}\left(\frac{1 \text{ day}}{8.64 \times 10^4 \text{ s}}\right) = 7.27 \times 10^{-5} \text{ rad/s}$$

(a) For a person on the equator, $r = R_E = 6.38 \times 10^6$ m, so

$$a_c = r\omega^2 = \left(6.38 \times 10^6 \text{ m}\right)\left(7.27 \times 10^{-5} \text{ rad/s}\right)^2 = \boxed{3.37 \times 10^{-2} \text{ m/s}^2}$$

(b) For a person at the North Pole, $r = 0 \quad \Rightarrow \quad \boxed{a_c = 0}$.

(c) The centripetal acceleration of an object is directed toward the center of the circular path the object is following. Thus, the forces involved in producing this acceleration are all forces acting on the object which have a component along the radius line of the circular path. These forces are $\boxed{\text{the gravitational force and the normal force.}}$

7.16 The radius of the cylinder is $r = 2.5 \text{ mi}\left(\frac{1609 \text{ m}}{1 \text{ mi}}\right) = 4.0 \times 10^3 \text{ m}$. Thus, from $a_c = r\omega^2$, the required angular velocity is

$$\omega = \sqrt{\frac{a_c}{r}} = \sqrt{\frac{9.80 \text{ m/s}^2}{4.0 \times 10^3 \text{ m}}} = \boxed{4.9 \times 10^{-2} \text{ rad/s}}$$

7.17 The final angular velocity is

$$\omega_f = 78 \frac{\text{rev}}{\text{min}}\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 8.17 \text{ rad/s}$$

and the radius of the disk is

$$r = 5.0 \text{ in}\left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right) = 12.7 \text{ cm} = 0.127 \text{ m}$$

(a) The tangential acceleration of the bug as the disk speeds up is

$$a_t = r\alpha = r\left(\frac{\Delta\omega}{\Delta t}\right) = \left(0.127 \text{ m}\right)\left(\frac{8.17 \text{ rad/s}}{3.0 \text{ s}}\right) = \boxed{0.35 \text{ m/s}^2}$$

(b) The final tangential speed of the bug is

$$v_t = r\,\omega_f = \left(0.127 \text{ m}\right)\left(8.17 \text{ rad/s}\right) = \boxed{1.0 \text{ m/s}}$$

(c) At $t = 1.0$ s $\omega = \omega_i + \alpha t = 0 + \left(\frac{8.17 \text{ rad/s}}{3.0 \text{ s}}\right)\left(1.0 \text{ s}\right) = 2.7 \text{ rad/s}$

Thus, $a_t = r\alpha = \boxed{0.35\ \text{m/s}^2}$ as above, while the radial acceleration is

$$a_c = r\omega^2 = (0.127\ \text{m})(2.7\ \text{rad/s})^2 = \boxed{0.94\ \text{m/s}^2}$$

The total acceleration is $a = \sqrt{a_c^2 + a_t^2} = \boxed{1.0\ \text{m/s}^2}$, and the angle this acceleration makes with the direction of $\vec{\mathbf{a}}_c$ is

$$\theta = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{0.35}{0.94}\right) = \boxed{20°}$$

7.18 The normal force exerted by the wall behind the person's back will supply the necessary centripetal acceleration, or

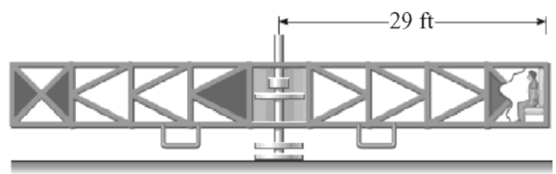


Figure P7.18

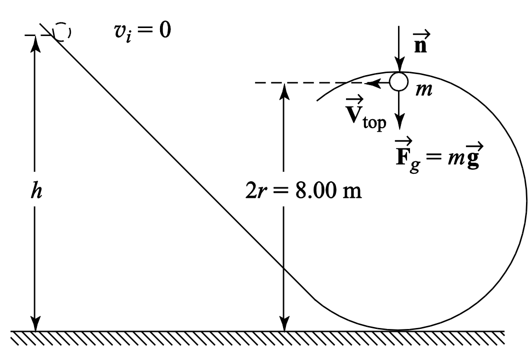
$$n = ma_c = mr\omega^2$$

where $r = 29$ ft is the radius of the circular path followed by the person.

If it is desired to have $n = 20 \times weight = 20mg$, then it is necessary that $\cancel{m}r\omega^2 = 20\cancel{m}g$, or

$$\omega = \sqrt{\frac{20g}{r}} = \sqrt{\frac{20(9.8\ \text{m/s}^2)}{(29\ \cancel{\text{ft}})(1\ \text{m}/3.281\ \cancel{\text{ft}})}} = 4.7\ \frac{\cancel{\text{rad}}}{\cancel{\text{s}}}\left(\frac{1\ \text{rev}}{2\pi\ \cancel{\text{rad}}}\right)\left(\frac{60\ \cancel{\text{s}}}{1\ \text{min}}\right) = \boxed{45\ \text{rev/min}}$$

7.19 The total force, directed toward the center of the circular path, acting on the rider at the top of the loop is the sum of the normal force and the gravitation force. If the magnitude of the normal force (exerted on the rider by the seat) is to have a magnitude equal to the rider's weight, the total centripetal force is then



$$F_c = n + F_g = mg + mg = 2mg$$

Also, $F_c = m\,v_{top}^2/r$ so we solve for the needed speed at the top of the loop as

$$\frac{\cancel{m} v_{\text{top}}^2}{r} = 2\cancel{m}g \qquad \text{or} \qquad v_{\text{top}}^2 = 2rg$$

Ignoring any friction and using conservation of energy from when the coaster starts from rest ($v_i = 0$) at height h until it reaches the top of the loop gives

$$\frac{1}{2}\cancel{m}v_i^2 + \cancel{m}gh = \frac{1}{2}\cancel{m}v_{\text{top}}^2 + \cancel{m}g(2r) \qquad \text{or} \qquad 0 + gh = \frac{1}{2}(2rg) + g(2r)$$

and reduces to $h = 3r = 3(4.00 \text{ m}) = \boxed{12.0 \text{ m}}$.

7.20 (a) The natural tendency of the coin is to move in a straight line (tangent to the circular path of radius 15.0 cm), and hence, go farther from the center of the turntable. To prevent this, $\boxed{\text{the force of static friction}}$ must act toward the center of the turntable and supply the needed centripetal force. When the necessary centripetal force exceeds the maximum value of the static friction force, $(f_s)_{\max} = \mu_s n = \mu_s mg$, the coin begins to slip.

(b) When the turntable has angular speed, ω the required centripetal force is $F_c = mr\omega^2$. Thus, if the coin is not to slip, it is necessary that $mr\omega^2 \leq \mu_s mg$, or

$$\omega \leq \sqrt{\frac{\mu_s g}{r}} = \sqrt{\frac{(0.350)(9.80 \text{ m/s}^2)}{0.150 \text{ m}}} = 4.78 \text{ rad/s}$$

With a constant angular acceleration of $\alpha = 0.730 \text{ rad/s}^2$, the time required to reach the critical angular speed is

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{4.78 \text{ rad/s} - 0}{0.730 \text{ rad/s}^2} = \boxed{6.55 \text{ s}}$$

7.21 (a) From $\Sigma F_r = ma_c$, we have

$$T = m\left(\frac{v_t^2}{r}\right) = \frac{(55.0 \text{ kg})(4.00 \text{ m/s})^2}{0.800 \text{ m}} = 1.10 \times 10^3 \text{ N} = \boxed{1.10 \text{ kN}}$$

(b) The tension is $\boxed{\text{larger than her weight}}$ by a factor of

$$\frac{T}{mg} = \frac{1.10 \times 10^3 \text{ N}}{(55.0 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{2.04 \text{ times}}$$

7.22 (a) The centripetal acceleration is $a_c = v_t^2/r$. Thus, when $a_c = a_t = 0.500$ m/s^2, we have

$$v_t = \sqrt{r\,a_c} = \sqrt{(400 \text{ m})(0.500 \text{ m/s}^2)} = \sqrt{200} \text{ m/s} = \boxed{14.1 \text{ m/s}}$$

(b) At this time,

$$t = \frac{v_t - v_i}{a_t} = \frac{\sqrt{200} \text{ m/s} - 0}{0.500 \text{ m/s}^2} = 28.3 \text{ s}$$

and the linear displacement is

$$s = (v_t)_{\text{av}}\, t = \left(\frac{v_t + v_i}{2}\right) t = \left(\frac{\sqrt{200} \text{ m/s} + 0}{2}\right)(28.3 \text{ s}) = \boxed{200 \text{ m}}$$

(c) The time is $t = \boxed{28.3 \text{ s}}$ as found in part (b) above.

7.23 Friction between the tires and the roadway is capable of giving the truck a maximum centripetal acceleration of

$$a_{c,\max} = \frac{v_{t,\max}^2}{r} = \frac{(32.0 \text{ m/s})^2}{150 \text{ m}} = 6.83 \text{ m/s}^2$$

If the radius of the curve changes to 75.0 m, the maximum safe speed will be

$$v_{t,\max} = \sqrt{r\,a_{c,\max}} = \sqrt{(75.0 \text{ m})(6.83 \text{ m/s}^2)} = \boxed{22.6 \text{ m/s}}$$

7.24 Since $F_c = m\frac{v_t^2}{r} = m\,r\,\omega^2$, the needed angular velocity is

$$\omega = \sqrt{\frac{F_c}{mr}} = \sqrt{\frac{4.0 \times 10^{-11} \text{ N}}{(3.0 \times 10^{-16} \text{ kg})(0.150 \text{ m})}}$$

$$= (9.4 \times 10^2 \text{ rad/s})\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{1.5 \times 10^2 \text{ rev/s}}$$

7.25 (a) $a_c = r\,\omega^2 = (2.00 \text{ m})(3.00 \text{ rad/s})^2 = \boxed{18.0 \text{ m/s}^2}$

(b) $F_c = m\,a_c = (50.0\text{ kg})(18.0\text{ m/s}^2) = \boxed{900\text{ N}}$

(c) We know the centripetal acceleration is produced by the force of friction. Therefore, the needed static friction force is $f_s = 900$ N. Also, the normal force is $n = mg = 490$ N. Thus, the minimum coefficient of friction required is

$$\mu_s = \frac{(f_s)_{\max}}{n} = \frac{900\text{ N}}{490\text{ N}} = \boxed{1.84}$$

So large a coefficient of friction is unreasonable, and she will not be able to stay on the merry-go-round.

