PROBLEM SOLUTIONS

6.1 Use $p = m\upsilon$

(a) $p = (1.67 \times 10^{-27}\text{ kg})(5.00 \times 10^{6}\text{ m/s}) = \boxed{8.35 \times 10^{-21}\text{ kg m/s}}$

(b) $p = (1.50 \times 10^{-2}\text{ kg})(3.00 \times 10^{2}\text{ m/s}) = \boxed{4.50\text{ kg m/s}}$

(c) $p = (75.0\text{ kg})(10.0\text{ m/s}) = \boxed{750\text{ kg m/s}}$

(d) $p = (5.98 - 10^{24}\text{ kg})(2.98 \times 10^{4}\text{ m/s}) = \boxed{1.78 \times 10^{29}\text{ kg m/s}}$

6.2 From the impulse–momentum theorem,

$$F_{\text{av}}\left(\Delta t\right) = \Delta p = mv_f - mv_i$$

Thus,

$$F_{\text{av}} = \frac{m\left(v_f - v_i\right)}{\left(\Delta t\right)} = \frac{\left(55 \times 10^{-3}\text{ kg}\right)\left(2.0 \times 10^{2}\text{ ft/s} - 0\right)}{0.0020\text{ s} - 0}\left(\frac{1\text{ m/s}}{3.281\text{ ft/s}}\right) = \boxed{1.7\text{ kN}}$$

6.3 (a) If $p_{\text{ball}} = p_{\text{bullet}}$, then

$$v_{\text{ball}} = \frac{m_{\text{bullet}}v_{\text{bullet}}}{m_{\text{ball}}} = \frac{\left(3.00 \times 10^{-3}\text{ kg}\right)\left(1.50 \times 10^{3}\text{ m/s}\right)}{0.145\text{ kg}} = \boxed{31.0\text{ m/s}}$$

(b) The kinetic energy of the bullet is

$$KE_{\text{bullet}} = \frac{1}{2}m_{\text{bullet}}v_{\text{bullet}}^2 = \frac{\left(3.00 \times 10^{-3}\text{ kg}\right)\left(1.50 \times 10^{3}\text{ m/s}\right)^2}{2} = 3.38 \times 10^{3}\text{ J}$$

while that of the baseball is

$$KE_{\text{ball}} = \frac{1}{2}m_{\text{ball}}v_{\text{ball}}^2 = \frac{\left(0.145\text{ kg}\right)\left(31.0\text{ m/s}\right)^2}{2} = 69.7\text{ J}$$

The $\boxed{\text{bullet has the larger kinetic energy}}$ by a factor of 48.4.

6.4 (a) Since the ball was thrown straight upward, it is at rest momentarily ($\upsilon = 0$) at its maximum height. Therefore, $p = \boxed{0}$.

(b) The maximum height is found from $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$ with $\upsilon_y = 0$.

$0 = v_{0y}^2 + 2(-g)(\Delta y)_{\max}$. Thus,

$$(\Delta y)_{\max} = \frac{v_{0y}^2}{2g}$$

We need the velocity at $\Delta y = (\Delta y)_{\max}/2 = v_{0y}^2/4g$; thus $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$ gives

$$v_y^2 = v_{0y}^2 + 2(-g)\left(\frac{v_{0y}^2}{4g}\right) = \frac{v_{0y}^2}{2}, \qquad \text{or} \qquad v_y = \frac{v_{0y}}{\sqrt{2}} = \frac{15 \text{ m/s}}{\sqrt{2}}$$

Therefore,

$$p = mv_y = \frac{(0.10 \text{ kg})(15 \text{ m/s})}{\sqrt{2}} = \boxed{1.1 \text{ kg} \cdot \text{m/s}} \text{ upward}$$

6.5 (a) $I = F_{\text{av}}(\Delta t) = |\Delta p| = m|v_f - v_i| = (84.0 \text{ kg})|0 - 6.70 \text{ m/s}| = \boxed{563 \text{ kg} \cdot \text{m/s}}$

(b) $F_{\text{av}} = \dfrac{I}{\Delta t} = \dfrac{563 \text{ kg} \cdot \text{m/s}}{0.750 \text{ s}} = \boxed{751 \text{ N}}$

6.6 $$KE = \frac{mv^2}{2} = \frac{m^2v^2}{2m} = \frac{(mv)^2}{2m} = \boxed{\frac{p^2}{2m}}$$

