ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. Earth moves 2π radians around the Sun in 1 year. The average angular speed is then

$$\omega_{\text{av}} = \frac{2\pi \text{ rad}}{1 \text{ y}}\left(\frac{1 \text{ y}}{3.156 \times 10^7 \text{ s}}\right) = 1.99 \times 10^{-7} \text{ rad/s}$$

which is choice (e).

2. The angular displacement will be

$$\Delta\theta = \omega_{\text{av}} \cdot \Delta t = \left(\frac{\omega_f + \omega_i}{2}\right)\Delta t = \left(\frac{12.00 \text{ rad/s} + 4.00 \text{ rad/s}}{2}\right)(4.00 \text{ s}) = 32.0 \text{ rad}$$

which matches choice (d).

3. The wheel has a radius of 0.500 m and made 320 revolutions. The distance traveled is

$$s = r\theta = (0.500 \text{ m})(320 \text{ rev})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 1.00 \times 10^3 \text{ m} = 1.00 \text{ km}$$

so choice (c) is the correct answer.

4. At the top of the circular path, both the tension in the string and the gravitational force act downward, toward the center of the circle, and together supply the needed centripetal force. Thus, $F_c = T + mg = mr\omega^2$ or

$$T = m\left(r\omega^2 - g\right) = (0.400 \text{ kg})\left[(0.500 \text{ m})(8.00 \text{ rad/s})^2 - 9.80 \text{ m/s}^2\right] = 8.88 \text{ N}$$

making (a) the correct choice for this question.

5. The required centripetal force is $F_c = ma_c = mv^2/r = mr\omega^2$. When are both constant, the centripetal force is directly proportional to the radius of the circular path. Thus, as the rider moves toward the center of the merry-go-round, the centripetal force decreases and the correct choice is (c).

6. Any object moving in a circular path undergoes a constant change in the direction of its velocity. This change in the direction of velocity is an acceleration, always directed toward the center of the path, called the centripetal acceleration, $a_c = v^2/r = r\omega^2$. The tangential speed of the object is $v_t = r\omega$, where ω is the angular velocity. If ω is not constant, the object will have both an angular acceleration, $\alpha_{\text{av}} = \Delta\omega/\Delta t$, and a tangential acceleration, $a_t = r\alpha$. The only untrue statement among the listed choices is (b). Even when ω is constant, the object still has centripetal acceleration.

7. According to Newton's law of universal gravitation, the gravitational force one body exerts on the other decreases as the distance separating the two bodies increases. When on Earth's surface, the astronaut's distance from the center of the Earth is Earth's radius $r_0 = R_E$. If h is the altitude at which the station orbits above the surface, her distance from Earth's center when on the station is $r' = R_E + h > r_0$. Thus, she experiences a smaller force while on the space station and (c) is the correct choice.

8. The mass of a spherical body of radius R and density ρ is $M = \rho V = \rho(4\pi R^3/3)$. The escape velocity from the surface of this body may then be written in either of the following equivalent forms:

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} \qquad \text{and} \qquad v_{\text{esc}} = \sqrt{\frac{2G}{R}\left(\frac{4\pi\rho R^3}{3}\right)} = \sqrt{\frac{8\pi\rho G R^2}{3}}$$

We see that the escape velocity depends on the three properties (mass, density, and radius) of the planet. Also, the weight of an object on the surface of the planet is $F_g = mg = GMm/R^2$, giving

$$g = GM/R^2 = \frac{G}{R^2}\left[\rho\left(\frac{4\pi R^3}{3}\right)\right] = \frac{4}{3}\pi\rho G R$$

The acceleration of gravity at the planet surface then depends on the same properties as does the escape velocity. Changing the value of g would necessarily change the escape velocity. Of the listed quantities, the only one that does not affect the escape velocity is choice (e), the mass of the object on the planet's surface.

9. The satellite experiences a gravitational force, always directed toward the center of its orbit, and supplying the centripetal force required to hold it in its orbit. This force gives the satellite a centripetal acceleration, even if it is moving with constant angular speed. At each point on the circular orbit, the gravitational force is directed along a radius line of the path, and is perpendicular to the motion of the satellite, so this force does no work on the satellite. Therefore, the only true statement among the listed choices is (d).

10. In a circular orbit, the gravity force is always directed along a radius line of the circle, and hence, perpendicular to the object's velocity which is tangential to the circle. In an elliptical orbit, the gravity force is always directed toward the center of the Earth, located at one of the foci of the orbit. This means that it is perpendicular to the velocity, which is always tangential to the orbit, only at the two points where the object crosses the major axis of the ellipse. These are the points where the object is nearest to and farthest from Earth. Since the gravity force is a conservative force, the total energy (kinetic plus gravitational potential energy) of the object is constant as it moves around the orbit. This means that it has maximum kinetic energy (and hence, greatest speed) when its potential energy is lowest (i.e., when it is closest to Earth. The only true statements among the listed choices are (a) and (b).

11. The weight of an object of mass m at the surface of a spherical body of mass M and radius R is $F_g = mg = GMm/R^2$. Thus, the acceleration of gravity at the surface is $g = GM/R^2$.

For Earth,

$$g_E = \frac{GM_E}{R_E^2}$$

and for the planet,

$$g_p = \frac{GM_p}{R_p^2} = \frac{G(2M_E)}{(2R_E)^2} = \frac{1}{2}\left(\frac{GM_E}{R_E^2}\right) = \frac{1}{2}g_E = 0.5g_E$$

meaning that choice (b) is the correct response.

12. The total gravitational potential energy of this set of 4 particles is the sum of the gravitational energies of each distinct pair of particles in the set of four. There are six distinct pairs in a set of four particles, which are: 1 & 2, 1 & 3, 1 & 4, 2 & 3, 2 & 4, and 3 & 4. Therefore, the correct answer to this question is (b).

13. We assume that the elliptical orbit is so elongated that Sun, at one foci, is almost at one end of the major axis. If the period, T, is expressed in years and the semi-major axis, a, in astronomical units (AU), Kepler's third law states that $T^2 = a^3$. Thus, for Halley's comet, with a period of $T = 76$ y, the semi-major axis of its orbit is

$$a = \sqrt[3]{(76)^2} = 18 \text{ AU}$$

The length of the major axis, and the approximate maximum distance from the Sun, is $2a = 36$ AU, making the correct answer for this question choice (e).