7.26 (a) The only force acting on the astronaut is $\boxed{\text{the normal force}}$ exerted on him by the "floor" of the cabin.

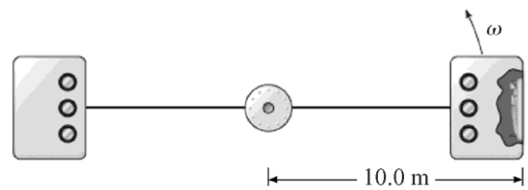


Figure P7.26

(b) $F_c = \dfrac{m v_t^2}{r} = n$

(c) If, $n = \dfrac{1}{2} m g_E$ then

$$n = \frac{1}{2}(60.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{294\text{ N}}$$

(d) From the equation in Part (b),

$$v_t = \sqrt{\frac{nr}{m}} = \sqrt{\frac{(294\text{ N})(10.0\text{ m})}{60.0\text{ kg}}} = \boxed{7.00\text{ m/s}}$$

(e) Since $v_1 = r\omega$, we have

$$\omega = \frac{v_t}{r} = \frac{7.00\text{ m/s}}{10.0\text{ m}} = \boxed{0.700\text{ rad/s}}$$

(f) The period of rotation is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.700\text{ rad/s}} = \boxed{8.98\text{ s}}$$

(g) Upon standing, the astronaut's head is moving slower than his feet because his head is closer to the axis of rotation. When standing, the radius of the circular path followed by the head is r_{head} = 10.0 m – 1.80 m = 8.20 m, and the tangential speed of the head is

$$\left(v_t\right)_{\text{head}} = r_{\text{head}}\omega = \left(8.20\ \text{m}\right)\left(0.700\ \text{rad/s}\right) = \boxed{5.74\ \text{m/s}}$$

7.27 (a) Since the 1.0-kg mass is in equilibrium, the tension in the string is

$$T = mg = \left(1.0\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right) = \boxed{9.8\ \text{N}}$$

(b) The tension in the string must produce the centripetal acceleration of the puck. Hence, $F_c = T = \boxed{9.8\ \text{N}}$.

(c) From $F_c = m_{\text{puck}}\left(\dfrac{v_t^2}{r}\right)$, we find $v_t = \sqrt{\dfrac{r\,F_c}{m_{\text{puck}}}} = \sqrt{\dfrac{\left(1.0\ \text{m}\right)\left(9.8\ \text{N}\right)}{0.25\ \text{kg}}} = \boxed{6.3\ \text{m/s}}$.

7.28 (a) Since the mass m_2 hangs in equilibrium on the end of the string,

$$\Sigma F_y = T - m_2 g = 0 \qquad \text{or} \qquad \boxed{T = m_2 g}$$

(b) The puck moves in a circular path of radius R and must have an acceleration directed toward the center equal to $a_c = v_t^2/R$. The only force acting on the puck and directed toward the center is the tension in the string. Newton's second law requires

$$\Sigma F_{\substack{\text{toward}\\\text{center}}} = m_1 a_c \qquad \text{giving} \qquad \boxed{T = m_1\frac{v_t^2}{R}}$$

(c) Combing the results from (a) and (b) gives

$$m_1\frac{v_t^2}{R} = m_2 g \qquad \text{or} \qquad \boxed{v_t = \sqrt{\frac{m_2 g R}{m_1}}}$$

(d) Substitution of the numeric data from problem 7.27 into the results for (a) and (c) shown above will yield the answers given for that problem.

7.29 (a) The $\boxed{\text{force of static friction}}$ acting toward the road's center of curvature must supply the briefcase's required centripetal acceleration. The condition that it be able to meet this need is that $F_c = m\,v_t^2/r \le \left(f_s\right)_{\text{max}} = \mu_s mg$, or $\mu_s \ge v_t^2$/rg. When the tangential

$\boxed{\text{speed becomes large enough that } \mu_s \geq v_t^2/rg}$ the briefcase will begin to slide.

(b) As discussed above, the briefcase starts to slide when $\mu_s \geq v_t^2/rg$. If this occurs at the speed, v_t= 15.0 m/s, the coefficient of static friction must be

$$\mu_s = \frac{(15.0 \text{ m/s})^2}{(62.0 \text{ m})(9.80 \text{ m/s}^2)} = \boxed{0.370}$$

7.30 (a) The external forces acting on the water are $\boxed{\text{the gravitational force}}$ and $\boxed{\text{the contact force exerted on the water by the pail}}$.

(b) The $\boxed{\text{contact force exerted by the pail}}$ is the most important in causing the water to move in a circle. If the gravitational force acted alone, the water would follow the parabolic path of a projectile.

(c) When the pail is inverted at the top of the circular path, it cannot hold the water up to prevent it from falling out. If the water is not to spill, the pail must be moving fast enough that the required centripetal force is at least as large as the gravitational force. That is, we must have

$$m\frac{v^2}{r} \geq mg \qquad \text{or} \qquad v \geq \sqrt{rg} = \sqrt{(1.00 \text{ m})(9.80 \text{ m/s}^2)} = \boxed{3.13 \text{ m/s}}$$

(d) If the pail were to suddenly disappear when is it at the top of the circle and moving at 3.13 m/s, $\boxed{\text{he water would follow the parabolic are of a projectile}}$ launched with initial velocity components of $v_{0x} = 3.13 \text{ m/s}$, $v_{0y} = 0$

7.31 (a) The centripetal acceleration is

$$a_c = r\,\omega^2 = (9.00 \text{ m})\left[\left(4.00 \ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\,\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\right]^2 = \boxed{1.58 \text{ m/s}^2}$$

(b) At the bottom of the circular path, the normal force exerted by the seat must support the weight and also produce the centripetal acceleration. Thus,

$$n = m(g + a_c) = (40.0 \text{ kg})\left[(9.80 + 1.58) \text{ m/s}^2\right] = \boxed{455 \text{ N upward}}$$

(c) At the top of the path, the weight must offset the normal force of the seat plus supply the needed centripetal acceleration. Therefore, $mg = n + ma_c$, or

$$n = m(g - a_c) = (40.0 \text{ kg})\left[(9.80 - 1.58) \text{ m/s}^2\right] = \boxed{329 \text{ N upward}}$$

(d) At a point halfway up, the seat exerts an upward vertical component equal to the child's weight (392 N) and a component toward the center having magnitude $F_c = ma_c = (40.0 \text{ kg})(1.58 \text{ m/s}^2) = 63.2 \text{ N}$. The total force exerted by the seat is

$$F_R = \sqrt{(392 \text{ N})^2 + (63.2 \text{ N})^2} = 397 \text{ N} \text{ directed inward and at}$$

$$\theta = \tan^{-1}\left(\frac{392 \text{ N}}{63.2 \text{ N}}\right) = \boxed{80.8° \text{ above the horizontal}}$$

7.32 (a) At A, the track supports the weight and supplies the centripetal acceleration. Thus,

$$n = mg + m\frac{v_t^2}{r} = (500 \text{ kg})\left[9.80 \text{ m/s}^2 + \frac{(20.0 \text{ m/s})^2}{10 \text{ m}}\right] = \boxed{25 \text{ kN}}$$

(b) At B, the weight must offset the normal force exerted by the track and produce the needed centripetal acceleration, or $mg = n + m\,v_t^2/r$. If the car is on the verge of leaving the track, then $n = 0$ and $m\,g = m\,v_t^2/r$. Hence,

$$v_t = \sqrt{r\,g} = \sqrt{(15 \text{ m})(9.80 \text{ m/s}^2)} = \boxed{12 \text{ m/s}}$$

7.33 At the half-way point the spaceship is 1.92×10^8 from both bodies. The force exerted on the ship by the Earth is directed toward the Earth and has magnitude

$$F_E = \frac{Gm_E m_s}{r^2}$$
$$= \frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(3.00 \times 10^4 \text{ kg})}{(1.92 \times 10^8 \text{ m})^2} = 325 \text{ N}$$

The force exerted on the ship by the Moon is directed toward the Moon and has a magnitude of

$$F_M = \frac{Gm_M m_s}{r^2}$$
$$= \frac{\left(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.36 \times 10^{22}\ \text{kg}\right)\left(3.00 \times 10^4\ \text{kg}\right)}{\left(1.92 \times 10^8\ \text{m}\right)^2} = 4.00\ \text{N}$$

The resultant force is (325 N – 4.00 N) = $\boxed{\text{321 N directed toward Earth}}$.

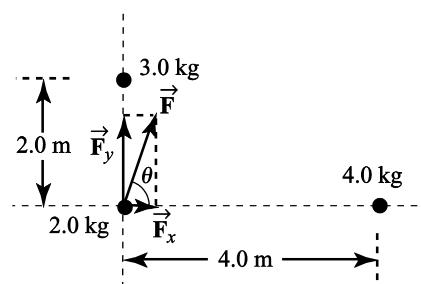
7.34 The radius of the satellite's orbit is

$$r = R_E + h = 6.38 \times 10^6\ \text{m} + 2.00 \times 10^6\ \text{m} = 8.38 \times 10^6\ \text{m}$$

(a) $$PE_g = -\frac{GM_E\, m}{r}$$
$$= -\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98 \times 10^{24}\ \text{kg}\right)\left(100\ \text{kg}\right)}{8.38 \times 10^6\ \text{m}} = \boxed{-4.76 \times 10^9\ \text{J}}$$

(b) $$F = \frac{GM_E\, m}{r^2} = \left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98 \times 10^{24}\ \text{kg}\right)\left(100\ \text{kg}\right)}{\left(8.38 \times 10^6\ \text{m}\right)^2} = \boxed{568\ \text{N}}\ a$$

7.35 The forces exerted on the 2.0-kg mass by the other bodies are F_x and F_y shown in the diagram at the right. The magnitudes of these forces are



$$F_x = \frac{\left(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.0\ \text{kg}\right)\left(4.0\ \text{kg}\right)}{\left(4.0\ \text{m}\right)^2}$$
$$= 3.3 \times 10^{-11}\ \text{N}$$

and

$$F_y = \frac{\left(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.0\ \text{kg}\right)\left(3.0\ \text{kg}\right)}{\left(2.0\ \text{m}\right)^2} = 1.0 \times 10^{-10}\ \text{N}$$

The resultant force exerted on the 2.0-kg mass is $F = \sqrt{F_x^2 + F_y^2} = \boxed{1.1 \times 10^{-10}\ \text{N}}$ directed at

$\theta = \tan^{-1}(F_y/F_x) = \tan^{-1}(3.0) = \boxed{72° \text{ above the } +x - \text{axis}}$.