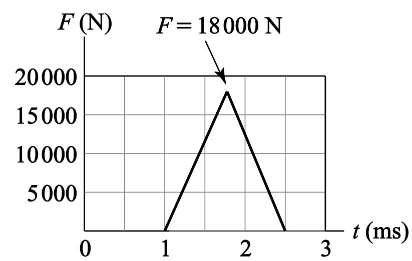
6.7 From problem 6.6, $KE = p^2/2m$, and hence, $p = \sqrt{2m(KE)}$. Thus,

$$m = \frac{p^2}{2 \cdot KE} = \frac{(25.0 \text{ kg} \cdot \text{m/s})^2}{2(275 \text{ J})} = \boxed{1.14 \text{ kg}}$$

and

$$v = \frac{p}{m} = \frac{\sqrt{2m(KE)}}{m} = \sqrt{\frac{2(KE)}{m}} = \sqrt{\frac{2(275\text{ J})}{1.14\text{ kg}}} = \boxed{22.0\text{ m/s}}$$

6.8 (a) The impulse delivered by a force is equal to the area under the force versus time curve. From the figure at the right, this is seen to be a triangular area having a base of 1.50 ms = of 1.50×10^{-3} s and altitude of 18 000 N. Thus,



$$I = \frac{1}{2}\left(1.50 \times 10^{-3}\text{ s}\right)\left(18\ 000\text{ N}\right) = \boxed{13.5\text{ N}\cdot\text{s}}$$

(b) $$F_{av} = \frac{I}{\Delta t} = \frac{13.5\text{ N}\cdot\text{s}}{1.50 \times 10^{-3}\text{ s}} = 9.00 \times 10^{3}\text{ N} = \boxed{9.00\text{ kN}}$$

6.9 (a) We choose the positive direction to be the direction of the final velocity of the ball.

$$I = \Delta p = m\left(v_f - v_i\right) = \left(0.280\text{ kg}\right)\left[+22.0\text{ m/s} - \left(-15.0\text{ m/s}\right)\right]$$

or

$$I = +10.4\text{ kg}\cdot\text{m/s} = \boxed{10.4\text{ kg}\cdot\text{m/s in the direction of the final velocity}}$$

(b) The average force the player exerts on the ball is

$$F_{av} = \frac{I}{\Delta t} = \frac{10.4\text{ kg}\cdot\text{m/s}}{0.060\ 0\text{ s}} = \boxed{173\text{ N}}$$

By Newton's third law, the ball exerts a force of equal magnitude back on the player's fist.

6.10 (a) $\left|F_{av}\right| = \dfrac{|I|}{\Delta t}$, where I is the impulse the man must deliver to the child: $|I| = m_{\text{child}}\left|v_f - v_0\right|$.

$$\left|F_{av}\right| = \frac{m_{\text{child}}\left|v_f - v_0\right|}{\Delta t} = \frac{\left(12.0\text{ kg}\right)\left|0 - 120\text{ mi/h}\right|}{0.10\text{ s}}\left(\frac{0.447\text{ m/s}}{1\text{ mi/h}}\right) = \boxed{6.4 \times 10^{3}\text{ N}}$$

or

$$\left|F_{\text{av}}\right| = \left(6.4 \times 10^3 \text{ N}\right)\left(\frac{0.224\ 8 \text{ lb}}{1 \text{ N}}\right) = \boxed{1.4 \times 10^3 \text{ lb}}$$

(b) It is unlikely that the man has sufficient arm strength to guarantee the safety of the child during a collision. The violent forces during the collision would tear the child from his arms.

(c) The laws are soundly based on physical principles: always wear a seat belt when in a car.