7.36 (a) The density of the white dwarf would be

$$\rho = \frac{M}{V} = \frac{M_{\text{sun}}}{V_{\text{Earth}}} = \frac{M_{\text{sun}}}{4\pi R_E^3/3} = \frac{3M_{\text{sun}}}{4\pi R_E^3}$$

and using data from Table 7.3,

$$\rho = \frac{3\left(1.991 \times 10^{30}\ \text{kg}\right)}{4\pi\left(6.38 \times 10^{6}\ \text{m}\right)^3} = \boxed{1.83 \times 10^{9}\ \ \text{kg/m}^3}$$

(b) $F_g = mg = GMm/r^2$, so the acceleration of gravity on the surface of the white dwarf would be

$$g = \frac{GM_{\text{sun}}}{R_E^2} = \frac{\left(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991 \times 10^{30}\ \text{kg}\right)}{\left(6.38 \times 10^{6}\ \text{m}\right)^2} = \boxed{3.26 \times 10^{6}\ \ \text{m/s}^2}$$

(c) The general expression for the gravitational potential energy of an object of mass *m* at distance *r* from the center of a spherical mass *M* is $PE = -GMm/r$. Thus, the potential energy of a 1.00-kg mass on the surface of the white dwarf would be

$$\begin{aligned} PE &= -\frac{GM_{\text{sun}}\left(1.00\ \text{kg}\right)}{R_E} \\ &= -\frac{\left(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991 \times 10^{30}\ \text{kg}\right)\left(1.00\ \text{kg}\right)}{6.38 \times 10^{6}\ \text{m}} = \boxed{-2.08 \times 10^{13}\ \text{J}} \end{aligned}$$

7.37 (a) At the midpoint between the two masses, the forces exerted by the 200-kg and 500-kg masses are oppositely directed, so from $F = \frac{GMm}{r^2}$ and $r_1 = r_2 = r$, we have

$$\Sigma F = \frac{GMm_1}{r_1^2} - \frac{GMm_2}{r_2^2} = \frac{GM}{r^2}\left(m_1 - m_2\right)$$

or

$$\begin{aligned} \Sigma F &= \frac{\left(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(50.0\ \text{kg}\right)\left(500\ \text{kg} - 200\ \text{kg}\right)}{\left(0.200\ \text{m}\right)^2} \\ &= \boxed{2.50 \times 10^{-5}\ \text{N}}\ \text{toward the 500-kg mass} \end{aligned}$$

(b) At a point between the two masses and distance *d* from the 500-kg mass, the net force will be zero when

$$\frac{G(50.0\text{ kg})(200\text{ kg})}{(0.400\text{ m} - d)^2} = \frac{G(50.0\text{ kg})(500\text{ kg})}{d^2} \qquad \text{or} \qquad d = \boxed{0.245\text{ m}}$$

Note that the above equation yields a second solution $d = 1.09$ m. At that point, the two gravitational forces do have equal magnitudes, but are in the same direction and cannot add to zero.

7.38 The equilibrium position lies between the Earth and the Sun on the line connecting their centers. At this point, the gravitational forces exerted on the object by the Earth and Sun have equal magnitudes and opposite directions. Let this point be located distance r from the center of the Earth. Then, its distance from the Sun is $(1.496 \times 10^{11}\text{ m} - r)$, and we may determine the value of r by requiring that

$$\frac{G\, m_E\, m}{r^2} = \frac{G\, m_S\, m}{(1.496 \times 10^{11}\text{ m} - r)^2}$$

where m_E and m_S are the masses of the Earth and Sun respectively. This reduces to

$$\frac{(1.496 \times 10^{11}\text{ m} - r)}{r} = \sqrt{\frac{m_S}{m_E}} = 577$$

or 1.496×10^{11} m = 578 r, which yields r = $\boxed{2.59 \times 10^{8}\text{ m from center of the Earth}}$.

7.39 (a) When the rocket engine shuts off at an altitude of 250 km, we may consider the rocket to be beyond Earth's atmosphere. Then, its mechanical energy will remain constant from that instant until it comes to rest momentarily at the maximum altitude. That is, $KE_f + PE_f = KE_i + PE_i$ or

$$0 - \frac{GM_E \cancel{m}}{r_{\max}} = \frac{1}{2}\cancel{m} v_i^2 - \frac{GM_E \cancel{m}}{r_i} \qquad \text{or} \qquad \frac{1}{r_{\max}} = -\frac{v_i^2}{2GM_E} + \frac{1}{r_i}$$

With $r_i = R_E + 250\text{ km} = 6.38 \times 10^6\text{ m} + 250 \times 10^3\text{ m} = 6.63 \times 10^6\text{ m}$ and $v_i = 6.00\text{ km/s} = 6.00 \times 10^3\text{ m/s}$, this gives

$$\frac{1}{r_{\max}} = -\frac{(6.00 \times 10^3\text{ m/s})^2}{2(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})} + \frac{1}{6.63 \times 10^6\text{ m}} = 1.06 \times 10^{-7}\text{ m}^{-1}$$

or $r_{\max} = 9.46 \times 10^6$ m The maximum altitude above Earth's surface is then

$$h_{\max} = r_{\max} - R_E = 9.46 \times 10^6\text{ m} - 6.38 \times 10^6\text{ m} = 3.08 \times 10^6\text{ m} = \boxed{3.08 \times 10^3\text{ km}}$$

(b) If the rocket were fired from a launch site on the equator, it would have a significant eastward component of velocity because of the Earth's rotation about its axis. Hence, compared to being fired from the South Pole, the rocket's initial speed would be greater, and $\boxed{\text{the rocket would travel farther from Earth}}$.

7.40 We know that $m_1 + m_2 = 5.00 \text{ kg}$, or $m_2 = 5.00 \text{ kg} - m_1$

$$F = \frac{G\,m_1\,m_2}{r^2} \Rightarrow 1.00 \times 10^{-8} \text{ N} = \left(6.67 \times 10^{-11} \ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) \frac{m_1\left(5.00 \text{ kg} - m_1\right)}{\left(0.200 \text{ m}\right)^2}$$

$$\left(5.00 \text{ kg}\right) m_1 - m_1^2 = \frac{\left(1.00 \times 10^{-8} \text{ N}\right)\left(0.200 \text{ m}\right)^2}{6.67 \times 10^{-11} \ \text{N} \cdot \text{m}^2/\text{kg}^2} = 6.00 \text{ kg}^2$$

Thus, $m_1^2 - \left(5.00 \text{ kg}\right) m_1 + 6.00 \text{ kg}^2 = 0$, or $\left(m_1 - 3.00 \text{ kg}\right)\left(m_1 - 2.00 \text{ kg}\right) = 0$ giving

$\boxed{m_1 = 3.00 \text{ kg, so } m_2 = 2.00 \text{ kg}}$.

The answer $m_1 - 2.00$ kg and $m_2 = 3.00$ kg is physically equivalent.

7.41 (a) The gravitational force must supply the required centripetal acceleration, so

$$\frac{G\,m_E\,m}{r^2} = m\left(\frac{v_t^2}{r}\right)$$

This reduces to

$$r = \frac{G\,m_E}{v_t^2}$$

which gives

$$r = \left(6.67 \times 10^{-11} \ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) \frac{\left(5.98 \times 10^{24} \text{ kg}\right)}{\left(5\,000 \text{ m/s}\right)^2} = 1.595 \times 10^7 \text{ m}$$

The altitude above the surface of the Earth is then

$$h = r - R_E = 1.595 \times 10^7 \text{ m} - 6.38 \times 10^6 \text{ m} = \boxed{9.57 \times 10^6 \text{ m}}$$

(b) The time required to complete one orbit is

$$T = \frac{\text{circumference of orbit}}{\text{orbital speed}} = \frac{2\pi\left(1.595 \times 10^7 \text{ m}\right)}{5\,000 \text{ m/s}} = 2.00 \times 10^4 \text{ s} = \boxed{5.57 \text{ h}}$$

7.42 For an object in orbit about Earth, Kepler's third law gives the relation between the orbital period T and the average radius of the orbit ("semi-major axis") as

$$T^2 = \left(\frac{4\pi^2}{GM_E}\right) r^3$$

Thus, if the average radius is

$$r = \frac{r_{\text{min}} + r_{\text{max}}}{2} = \frac{6\,670 \text{ km} + 385\,000 \text{ km}}{2} = 1.96 \times 10^5 \text{ km} = 1.96 \times 10^8 \text{ m}$$

the period (time for a round trip from Earth to the Moon) would be

$$T = 2\pi\sqrt{\frac{r^3}{GM_E}} = 2\pi\sqrt{\frac{\left(1.96 \times 10^8 \text{ m}\right)^3}{\left(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24} \text{ kg}\right)}} = 8.63 \times 10^5 \text{ s}$$

The time for a one way trip from Earth to the Moon is then

$$\Delta t = \frac{1}{2}T = \frac{8.63 \times 10^5 \text{ s}}{2}\left(\frac{1 \text{ day}}{8.64 \times 10^4 \text{ s}}\right) = \boxed{4.99 \text{ d}}$$

7.43 The gravitational force exerted on Io by Jupiter provides the centripetal acceleration, so

$$m\left(\frac{v_t^2}{r}\right) = \frac{G\,M\,m}{r^2}, \text{ or } M = \frac{r\,v_t^2}{G}$$

The orbital speed of Io is

$$v_t = \frac{2\,\pi\,r}{T} = \frac{2\,\pi\left(4.22 \times 10^8 \text{ m}\right)}{\left(1.77 \text{ days}\right)\left(86\,400 \text{ s/day}\right)} = 1.73 \times 10^4 \text{ m/s}$$

Thus,

$$M = \frac{\left(4.22 \times 10^8 \text{ m}\right)\left(1.73 \times 10^4 \text{ m/s}\right)^2}{6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2} = \boxed{1.90 \times 10^{27} \text{ kg}}$$

7.44 (a) The satellite moves in an orbit of radius $r = 2R_E$ and the gravitational force supplies the required centripetal acceleration. Hence, $m\left(v_t^2/2\,R_E\right) = G\,m_E\,m/\left(2\,R_E\right)^2$, or

$$v_t = \sqrt{\frac{G\,m_E}{2\,R_E}} = \sqrt{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98\times10^{24}\ \text{kg}\right)}{2\left(6.38\times10^{6}\ \text{m}\right)}} = \boxed{5.59\times10^{3}\ \text{m/s}}$$

(b) The period of the satellite's motion is

$$T = \frac{2\,\pi\,r}{v_t} = \frac{2\,\pi\left[2\left(6.38\times10^{6}\ \text{m}\right)\right]}{5.59\times10^{3}\ \text{m/s}} = 1.43\times10^{4}\ \text{s} = \boxed{3.98\ \text{h}}$$

(c) The gravitational force acting on the satellite is $F = G\,m_E\,m/r^2$, or

$$F = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98\times10^{24}\ \text{kg}\right)\left(600\ \text{kg}\right)}{\left[2\left(6.38\times10^{6}\ \text{m}\right)\right]^2} = \boxed{1.47\times10^{3}\ \text{N}}$$

7.45 The radius of the satellite's orbit is

$$r = R_E + h = 6.38\times10^{6}\ \text{m} + 200\times10^{3}\ \text{m} = 6.58\times10^{6}\ \text{m}$$

(a) Since the gravitational force provides the centripetal acceleration,

$$m\left(\frac{v_t^2}{r}\right) = \frac{G\,m_E\,m}{r^2}$$

or

$$v_t = \sqrt{\frac{G\,m_E}{r}} = \sqrt{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.58\times10^{6}\ \text{m}\right)}} = 7.79\times10^{3}\ \text{m/s}$$

Hence, the period of the orbital motion is

$$T = \frac{2\,\pi\,r}{v_t} = \frac{2\,\pi\left(6.58\times10^{6}\ \text{m}\right)}{7.79\times10^{3}\ \text{m/s}} = 5.31\times10^{3}\ \text{s} = \boxed{1.48\ \text{h}}$$

(b) The orbital speed is $v_t = \boxed{7.79 \times 10^3 \text{ m/s}}$ as computed above.

(c) Assuming the satellite is launched from a point on the equator of the Earth, its initial speed is the rotational speed of the launch point, or

$$v_i = \frac{2\pi R_E}{1 \text{ day}} = \frac{2\pi\left(6.38 \times 10^6 \text{ m}\right)}{86\,400 \text{ s}} = 464 \text{ m/s}$$

The work–kinetic energy theorem gives the energy input required to place the satellite in orbit as $W_{nc} = \left(KE + PE_g\right)_f - \left(KE + PE_g\right)_i$, or

$$W_{nc} = \left(\frac{1}{2}mv_t^2 - \frac{GM_E m}{r}\right) - \left(\frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E}\right) = m\left[\frac{v_t^2 - v_i^2}{2} + GM_E\left(\frac{1}{R_E} - \frac{1}{r}\right)\right]$$

Substitution of appropriate numeric values into this result gives the minimum energy input as $W_{nc} = \boxed{6.43 \times 10^9 \text{ J}}$.

7.46 A synchronous satellite will have an orbital period equal to Jupiter's rotation period, so the satellite can have the red spot in sight at all times. Thus, the desired orbital period is

$$T = 9.84 \text{ h}\left(\frac{3\,600 \text{ s}}{1 \text{ h}}\right) = 3.54 \times 10^4 \text{ s}$$

Kepler's third law gives the period of a satellite in orbit around Jupiter as

$$T^2 = \frac{4\pi^2}{GM_{\text{Jupiter}}} r^3$$

The required radius of the circular orbit is therefore

$$r = \left(\frac{GM_{\text{Jupiter}}T^2}{4\pi^2}\right)^{1/3} = \left[\frac{\left(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.90 \times 10^{27} \text{ kg}\right)\left(3.54 \times 10^4 \text{ s}\right)^2}{4\pi^2}\right]^{1/3} = 1.59 \times 10^8 \text{ m}$$

and the altitude of the satellite above Jupiter's surface should be

$$h = r - R_{\text{Jupiter}} = 1.59 \times 10^8 \text{ m} - 6.99 \times 10^7 \text{ m} = \boxed{8.91 \times 10^7 \text{ m}}$$

7.47 The gravitational force on mass located at distance r from the center of the Earth is $F_g = mg = GM_E m/r^2$. Thus, the acceleration of gravity at this location is $g = GM_E/r^2$. If $g = 9.00\ \text{m/s}^2$ at the location of the satellite, the radius of its orbit must be

$$r = \sqrt{\frac{GM_E}{g}} = \sqrt{\frac{\left(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24}\ \text{kg}\right)}{9.00\ \text{m/s}^2}} = 6.66 \times 10^6\ \text{m}$$

From Kepler's third law for Earth satellites, $T^2 = 4\pi^2 r^3/G\,M_E S$, the period is found to be

$$T = 2\pi\sqrt{\frac{r^3}{GM_E}} = 2\pi\sqrt{\frac{\left(6.66 \times 10^6\ \text{m}\right)^3}{\left(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24}\ \text{kg}\right)}} = 5.41 \times 10^3\ \text{s}$$

or

$$T = \left(5.41 \times 10^3\ \text{s}\right)\left(\frac{1\ \text{h}}{3\ 600\ \text{s}}\right) = \boxed{1.50\ \text{h} = 90.0\ \text{min}}$$

7.48 The gravitational force on a small parcel of material at the star's equator supplies the centripetal acceleration, or

$$\frac{G\,M_s\,m}{R_s^2} = m\left(\frac{v_t^2}{R_s}\right) = m\left(R_s\,\omega^2\right)$$

Hence, $\omega = \sqrt{G\,M_s/R_s^3}$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left[2\left(1.99 \times 10^{30}\ \text{kg}\right)\right]}{\left(10.0 \times 10^3\ \text{m}\right)^3}} = \boxed{1.63 \times 10^4\ \text{rad/s}}$$

7.49 (a) $$\omega = \frac{v_t}{r} = \frac{\left(98.0\ \text{mi/h}\right)\left(\frac{0.447\ \text{m/s}}{1\ \text{mi/h}}\right)}{0.742\ \text{m}} = 59.0\ \frac{\text{rad}}{\text{s}}\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = \boxed{9.40\ \text{rev/s}}$$

(b) $$\alpha = \frac{\omega^2 - \omega_i^2}{2\ \Delta\theta} = \frac{\left(9.40\ \text{rev/s}\right)^2 - 0}{2\left(1\ \text{rev}\right)} = \boxed{44.2\ \text{rev/s}^2}$$

$$a_c = \frac{v_t^2}{r} = \frac{\left[(98.0\ \text{mi/h})\left(\frac{0.447\ \text{m/s}}{1\ \text{mi/h}}\right)\right]^2}{0.742\ \text{m}} = \boxed{2.59 \times 10^3\ \text{m/s}^2}$$

$$a_t = r\alpha = (0.742\ \text{m})\left[44.2\ \frac{\text{rev}}{\text{s}^2}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\right] = \boxed{206\ \text{m/s}^2}$$

(c) In the radial direction at the release point, the hand supports the weight of the ball and also supplies the centripetal acceleration. Thus, $F_r = mg + ma_r = m(g + a_r)$, or

$$F_r = (0.198\ \text{kg})(9.80\ \text{m/s}^2 + 2.59 \times 10^3\ \text{m/s}^2) = \boxed{514\ \text{N}}$$

In the tangential direction, the hand supplies only the tangential acceleration, so

$$F_t = ma_t = (0.198\ \text{kg})(206\ \text{m/s}^2) = \boxed{40.8\ \text{N}}$$

7.50 (a) $$\omega_i = \frac{v_t}{r_i} = \frac{1.30\ \text{m/s}}{2.30 \times 10^{-2}\ \text{m}} = \boxed{56.5\ \text{rad/s}}$$

(b) $$\omega_f = \frac{v_t}{r_f} = \frac{1.30\ \text{m/s}}{5.80 \times 10^{-2}\ \text{m}} = \boxed{22.4\ \text{rad/s}}$$

(c) The duration of the recording is

$$\Delta t = (74\ \text{min})(60\ \text{s/min}) + 33\ \text{s} = 4\,473\ \text{s}$$

Thus,

$$\alpha_{\text{av}} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{(22.4 - 56.5)\ \text{rad/s}}{4\,473\ \text{s}} = \boxed{-7.62 \times 10^{-3}\ \text{rad/s}^2}$$

(d) $$\Delta\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{(22.4\ \text{rad/s})^2 - (56.5\ \text{rad/s})^2}{2(-7.62 \times 10^{-3}\ \text{rad/s}^2)} = \boxed{1.77 \times 10^5\ \text{rad}}$$

(e) The track moves past the lens at a constant speed of $v_t = 1.30$ m/s for 4 473 seconds. Therefore, the length of the spiral track is

$$\Delta s = v_t(\Delta t) = (1.30 \text{ m/s})(4\ 473 \text{ s}) = 5.81 \times 10^3 \text{ m} = \boxed{5.81 \text{ km}}$$

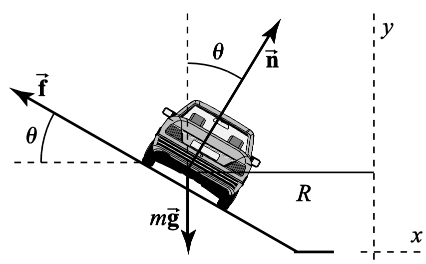
7.51 The angular velocity of the ball is $\omega = 0.500 \text{ rev/s} = \pi \text{ rad/s}$.

(a) $v_t = r\omega = (0.800 \text{ m})(\pi \text{ rad/s}) = \boxed{2.51 \text{ m/s}}$

(b) $a_c = \dfrac{v_t^2}{r} = r\omega^2 = (0.800 \text{ m})(\pi \text{ rad/s})^2 = \boxed{7.90 \text{ m/s}^2}$

(c) We imagine that the weight of the ball is supported by a frictionless platform. Then, the rope tension need only produce the centripetal acceleration. The force required to produce the needed centripetal acceleration is $F = m\left(v_t^2/r\right)$. Thus, if the maximum force the rope can exert is 100 N, the maximum tangential speed of the ball is

$$\left(v_t\right)_{\max} = \sqrt{\frac{r F_{\max}}{m}} = \sqrt{\frac{(0.800 \text{ m})(100 \text{ N})}{5.00 \text{ kg}}} = \boxed{4.00 \text{ m/s}}$$



7.52 (a) When the car is about to slip down the incline, the friction force, $\vec{\mathbf{f}}$, is directed up the incline as shown and has the magnitude $f = \mu n$. Thus,

$$\Sigma F_y = n\cos\theta + \mu n \sin\theta - mg = 0$$

or

$$n = \frac{mg}{\cos\theta + \mu\sin\theta} \qquad [1]$$

Also, $\Sigma F_x = n\sin\theta - \mu n\cos\theta = m\left(v_{\min}^2/R\right)$, or

$$v_{\min} = \sqrt{\frac{nR}{m}\left(\sin\theta - \mu\cos\theta\right)} \qquad [2]$$

Substituting equation [1] into [2] gives

$$v_{\min} = \sqrt{R\,g\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)} = \boxed{\sqrt{R\,g\left(\frac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)}}$$