6.11 The velocity of the ball just before impact is found from $v_y^2 = v_{0y}^2 + 2a_y\Delta y$ as

$$v_1 = -\sqrt{v_{0y}^2 + 2a_y\Delta y} = -\sqrt{0 + 2\left(-9.80 \text{ m/s}^2\right)\left(-1.25 \text{ m}\right)} = -4.95 \text{ m/s}$$

and the rebound velocity with which it leaves the floor is

$$v_2 = +\sqrt{v_f^2 - 2a_y\Delta y} = +\sqrt{0 - 2\left(-9.80 \text{ m/s}^2\right)\left(+0.960 \text{ m}\right)} = +4.34 \text{ m/s}$$

The impulse given the ball by the floor is then

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}\Delta t = \Delta\left(m\vec{\mathbf{v}}\right) = m\left(\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1\right)$$

$$= \left(0.150 \text{ kg}\right)\left[+4.34 \text{ m/s} - \left(-4.95 \text{ m/s}\right)\right] = +1.39 \text{ N}\cdot\text{s} = \boxed{1.39 \text{ N}\cdot\text{s upward}}$$

6.12 Take the direction of the ball's final velocity (toward the net) to be the $+x$-direction.

(a) $I = \Delta p = m\left(v_f - v_i\right) = \left(0.0600 \text{ kg}\right)\left[40.0 \text{ m/s} - \left(-50.0 \text{ m/s}\right)\right]$, giving

$$I = +5.40 \text{ kg}\cdot\text{m/s} = \boxed{5.40 \text{ N}\cdot\text{s toward the net}}$$

(b) $$Work = \Delta KE = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

$$= \frac{\left(0.0600 \text{ kg}\right)\left[\left(40.0 \text{ m/s}\right)^2 - \left(50.0 \text{ m/s}\right)^2\right]}{2} = \boxed{-27.0 \text{ J}}$$

6.13 $I = F_{\text{av}}\left(\Delta t\right) = \Delta p = m\left(\Delta v\right)$

Thus, $|I| = m|\Delta v| = (70.0\text{ kg})(5.20\text{ m/s} - 0) = \boxed{364\text{ kg}\cdot\text{m/s}}$, and

$$F_{\text{av}} = \frac{I}{\Delta t} = \frac{364\text{ kg}\cdot\text{m/s}}{0.832\text{ s}} = 438\text{ kg}\cdot\text{m/s}^2$$

or

$$\vec{\mathbf{F}}_{\text{av}} = \boxed{438\text{ N directed forward}}$$

6.14 Choose toward the east as the positive direction.

(a) The impulse delivered to the ball as it is caught is

$$\vec{\mathbf{I}} = \Delta\vec{\mathbf{p}} = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i = 0 - (0.500\text{ kg})(+15.0\text{ m/s}) = -7.50\text{ kg}\cdot\text{m/s}$$

or

$$\vec{\mathbf{I}} = \boxed{7.50\text{ kg}\cdot\text{m/s westward}}$$

(b) The average force exerted by the ball on the receiver is the negative of the average force exerted by the receiver on the ball, or

$$\left(\vec{\mathbf{F}}_{\text{av}}\right)_{\text{receiver}} = -\left(\vec{\mathbf{F}}_{\text{av}}\right)_{\text{ball}} = -\frac{\vec{\mathbf{I}}}{\Delta t} = -\left(\frac{-7.50\text{ kg}\cdot\text{m/s}}{0.0200\text{ s}}\right) = +375\text{ N}$$

$$\left(\vec{\mathbf{F}}_{\text{av}}\right)_{\text{receiver}} = \boxed{375\text{ N eastward}}$$

6.15 (a) The impulse equals the area under the F versus t graph. This area is the sum of the area of the rectangle plus the area of the triangle. Thus,

$$I = (2.0\text{ N})(3.0\text{ s}) + \frac{1}{2}(2.0\text{ N})(2.0\text{ s}) = \boxed{8.0\text{ N}\cdot\text{s}}$$

(b) $I = F_{\text{av}}(\Delta t) = \Delta p = m(v_f - v_i)$

$$8.0\text{ N}\cdot\text{s} = (1.5\text{ kg})v_f - 0,\quad\text{giving}\quad v_f = \boxed{5.3\text{ m/s}}$$

(c) $I = F_{av}(\Delta t) = \Delta p = m\,(v_f - v_i)$, so $v_f = v_i + \dfrac{I}{m}$

$$v_f = -2.0 \text{ m/s} + \frac{8.0 \text{ N}\cdot\text{s}}{1.5 \text{ kg}} = \boxed{3.3 \text{ m/s}}$$

6.16 (a) Impulse = area under curve = (two triangular areas of altitude 4.00 N and base 2.00 s) + (one rectangular area of width 1.00 s and height of 4.00 N.) Thus,

$$I = 2\left[\frac{(4.00 \text{ N})(2.00 \text{ s})}{2}\right] + (4.00 \text{ N})(1.00 \text{ s}) = \boxed{12.0 \text{ N}\cdot\text{s}}$$

(b) $I = F_{av}(\Delta t) = \Delta p = m\left(v_f - v_i\right)$, so $v_f = v_i + \dfrac{I}{m}$

$$v_f = 0 + \frac{12.0 \text{ N}\cdot\text{s}}{2.00 \text{ kg}} = \boxed{6.00 \text{ m/s}}$$

(c) $v_f = v_i + \dfrac{I}{m} = -2.00 \text{ m/s} + \dfrac{12.0 \text{ N}\cdot\text{s}}{2.00 \text{ kg}} = \boxed{4.00 \text{ m/s}}$

6.17 (a) The impulse is the area under the curve between 0 and 3.0 s. This is

$$I = (4.0 \text{ N})(3.0 \text{ s}) = \boxed{12 \text{ N}\cdot\text{s}}$$

(b) The area under the curve between 0 and 5.0 s is

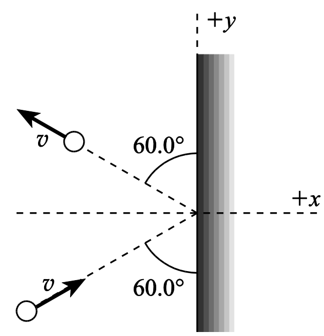
$$I = (4.0 \text{ N})(3.0 \text{ s}) + (-2.0 \text{ N})(2.0 \text{ s}) = \boxed{8.0 \text{ N}\cdot\text{s}}$$

(c) $I = F_{av}(\Delta t) = \Delta p = m\left(v_f - v_i\right)$, so $v_f = v_i + \dfrac{I}{m}$

At 3.0 s: $v_f = v_i + \dfrac{I}{m} = 0 + \dfrac{12 \text{ N}\cdot\text{s}}{1.50 \text{ kg}} = \boxed{8.0 \text{ m/s}}$

At 5.0 s: $v_f = v_i + \dfrac{I}{m} = 0 + \dfrac{8.0\ \text{N} \cdot \text{s}}{1.50\ \text{kg}} = \boxed{5.3\ \text{m/s}}$

6.18 $\vec{\mathbf{F}}_{\text{av}} = \dfrac{\Delta\vec{\mathbf{p}}}{\Delta t}$ so $\left(F_{\text{av}}\right)_x = \dfrac{\Delta p_x}{\Delta t}$ $\left(F_{\text{av}}\right)_y = \dfrac{\Delta p_y}{\Delta t}$



$$\left(F_{\text{av}}\right)_y = \frac{m\left[\left(v_y\right)_f - \left(v_y\right)_i\right]}{\Delta t} = \frac{m\left[v\cos 60.0° - v\cos 60.0°\right]}{\Delta t} = 0$$

$$\left(F_{\text{av}}\right)_x = \frac{m\left[\left(\upsilon_x\right)_f - \left(\upsilon_x\right)_i\right]}{\Delta t} = \frac{m\left[\left(-\upsilon\sin 60.0°\right) - \left(+\upsilon\sin 60.0°\right)\right]}{\Delta t}$$

$$= \frac{-2mv\sin 60.0°}{\Delta t} = \frac{-2\left(3.00\ \text{kg}\right)\left(10.0\ \text{m/s}\right)\sin 60.0°}{0.200\ \text{s}}$$

$$= -260\ \text{N}$$

$\vec{\mathbf{F}}_{\text{av}} = \boxed{260\ \text{N in the negative } x\text{-direction or perpendicular to the wall}}$

6.19 (a) $\Delta t = \dfrac{\Delta x}{v_{\text{av}}} = \dfrac{2\left(\Delta x\right)}{v_f + v_i} = \dfrac{2\left(1.20\ \text{m}\right)}{0 + 25.0\ \text{m/s}} = \boxed{9.60 \times 10^{-2}\ \text{s}}$

(b) $F_{\text{av}} = \dfrac{\Delta p}{\Delta t} = \dfrac{m\left(\Delta v\right)}{\Delta t} = \dfrac{\left(1\,400\ \text{kg}\right)\left(25.0\ \text{m/s}\right)}{9.60 \times 10^{-2}\ \text{s}} = \boxed{3.65 \times 10^5\ \text{N}}$