If the car is about to slip up the incline, $f = \mu n$ is directed down the slope (opposite to what is shown in the sketch). Then, $\Sigma F_y = n\cos\theta - \mu n\sin\theta - mg = 0$, or

$$n = \frac{mg}{\cos\theta - \mu\sin\theta} \qquad [3]$$

Also, $\Sigma F_x = n\sin\theta + \mu n\cos\theta = m\left(v_{\max}^2/R\right)$

or

$$v_{\max} = \sqrt{\frac{n\,R}{m}\left(\sin\theta + \mu\cos\theta\right)} \qquad [4]$$

Combining equations [3] and [4] gives

$$v_{\max} = \sqrt{R\,g\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)} = \boxed{\sqrt{R\,g\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}}$$

(b) If $R = 100$ m, $\theta = 10°$, and $\mu = 0.10$, the lower and upper limits of safe speeds are

$$v_{\min} = \sqrt{(100\ m)(9.8\ m/s^2)\left(\frac{\tan 10° - 0.10}{1 + 0.10\tan 10°}\right)} = \boxed{8.6\ \text{m/s}}$$

and

$$v_{\max} = \sqrt{(100\ \text{m})(9.8\ \text{m/s}^2)\left(\frac{\tan 10° + 0.10}{1 - 0.10\tan 10°}\right)} = \boxed{17\ \text{m/s}}$$

7.53 The radius of the satellite's orbit is

$$r = R_E + h = 6.38 \times 10^6 \text{ m} + \left(1.50 \times 10^2 \text{ mi}\right)\left(1\,609 \text{ m}/1 \text{ mi}\right) = 6.62 \times 10^6 \text{ m}$$

(a) The required centripetal acceleration is produced by the gravitational force, so

$$m\left(\frac{v_t^2}{r}\right) = \frac{G M_E m}{r^2},$$

which gives

$$v_t = \sqrt{\frac{G M_E}{r}}$$

$$v_t = \sqrt{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98 \times 10^{24} \text{ kg}\right)}{6.62 \times 10^6 \text{ m}}} = \boxed{7.76 \times 10^3 \text{ m/s}}$$

(b) The time for one complete revolution is

$$T = \frac{2\pi r}{v_t} = \frac{2\pi\left(6.62 \times 10^6 \text{ m}\right)}{7.76 \times 10^3 \text{ m/s}} = 5.36 \times 10^3 \text{ s} = \boxed{89.3 \text{ min}}$$

7.54 (a) At the lowest point on the path, the net upward force (i.e., the force directed toward the center of the path and supplying the centripetal acceleration) is $\Sigma F_{up} = T - mg = m\left(v_t^2/r\right)$, so the tension in the cable is

$$T = m\left(g + \frac{v_t^2}{r}\right) = \left(0.400 \text{ kg}\right)\left(9.80 \text{ m/s}^2 + \frac{\left(3.00 \text{ m/s}\right)^2}{0.800 \text{ m}}\right) = \boxed{8.42 \text{ N}}$$

(b) Using conservation of mechanical energy, $\left(KE + PE_g\right)_f = \left(KE + PE_g\right)_i$, as the bob goes from the lowest to the highest point on the path gives

$$0 + mg\left[L\left(1 - \cos\theta_{max}\right)\right] = \frac{1}{2}mv_i^2 + 0, \text{ or } \quad \cos\theta_{max} = 1 - \frac{v_i^2}{2gL}$$

$$\theta_{max} = \cos^{-1}\left(1 - \frac{v_i^2}{2gL}\right) = \cos^{-1}\left(1 - \frac{\left(3.00 \text{ m/s}\right)^2}{2\left(9.80 \text{ m/s}^2\right)\left(0.800 \text{ m}\right)}\right) = \boxed{64.8°}$$

(c) At the highest point on the path, the bob is at rest and the net radial force is

$$\Sigma F_r = T - mg\cos\theta_{\max} = m\left(\frac{v_t^2}{r}\right) = 0$$

Therefore,

$$T = mg\cos\theta_{\max} = (0.400\ \text{kg})(9.80\ \text{m/s}^2)\cos(64.8°) = \boxed{1.67\ \text{N}}$$

7.55 (a) When the car is at the top of the arc, the normal force is upward and the weight downward. The net force directed downward, toward the center of the circular path and hence supplying the centripetal acceleration, is $\Sigma F_{\text{down}} = mg - n = m\left(v_t^2/r\right)$.

Thus, the normal force is $\boxed{n = m\left(g - v_t^2/r\right)}$.

(b) If $r = 30.0$ m and $n \to 0$, then $g - \dfrac{v_t^2}{r} \to 0$ or the speed of the car must be

$$v_t = \sqrt{rg} = \sqrt{(30.0\ \text{m})(9.80\ \text{m/s}^2)} = \boxed{17.1\ \text{m/s}}$$

7.56 The escape speed from the surface of a planet of radius R and mass M is given by

$$v_e = \sqrt{\frac{2GM}{R}}$$

If the planet has uniform density, ρ, the mass is given by

$$M = \rho(\mathit{volume}) = \rho\left(4\pi R^3/3\right) = 4\pi\rho R^3/3$$

The expression for the escape speed then becomes

$$v_e = \sqrt{\frac{2G}{R}\left(\frac{4\pi\rho R^3}{3}\right)} = \left(\sqrt{\frac{8\pi\rho G}{3}}\right)R = \boxed{(\mathit{constant})R}$$

or the escape speed is directly proportional to the radius of the planet.

7.57 The speed the person has due to the rotation of the Earth is $v_t = r\,\omega$ where r is the distance from the rotation axis and ω is the angular velocity of rotation.

The person's apparent weight, $(F_g)_{apparent}$, equals the magnitude of the upward normal force exerted on him by the scales. The true weight, $(F_g)_{true} = mg$, is directed downward. The net downward force produces the needed centripetal acceleration, or

$$\Sigma F_{down} = -n + \left(F_g\right)_{true} = -\left(F_g\right)_{apparent} + \left(F_g\right)_{true} = m\left(\frac{v_t^2}{r}\right) = m\,r\,\omega^2$$

(a) At the equator, $r = R_E$, so $\boxed{\left(F_g\right)_{true} = \left(F_g\right)_{apparent} + m\,R_E\,\omega^2} > \left(F_g\right)_{apparent}$.

(b) At the equator, it is given that $r\,\omega^2 = 0.0340\ \text{m/s}^2$, so the apparent weight is

$$\left(F_g\right)_{apparent} = \left(F_g\right)_{true} - mr\,\omega^2 = \left(75.0\ \text{kg}\right)\left[\left(9.80 - 0.0340\right)\ \text{m/s}^2\right] = \boxed{732\ \text{N}}$$

At either pole, $r = 0$ (the person is on the rotation axis) and

$$\left(F_g\right)_{apparent} = \left(F_g\right)_{true} = mg = \left(75.0\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) = \boxed{735\ \text{N}}$$

7.58 Choosing $y = 0$ and $PE_g = 0$ at the level of point B, applying the work–energy theorem to the block's motion gives $W_{nc} = \frac{1}{2}mv^2 + mgy - \frac{1}{2}mv_0^2 - mg\,(2R)$, or

$$v^2 = v_0^2 + \frac{2\,W_{nc}}{m} + 2\,g\left(2\,R - y\right) \qquad [1]$$

(a) At point A, $y = R$ and $W_{nc} = 0$ (no nonconservative force has done work on the block yet). Thus, $v_A^2 = v_0^2 + 2\,gR$. The normal force exerted on the block by the track must supply the centripetal acceleration at point A, so

$$n_A = m\left(\frac{v_A^2}{R}\right) = m\left(\frac{v_0^2}{R} + 2\,g\right)$$

$$= \left(0.50\ \text{kg}\right)\left(\frac{\left(4.0\ \text{m/s}\right)^2}{1.5\ \text{m}} + 2\left(9.8\ \text{m/s}^2\right)\right) = \boxed{15\ \text{N}}$$

At point B, $y = 0$ and W_{nc} is still zero. Thus, $v_B^2 = v_0^2 + 4gR$. Here, the normal force must supply the centripetal acceleration *and* support the weight of the block. Therefore,

$$n_B = m\left(\frac{v_B^2}{R}\right) + mg = m\left(\frac{v_0^2}{R} + 5g\right)$$

$$= (0.50\ \text{kg})\left(\frac{(4.0\ \text{m/s})^2}{1.5\ \text{m}} + 5(9.8\ \text{m/s}^2)\right) = \boxed{30\ \text{N}}$$

(b) When the block reaches point C, $y = 2R$ and $W_{nc} = -f_k L = -\mu_k(mg)L$. At this point, the normal force is to be zero, so the weight alone must supply the centripetal acceleration. Thus, $m\left(v_c^2/R\right) = mg$, or the required speed at point C is $v_c^2 = Rg$. Substituting this into equation [1] yields $Rg = v_0^2 - 2\mu_k gL + 0$, or

$$\mu_k = \frac{v_0^2 - Rg}{2gL} = \frac{(4.0\ \text{m/s})^2 - (1.5\ \text{m})(9.8\ \text{m/s}^2)}{2(9.8\ \text{m/s}^2)(0.40\ \text{m})} = \boxed{0.17}$$

7.59 Define the following symbols: M_m = mass of moon, M_e = mass of the Earth, R_m = radius of moon, R_e = radius of the Earth, and r = radius of the Moon's orbit around the Earth.

We interpret "lunar escape speed" to be the escape speed from the surface of a stationary moon alone in the universe. Then,

$$v_{\text{launch}} = 2\,v_{\text{escape}} = 2\sqrt{\frac{2GM_m}{R_m}} \quad \text{or} \quad v_{\text{launch}}^2 = \frac{8GM_m}{R_m}$$

Applying conservation of mechanical energy from launch to impact gives

$$\frac{1}{2}m\,v_{\text{impact}}^2 + \left(PE_g\right)_f = \frac{1}{2}m\,v_{\text{launch}}^2 + \left(PE_g\right)_i, \text{ or}$$

$$v_{\text{impact}} = \sqrt{v_{\text{launch}}^2 + \frac{2}{m}\left[\left(PE_g\right)_i - \left(PE_g\right)_f\right]}$$

The needed potential energies are

$$\left(PE_g\right)_i = -\frac{GM_m m}{R_m} - \frac{GM_e m}{r} \quad \text{and} \quad \left(PE_g\right)_f = -\frac{GM_e m}{R_e} - \frac{GM_m m}{r}$$

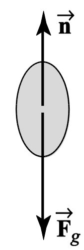
Using these potential energies and the expression for v^2_{launch} from above, the equation for the impact speed reduces to

$$v_{\text{impact}} = \sqrt{2\,G\left(\frac{3\,M_m}{R_m} + \frac{M_e}{R_e} - \frac{(M_e - M_m)}{r}\right)}$$

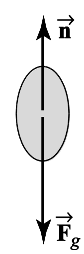
With numeric values of $G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$, $M_m = 7.36 \times 10^{22}\ \text{kgs}$, and $R_m = 1.74 \times 10^6\ \text{m}$, $R_e = 6.38 \times 10^6\ \text{m}$, $r = 3.84 \times 10^8\ \text{ms}$ we find

$$v_{\text{impact}} = 1.18 \times 10^4\ \text{m/s} = \boxed{11.8\ \text{km/s}}$$

7.60 (a) When the passenger is at the top, the radial forces producing the centripetal acceleration are the upward force of the seat and the downward force of gravity. The downward force must exceed the upward force to yield a net force toward the center of the circular path.



(b) At the lowest point on the path, the radial forces contributing to the centripetal acceleration are again the upward force of the seat and the downward force of gravity. However, the upward force must now exceed the downward force to yield a net force directed toward the center of the circular path.



(c) The seat must exert the greatest force on the passenger at the lowest point on the circular path.

(d) At the top of the loop, $\Sigma F_r = m\dfrac{v^2}{r} = F_g - n$

or

$$n = F_g - m\frac{v^2}{r} = m\left(g - \frac{v^2}{r}\right) = (70.0\ \text{kg})\left(9.80\ \text{m/s}^2 - \frac{(4.00\ \text{m/s})^2}{8.00\ \text{m}}\right) = \boxed{546\ \text{N}}$$

At the bottom of the loop, $\Sigma F_r = m\,(v^2/r) = n - F_g$

or

$$n = F_g + m\frac{v^2}{r} = m\left(g + \frac{v^2}{r}\right) = (70.0 \text{ kg})\left(9.80 \text{ m/s}^2 + \frac{(4.00 \text{ m/s})^2}{8.00 \text{ m}}\right) = \boxed{826 \text{ N}}$$

7.61 (a) In order to launch yourself into orbit by running, your running speed must be such that the gravitational force acting on you exactly equals the force needed to produce the centripetal acceleration. That is, $GMm/r^2 = m\, v_t^2/r$, where M is the mass of the asteroid and r is its radius. Since $M = density \times volume = \rho[(4/3)\pi r^3]$, this requirement becomes

$$G\rho\left(\frac{4}{3}\pi r^3\right)\frac{\cancel{m}}{r^2} = \frac{\cancel{m}\, v_t^2}{r} \qquad \text{or} \qquad r = \sqrt{\frac{3v_t^2}{4\pi G\rho}}.$$

The radius of the asteroid would then be

$$r = \sqrt{\frac{3(8.50 \text{ m/s})^2}{4\pi\left(6.673\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.10\times10^3 \text{ kg/m}^3\right)}} = 1.53\times10^4 \text{ m}$$

or $r = \boxed{15.3 \text{ km}}$.

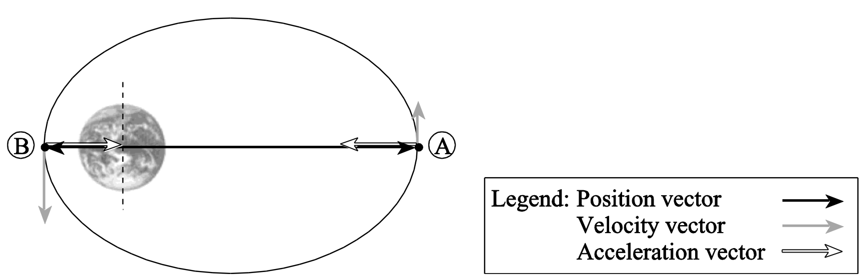
(b) The mass of the asteroid is given by

$$M = \rho\left(\frac{4}{3}\pi r^3\right) = \left(1.10\times10^3 \text{ kg/m}^3\right)\frac{4}{3}\pi\left(1.53\times10^4 \text{ m}\right)^3 = \boxed{1.66\times10^{16} \text{ kg}}$$

(c) Your period will be

$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{v_t} = \frac{2\pi\left(1.53\times10^4 \text{ m}\right)}{8.50 \text{ m/s}} = \boxed{1.13\times10^4 \text{ s}}$$

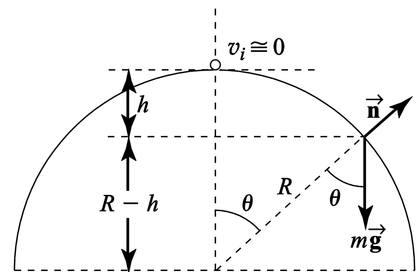
7.62 (a)



(b) The velocity vector at A is shorter than that at B. The gravitational force acting on the spacecraft is a conser vative force, so the total mechanical energy of the craft is constant. The gravitational potential energy at A is larger than at B. Hence, the kinetic energy (and therefore the velocity) at A must be less than at B.

(c) The acceleration vector at A is shorter than that at B. From Newton's second law, the acceleration of the spacecraft is directly proportional to the force acting on it. Since the gravitational force at A is weaker than that at B, the acceleration at A must be less than the acceleration at B.

7.63 Choosing $PE_s = 0$ at the top of the hill, the speed of the skier after dropping distance h is found using conservation of mechanical energy as



$$\frac{1}{2} m v_t^2 - m g h = 0 + 0 \text{, or } v_t^2 = 2 g h$$

The net force directed toward the center of the circular path, and providing the centripetal acceleration, is

$$\Sigma F_r = m g \cos\theta - n = m\left(\frac{v_t^2}{R}\right)$$

Solving for the normal force, after making the substitutions $v_t^2 = 2 g h$ and $\cos\theta = \dfrac{R-h}{R} = 1 - \dfrac{h}{R}$

gives $\quad n = m g\left(1 - \dfrac{h}{R}\right) - m\left(\dfrac{2 g h}{R}\right) = m g\left(1 - \dfrac{3h}{R}\right)$

The skier leaves the hill when $n \to 0$ This occurs when

$$1 - \frac{3h}{R} = 0 \quad \text{or} \quad \boxed{h = \frac{R}{3}}$$

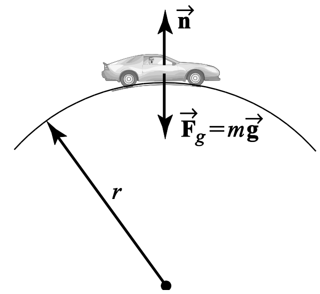
7.64 The centripetal acceleration of a particle at distance r from the axis is $a_c = v_t^2/r = r\omega^2$ If we are to have $a_c =$ 100g, then it is necessary that

$$r\omega^2 = 100g \quad \text{or} \quad \omega = \sqrt{\frac{100g}{r}}$$

The required rotation rate increases as r decreases. In order to maintain the required acceleration for all particles in the casting, we use the minimum value of r and find

$$\omega = \sqrt{\frac{100g}{r_{\min}}} = \sqrt{\frac{100\left(9.80 \text{ m/s}^2\right)}{2.10 \times 10^{-2} \text{ m}}} = 216 \frac{\text{rad}}{\text{s}}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{60.0 \text{ s}}{1 \text{ min}}\right) = \boxed{2.06 \times 10^3 \frac{\text{rev}}{\text{min}}}$$

7.65 The sketch at the right shows the car as it passes the highest point on the bump. Taking upward as positive, we have



$$\Sigma F_y = ma_y \Rightarrow n - mg = m\left(-\frac{v^2}{r}\right)$$

or

$$n = m\left(g - \frac{v^2}{r}\right)$$

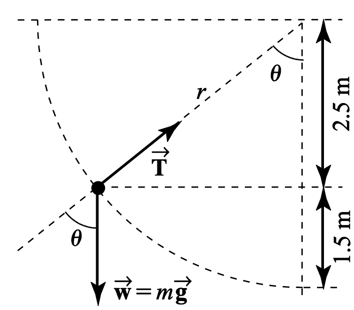
(a) If $v = 8.94 \text{ m/s}$, the normal force exerted by the road is

$$n = \left(1\,800 \text{ kg}\right)\left[9.80 \frac{\text{m}}{\text{s}^2} - \frac{\left(8.94 \text{ m/s}\right)^2}{20.4 \text{ m}}\right] = 1.06 \times 10^4 \text{ N} = \boxed{10.6 \text{ kN}}$$

(b) When the car is on the verge of losing contact with the road, $n = 0$. This gives $g = v^2/r$ and the speed must be

$$v = \sqrt{rg} = \sqrt{\left(20.4 \text{ m}\right)\left(9.80 \text{ m/s}^2\right)} = \boxed{14.1 \text{ m/s}}$$

7.66 When the rope makes angle θ with the vertical, the net force directed toward the center of the circular path is $\Sigma F_r = T - m g \cos\theta$ as shown in the sketch. This force supplies the needed centripetal acceleration, so



$$T - m g \cos\theta = m\left(\frac{v_t^2}{r}\right), \quad \text{or} \quad T = m\left(g\cos\theta + \frac{v_t^2}{r}\right)$$

Using conservation of mechanical energy, with $KE = 0$ at $\theta = 90°$ and $PE_g = 0$ at the bottom of the arc, the speed when the rope is at angle θ from the vertical is given by $\frac{1}{2} m v_t^2 + m g\left(r - r\cos\theta\right) = 0 + m g r$, or $v_t^2 = 2 g r \cos\theta$. The expression for the tension in the rope at angle θ then reduces to $T = 3 m g \cos\theta$.

(a) At the beginning of the motion, $\theta = 90°$ and $T = \boxed{0}$

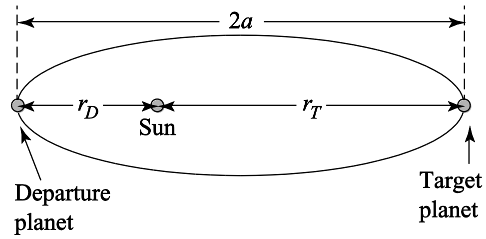
(b) At 1.5 m from the bottom of the arc, $\cos\theta = \dfrac{2.5\text{ m}}{r} = \dfrac{2.5\text{ m}}{4.0\text{ m}} = 0.63$ and the tension is

$$T = 3(70\text{ kg})(9.8\text{ m/s}^2)(0.63) = 1.3\times10^3\text{ N} = \boxed{1.3\text{ kN}}$$

(c) At the bottom of the arc, $\theta = 0°$ and $\cos\theta = 1.0$, so the tension is

$$T = 3(70\text{ kg})(9.8\text{ m/s}^2)(1.0) = 2.1\times10^3\text{ N} = \boxed{2.1\text{ kN}}$$

7.67 (a) The desired path is an elliptical trajectory with the Sun at one of the foci, the departure planet at the perihelion, and the target planet at the aphelion. The perihelion distance r_D is the radius of the departure planet's orbit, while the aphelion distance r_r is the radius of the target planet's orbit. The semi-major axis of the desired trajectory is then $a = (r_D + r_T)/2$.