(c) $a_{\text{av}} = \dfrac{\Delta v}{\Delta t} = \dfrac{25.0\ \text{m/s}}{9.60 \times 10^{-2}\ \text{s}} = 260\ \text{m/s}^2 = \left(260\ \text{m/s}^2\right)\left(\dfrac{1\ g}{9.80\ \text{m/s}^2}\right) = \boxed{26.6\ g}$

6.20 Choose the positive direction to be from the pitcher toward home plate.

(a) $\vec{\mathbf{I}} = \vec{\mathbf{F}}_{\text{av}}\left(\Delta t\right) = \Delta\vec{\mathbf{p}} = m\left(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i\right) = \left(0.15\ \text{kg}\right)\left[\left(-22\ \text{m/s}\right) - \left(20\ \text{m/s}\right)\right]$

$\vec{\mathbf{I}} = \vec{\mathbf{F}}_{\text{av}}\left(\Delta t\right) = -6.3\ \text{kg} \cdot \text{m/s}$ or $\boxed{6.3\ \text{kg} \cdot \text{m/s toward the pitcher}}$

(b) $\vec{\mathbf{F}}_{\text{av}} = \dfrac{\vec{\mathbf{I}}}{\Delta t} = \dfrac{-6.3\ \text{kg} \cdot \text{m/s}}{2.0 \times 10^{-3}\ \text{s}} = -3.2 \times 10^3\ \text{N}$

or

$$\vec{\mathbf{F}}_{\text{av}} = \boxed{3.2 \times 10^3 \text{ N toward the pitcher}}$$

6.21 Requiring that total momentum be conserved gives

$$\left(m_{\text{club}}\upsilon_{\text{club}} + m_{\text{ball}}\upsilon_{\text{ball}}\right)_f = \left(m_{\text{club}}\upsilon_{\text{club}} + m_{\text{ball}}\upsilon_{\text{ball}}\right)_i$$

or

$$(200 \text{ g})(40 \text{ m/s}) + (46 \text{ g})v_{\text{ball}} = (200 \text{ g})(55 \text{ m/s}) + 0$$

and

$$v_{\text{ball}} = \boxed{65 \text{ m/s}}$$

6.22 (a) The mass of the rifle is

$$m = \frac{w}{g} = \frac{30 \text{ N}}{9.80 \text{ m/s}^2} = \left(\frac{30}{9.8}\right) \text{ kg}$$

We choose the direction of the bullet's motion to be negative. Then, conservation of momentum gives

$$\left(m_{\text{rifle}}v_{\text{rifle}} + m_{\text{bullet}}v_{\text{bullet}}\right)_f = \left(m_{\text{rifle}}v_{\text{rifle}} + m_{\text{bullet}}v_{\text{bullet}}\right)_i$$

or

$$\left[(30/9.8) \text{ kg}\right]v_{\text{rifle}} + \left(5.0 \times 10^{-3} \text{ kg}\right)(-300 \text{ m/s}) = 0 + 0$$

and

$$v_{\text{rifle}} = \frac{9.8\left(5.0 \times 10^{-3} \text{ kg}\right)(300 \text{ m/s})}{30 \text{ kg}} = \boxed{0.49 \text{ m/s}}$$

(b) The mass of the man plus rifle is

$$m = \frac{730 \text{ N}}{9.80 \text{ m/s}^2} = 74.5 \text{ kg}$$

We use the same approach as in (a), to find

$$v = \left(\frac{5.0\times10^{-3}\ \text{kg}}{74.5\ \text{kg}}\right)(300\ \text{m/s}) = \boxed{2.0\times10^{-2}\ \text{m/s}}$$

6.23 The velocity of the girl relative to the ice, υ_{GI}, is $\upsilon_{GI} = \upsilon_{GP} + \upsilon_{PI}$ where υ_{GP} = velocity of girl relative to plank, and υ_{PI} = velocity of plank relative to ice. Since we are given that υ_{GP} = 1.50 m/s, this becomes

$$v_{GI} = 1.50\ \text{m/s} + v_{PI} \qquad [1]$$

(a) Conservation of momentum gives

$$m_G v_{GI} + m_P v_{PI} = 0 \text{ or } v_{PI} = -\left(\frac{m_G}{m_P}\right) v_{GI} \qquad [2]$$