If Earth is the departure planet, $r_D = 1.496\times10^{11}\text{ m} = 1.00\text{ AU}$.

With Mars as the target planet,

$$r_T = 2.28\times10^{11}\text{ m}\left(\frac{1\text{ AU}}{1.496\times10^{11}\text{ m}}\right) = 1.52\text{ AU}$$

Thus, the semi-major axis of the minimum energy trajectory is

$$a = \frac{r_D + r_T}{2} = \frac{1.00\text{ AU} + 1.52\text{ AU}}{2} = 1.26\text{ AU}$$

Kepler's third law, $T^2 = a^3$, then gives the time for a full trip around this path as

$$T = \sqrt{a^3} = \sqrt{(1.26\text{ AU})^3} = 1.41\text{ yr}$$

so the time for a one-way trip from Earth to Mars is

$$\Delta t = \frac{1}{2}T = \frac{1.41\text{ yr}}{2} = \boxed{0.71\text{ yr}}$$

(b) $\boxed{\text{This trip cannot be taken at just any time.}}$ The departure must be timed so that the spacecraft arrives at the aphelion when the target planet is located there.

7.68 (a) Consider the sketch at the right. At the bottom of the loop, the net force toward the center (i.e., the centripetal force) is

$$F_c = \frac{mv^2}{R} = n - F_g$$

so the pilot's apparent weight (normal force) is

$$n = F_g + \frac{mv^2}{R} = F_g + \frac{\left(F_g/g\right)v^2}{R} = F_g\left(1 + \frac{v^2}{gR}\right)$$

or

$$n = \left(712\text{ N}\right)\left(1 + \frac{\left(2.00 \times 10^2\text{ m/s}\right)^2}{\left(9.80\text{ m/s}^2\right)\left(3.20 \times 10^3\text{ m/s}^2\right)}\right)$$
$$= \boxed{1.62 \times 10^3\text{ N}}$$

(b) At the top of the loop, the centripetal force is $F_c = mv^2/R = n + F_g$, so the apparent weight is

$$n = \frac{mv^2}{R} - F_g = \frac{\left(F_g/g\right)v^2}{R} - F_g = F_g\left(\frac{v^2}{gR} - 1\right)$$
$$= \left(712\text{ N}\right)\left(\frac{\left(2.00 \times 10^2\text{ m/s}\right)^2}{\left(9.80\text{ m/s}^2\right)\left(3.20 \times 10^3\text{ m/s}^2\right)} - 1\right) = \boxed{196\text{ N}}$$

(c) With the right speed, the needed centripetal force at the top of the loop can be made exactly equal to the gravitational force. At this speed, the normal force exerted on the pilot by the seat (his apparent weight) will be zero, and the pilot will have the sensation of weightlessness.

(d) When $n = 0$ at the top of the loop, $F_c = mv^2/R = mg = F_g$, and the speed will be

$$v = \sqrt{\frac{mg}{m/R}} = \sqrt{Rg} = \sqrt{\left(3.20 \times 10^3\text{ m}\right)\left(9.80\text{ m/s}^2\right)} = \boxed{177\text{ m/s}}$$

7.69 (a) At the instant the mud leaves the tire and becomes a projectile, its velocity components are $v_{0x} = 0,\ v_{0y} = v_t = R\omega$. From $\Delta y = v_{0y}t + a_y t^2/2$ with $a_y = -g$, the time required for the mud to return to its starting point (with $\Delta y = 0$) is given by

$$0 = t\left(R\omega - \frac{gt}{2}\right)$$

for which the nonzero solution is

$$\boxed{t = \frac{2R\omega}{g}}$$

(b) The angular displacement of the wheel (turning at constant angular speed ω) in time t is $\Delta\theta = \omega t$. If the displacement is $\Delta\theta = 1 \text{ rev} = 2\pi \text{ rad}$ at $t = 2R\omega/g$, then

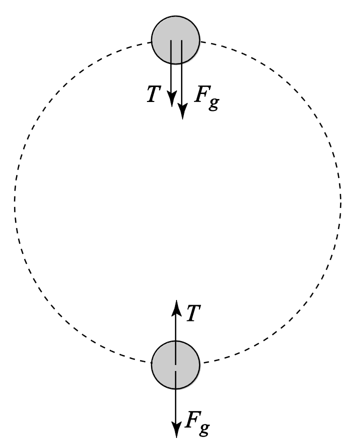
$$2\pi \text{ rad} = \omega\left(\frac{2R\omega}{g}\right) \qquad \text{or} \qquad \omega^2 = \frac{\pi g}{R} \qquad \text{and} \qquad \boxed{\omega = \sqrt{\frac{\pi g}{R}}}$$

7.70 (a) At each point on the vertical circular path, two forces are acting on the ball:

(1) $\boxed{\text{The downward gravitational force}}$ with constant magnitude $F_g = mg$

(2) $\boxed{\text{The tension force in the string, always directed toward the center of the path}}$

(b) The sketch at the right shows the forces acting on the ball when it is at the bottom of the circular path and when it is at the highest point on the path. Note that the gravitational force has the same magnitude and direction at each point on the circular path. The tension force varies in magnitude at different points and is always directed toward the center of the path.



(c) At the top of the circle, $F_c = mv^2/r = T + F_g$, or

$$T = \frac{mv^2}{r} - F_g = \frac{mv^2}{r} - mg = m\left(\frac{v^2}{r} - g\right)$$

$$= (0.275 \text{ kg})\left[\frac{(5.20 \text{ m/s})^2}{0.850 \text{ m}} - 9.80 \text{ m/s}^2\right] = 6.05 \text{ N}$$

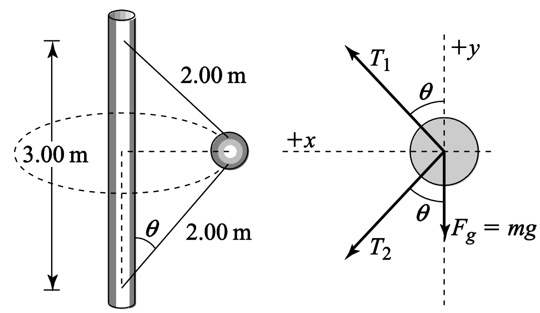
(d) At the bottom of the circle, $F_c = mv^2/r = T - F_g = T - mg$, and solving for the speed gives

$$v^2 = \frac{r}{m}\left(T - mg\right) = r\left(\frac{T}{m} - g\right) \qquad \text{and} \qquad v = \sqrt{r\left(\frac{T}{m} - g\right)}$$

If the string is at the breaking point at the bottom of the circle, then $T = 22.5$ N, and the speed of the object at this point must be

$$v = \sqrt{\left(0.850 \text{ m}\right)\left(\frac{22.5 \text{ N}}{0.275 \text{ kg}} - 9.80 \text{ m/s}^2\right)} = \boxed{7.82 \text{ m/s}}$$

7.71 From Figure (a) at the right, observe that the angle the strings make with the vertical is



(a) (b)

$$\theta = \cos^{-1}\left(\frac{1.50 \text{ m}}{2.00 \text{ m}}\right) = 41.4°$$

Also, the radius of the circular path is

$$r = \sqrt{\left(2.00 \text{ m}\right)^2 - \left(1.50 \text{ m}\right)^2} = 1.32 \text{ m}$$

Figure (b) gives a free-body diagram of the object with the $+y$-axis vertical and the $+x$-axis directed toward the center of the circular path.

(a) Since the object has zero vertical acceleration, Newton's second law gives

$$\Sigma F_y = T_1 \cos\theta - T_2 \cos\theta - mg = 0 \qquad \text{or} \qquad T_1 - T_2 = \frac{mg}{\cos\theta} \qquad [1]$$

In the horizontal direction, the object has the centripetal acceleration $a_c = v^2/r$ directed in the $+x$-direction (toward the center of the circular path). Thus,

$$\Sigma F_x = T_1 \sin\theta + T_2 \sin\theta = \frac{mv^2}{r} \qquad \text{or} \qquad T_1 + T_2 = \frac{mv^2}{r\sin\theta} \qquad [2]$$

Adding equations [1] and [2] gives

$$2T_1 = m\left(\frac{g}{\cos\theta} + \frac{v^2}{r\sin\theta}\right)$$

so the tension in the upper string is

$$T_1 = \frac{(4.00\text{ kg})}{2}\left[\frac{9.80\text{ m/s}^2}{\cos 41.4^\circ} + \frac{(6.00\text{ m/s}^2)^2}{(1.32\text{ m})\sin 41.4^\circ}\right] = \boxed{109\text{ N}}$$

(b) To compute the tension T_2 in the lower string, subtract equation [1] above from equation [2] to obtain

$$2T_2 = m\left(\frac{v^2}{r\sin\theta} - \frac{g}{\cos\theta}\right)$$

Thus,

$$T_2 = \frac{(4.00\text{ kg})}{2}\left[\frac{(6.00\text{ m/s}^2)^2}{(1.32\text{ m})\sin 41.4^\circ} - \frac{9.80\text{ m/s}^2}{\cos 41.4^\circ}\right] = \boxed{56.4\text{ N}}$$

7.72 The maximum lift force is $(F_L)_{\max} = Cv^2$, where $C = 0.018\text{ N}\cdot\text{s}^2/\text{m}^2$ and v is the flying speed. For the bat to stay aloft, the vertical component of the lift force must equal the weight, or $F_L\cos\theta = mg$ where θ is the banking angle. The horizontal component of this force supplies the centripetal acceleration needed to make a turn, or $F_L\sin\theta = m(v^2/r)$ where r is the radius of the turn.

(a) To stay aloft while flying at minimum speed, the bat must have $\theta = 0$ (to give $\cos\theta = (\cos\theta)_{\max} = 1$) and also use the maximum lift force possible at that speed. That is, we need

$$(F_L)_{\max}(\cos\theta)_{\max} = mg, \qquad \text{or} \qquad Cv_{\min}^2(1) = mg$$

Thus, we see that minimum flying speed is

$$v_{\min} = \sqrt{\frac{mg}{C}} = \sqrt{\frac{(0.031\text{ kg})(9.8\text{ m/s}^2)}{0.018\text{ N}\cdot\text{s}^2/\text{m}^2}} = \boxed{4.1\text{ m/s}}$$

(b) To maintain horizontal flight while banking at the maximum possible angle, we must have $(F_L)_{\max}\cos\theta_{\max} = mg$, or $Cv^2\cos\theta_{\max} = mg$. For $v = 10\text{ m/s}$, this yields

$$\cos\theta_{\max} = \frac{mg}{Cv^2} = \frac{(0.031\text{ kg})(9.8\text{ m/s}^2)}{(0.018\text{ N}\cdot\text{s}^2/\text{m}^2)(10\text{ m/s})^2} = 0.17 \qquad \text{or} \qquad \theta_{\max} = \boxed{80°}$$

(c) The horizontal component of the lift force supplies the centripetal acceleration in a turn, $F_L \sin\theta = mv^2/r$. Thus, the minimum radius turn possible is given by

$$r_{\min} = \frac{mv^2}{(F_L)_{\max}(\sin\theta)_{\max}} = \frac{m\cancel{v^2}}{C\cancel{v^2}\sin\theta_{\max}} = \frac{m}{C\sin\theta_{\max}}$$

where we have recognized that $\sin\theta$ has its maximum value at the largest allowable value of θ. For a flying speed of $v = 10$ m/s, the maximum allowable bank angle is $\theta_{\max} = 80°$ as found in part (b). The minimum radius turn possible at this flying speed is then

$$r_{\min} = \frac{0.031\text{ kg}}{(0.018\text{ N}\cdot\text{s}^2/\text{m}^2)\sin 80.0°} = \boxed{1.7\text{ m}}$$

(d) $\boxed{\text{No}}$. Flying slower actually increases the minimum radius of the achievable turns.

As found in part (c), $r_{\min} = m/C\sin\theta_{\max}$. To see how this depends on the flying speed, recall that the vertical component of the lift force must equal the weight or $F_L\cos\theta = mg$. At the maximum allowable bank angle, cos θ will be a minimum. This occurs when $F_L = (F_L)_{\max} = Cv^2$. Thus, $\cos\theta_{\max} = mg/Cv^2$ and

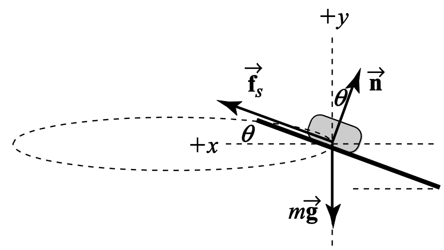
$$\sin\theta_{\max} = \sqrt{1-\cos^2\theta_{\max}} = \sqrt{1-\left(\frac{mg}{Cv^2}\right)^2}$$

This gives the minimum radius turn possible at flying speed v as

$$r_{\min} = \frac{m}{C\sqrt{1-\left(\frac{mg}{Cv^2}\right)^2}}$$

Decreasing the flying speed v will decrease the denominator of this expression, yielding a larger value for the minimum radius of achievable turns.

7.73 The angular speed of the luggage is $\omega = 2\pi/T$ where T is the time for one complete rotation of the carousel. The resultant force acting on the luggage must be directed toward the center of the horizontal circular path (that is, in the $+x$ direction). The magnitude of this resultant force must be



$$ma_c = m\left(\frac{v_t^2}{r}\right) = mr\omega^2$$

Thus,

$$\Sigma F_x = ma_x \Rightarrow f_s\cos\theta - n\sin\theta = ma_c \qquad [1]$$

and

$$\Sigma F_y = ma_y \Rightarrow f_s\sin\theta + n\cos\theta - mg = 0$$

or

$$n = \frac{mg - f_s\sin\theta}{\cos\theta} \qquad [2]$$

Substituting equation [2] into equation [1] gives

$$f_s\cos\theta - mg\tan\theta + f_s\left(\frac{\sin^2\theta}{\cos\theta}\right) = ma_c$$

or

$$f_s = \frac{ma_c + mg\tan\theta}{\cos\theta + \sin^2\theta/\cos\theta} \qquad [3]$$

(a) With $T = 38.0$ s and $r = 7.46$ m, we find that

$$\omega = 0.165 \text{ rad/s} \text{ and } ma_c = mr\omega^2 = (30.0 \text{ kg})(7.46 \text{ m})(0.165 \text{ rad/s})^2 = 6.09 \text{ N}$$

Equation [3] then gives the friction force as

$$f_s = \frac{6.09\text{ N} + (30.0\text{ kg})(9.80\text{ m/s}^2)\tan 20.0°}{\cos 20.0° + \dfrac{\sin^2 20.0°}{\cos 20.0°}} = \frac{113\text{ N}}{1.06} = \boxed{107\text{ N}}$$

(b) If $T = 34.0$ s and $r = 7.46$ m, then $\omega = 0.185$ rad/s and

$$ma_c = mr\omega^2 = (30.0\text{ kg})(7.94\text{ m/s}^2)(0.185\text{ rad/s})^2 = 8.15\text{ N}$$

From equation [1],

$$f_s = \frac{8.15\text{ N} + (30.0\text{ kg})(9.80\text{ m/s}^2)\tan 20.0°}{\cos 20.0° + \dfrac{\sin^2 20.0°}{\cos 20.0°}} = \frac{115\text{ N}}{1.06} = 108\text{ N}$$

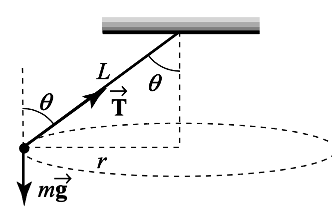
while equation [2] yields

$$n = \frac{(30.0\text{ kg})(9.80\text{ m/s}^2) - (108\text{ N})\sin 20.0°}{\cos 20.0°} = 273\text{ N}$$

Since the luggage is on the verge of slipping, $f_s = (f_s)_{\text{max}} = \mu_s n$ and the coefficient of static friction must be

$$\mu_s = \frac{f_s}{n} = \frac{108\text{ N}}{273\text{ N}} = \boxed{0.396}$$

7.74 The horizontal component of the tension in the cord is the only force directed toward the center of the circular path, so it must supply the centripetal acceleration. Thus,



$$T\sin\theta = m\left(\frac{v_t^2}{r}\right) = m\left(\frac{v_t^2}{L\sin\theta}\right)$$

or

$$T\sin^2\theta = \frac{m\,v_t^2}{L} \qquad [1]$$

Also, the vertical component of the tension must support the weight of the ball, or

$$T\cos\theta = m\,g \qquad [2]$$

(a) Dividing equation [1] by [2] gives

$$\frac{\sin^2\theta}{\cos\theta} = \frac{v_t^2}{L\,g}$$

or

$$v_t = \sin\theta\sqrt{\frac{L\,g}{\cos\theta}} \qquad [3]$$

With $L = 1.5$ m/s and $\theta = 30°$,

$$v_t = \sin 30°\sqrt{\frac{(1.5\text{ m})(9.8\text{ m/s}^2)}{\cos 30°}} = \boxed{2.1\text{ m/s}}$$

(b) From equation [3], with $\sin^2\theta = 1 - \cos^2\theta$, we find

$$\frac{1-\cos^2\theta}{\cos\theta} = \frac{v_t^2}{L\,g} \qquad \text{or} \qquad \cos^2\theta + \left(\frac{v_t^2}{L\,g}\right)\cos\theta - 1 = 0$$

Solving this quadratic equation for $\cos\theta$ gives

$$\cos\theta = -\left(\frac{v_t^2}{2\,L\,g}\right) \pm \sqrt{\left(\frac{v_t^2}{2\,L\,g}\right)^2 + 1}$$

If $L = 1.5$ m and $v_t = 4.0$ m/s, this yields solutions: $\cos\theta = -1.7$ (which is impossible), and $\cos\theta = +1.7$ (which is possible).

Thus, $\theta = \cos^{-1}(0.59) = \boxed{54°}$

(c) From equation [2], when $T = 9.8$ N and the cord is about to break, the angle is

$$\theta = \cos^{-1}\left(\frac{m\,g}{T}\right) = \cos^{-1}\left(\frac{(0.50\text{ kg})(9.8\text{ m/s}^2)}{9.8\text{ N}}\right) = 60°$$

Then equation [3] gives

$$v_t = \sin\theta\sqrt{\frac{L\,g}{\cos\theta}} = \sin 60°\sqrt{\frac{(1.5\text{ m})(9.8\text{ m/s}^2)}{\cos 60°}} = \boxed{4.7\text{ m/s}}$$

7.75 The normal force exerted on the person by the cylindrical wall must provide the centripetal acceleration, so $n = m\,(r\,\omega^2)$.

If the minimum acceptable coefficient of friction is present, the person is on the verge of slipping and the maximum static friction force equals the person's weight, or $(f_s)_{\max} = (\mu_s)_{\min}\, n = mg$.

Thus,

$$(\mu_s)_{\min} = \frac{mg}{n} = \frac{g}{r\,\omega^2} = \frac{9.80\ \text{m/s}^2}{(3.00\ \text{m})(5.00\ \text{rad/s})^2} = \boxed{0.131}$$

7.76 If the block will just make it through the top of the loop, the force required to produce the centripetal acceleration at point C must equal the block's weight, or . $m\,(v_c^2/R) = m\,g$

This gives $v_c = \sqrt{R\,g}$, as the required speed of the block at point C.

We apply the work–energy theorem in the form

$$W_{nc} = (\,KE + PE_{\,g}\,PE_s)_f - (\,KE + PE_{\,g}\,PE_{s.})_i$$

from when the block is first released until it reaches point C to obtain

$$f_k\left(\overline{AB}\right)\cos 180° = \frac{1}{2}m v_c^2 + mg(2\,R) + 0 - 0 - 0 - \frac{1}{2}kd^2$$

The friction force is $f_k = u_k\,(mg)$, and for minimum initial compression of the spring, $v_c^2 = Rg$ as found above. Thus, the work–energy equation reduces to

$$d_{\min} = \sqrt{\frac{2\,\mu_k mg\left(\overline{AB}\right) + mRg + 2\,mg(2\,R)}{k}} = \sqrt{\frac{mg\left(2\,\mu_k\,\overline{AB} + 5\,R\right)}{k}}$$

$$d_{\min} = \sqrt{\frac{(0.50\ \text{kg})(9.8\ \text{m/s}^2)\left[2(0.30)(2.5\ \text{m}) + 5(1.5\ \text{m})\right]}{78.4\ \text{N/m}}} = \boxed{0.75\ \text{m}}$$