Then, Equation [1] becomes

$$v_{GI} = 1.50\ \text{m/s} - \left(\frac{m_G}{m_P}\right) v_{GI} \text{ or } \left(1 + \frac{m_G}{m_P}\right) v_{GI} = 1.50\ \text{m/s}$$

giving

$$v_{GI} = \frac{1.50\ \text{m/s}}{1 + \left(\frac{45.0\ \text{kg}}{150\ \text{kg}}\right)} = \boxed{1.15\ \text{m/s}}$$

(b) Then, using [2] above,

$$v_{PI} = -\left(\frac{45.0\ \text{kg}}{150\ \text{kg}}\right)(1.15\ \text{m/s}) = -0.346\ \text{m/s}$$

or $v_{PI} = \boxed{0.346\ \text{m/s directed opposite to the girl's motion}}$.

6.24 We shall choose southward as the positive direction.

The mass of the man is

$$m = \frac{w}{g} = \frac{730\ \text{N}}{9.80\ \text{m/s}^2} = 74.5\ \text{kg}$$

Then, from conservation of momentum,

we find

$$\left(m_{\text{man}}v_{\text{man}} + m_{\text{book}}v_{\text{book}}\right)_f = \left(m_{\text{man}}v_{\text{man}} + m_{\text{book}}v_{\text{book}}\right)_i$$

or

$$(74.5\text{ kg})\,\upsilon_{\text{man}} + (1.2\text{ kg})(-5.0\text{ m/s}) = 0 + 0 \qquad \text{and} \qquad v_{\text{man}} = 8.1\times 10^{-2}\text{ m/s}$$

Therefore, the time required to travel the 5.0 m to shore is

$$t = \frac{\Delta x}{\upsilon_{man}} = \frac{5.0\text{ m}}{8.1\times 10^{-2}\text{ m/s}} = \boxed{62\text{ s}}$$

6.25 (a) Using subscript a for the astronaut and t for the tank, conservation of momentum gives $m_a\upsilon_{af} + m_t\upsilon_{tf} = m_a\upsilon_{ai} + m_t\upsilon_{ti}$. Since both astronaut and tank were initially at rest, this becomes

$$m_a v_{af} + m_t v_{tf} = 0 + 0 \text{ or } v_{af} = -\left(\frac{m_t}{m_a}\right)v_{tf}$$

The mass of the astronaut alone (after the oxygen tank has been discarded) is $m_a = 75.0$ kg. Taking toward the spacecraft as the positive direction, the velocity imparted to the astronaut is

$$v_{af} = -\left(\frac{12.0\text{ kg}}{75.0\text{ kg}}\right)(-8.00\text{ m/s}) = +1.28\text{ m/s}$$

and the distance she will move in 2.00 min is

$$d = v_{af}t = (1.28\text{ m/s})(120\text{ s}) = \boxed{154\text{ m}}$$

(b) By Newton's third law, when the astronaut exerts a force on the tank, the tank exerts a force back on the astronaut. This reaction force accelerates the astronaut towards the spacecraft.

6.26 (a) Using subscript c for the (flatcar + cannon) and p for the projectile, conservation of momentum in the horizontal direction gives $m_c\left(v_{cf}\right)_x + m_p\left(v_{pf}\right)_x = m_c\left(v_{ci}\right)_x + m_p\left(v_{pi}\right)_x$. Assuming that the flatcar, cannon, and projectile were initially at rest, $v_{ci} = v_{pi} = 0$, giving the initial recoil speed of the (flatcar + cannon) as

$$\left|\left(v_{cf}\right)_x\right| = \frac{m_p}{m_c}\left|\left(v_{pf}\right)_x\right| = \left(\frac{1.00\text{ ton}}{36.0\text{ ton}}\right)\left(1.00\times10^3\text{ m/s}\right)\cos 30.0^\circ = \boxed{24.1\text{ m/s}}$$

(b) The flatcar, cannon, and Earth undergo a change in momentum in the *y*-direction that is equal an opposite the vertical component of momentum imparted to the projectile. Because of the great mass of Earth, however, the effect goes unnoticed.

6.27 Consider the thrower first, with velocity after the throw of v_{thrower}. Applying conservation of momentum yields

$$\left(65.0\text{ kg}\right)v_{\text{thrower}} + \left(0.045\,0\text{ kg}\right)\left(30.0\text{ m/s}\right) = \left(65.0\text{ kg} + 0.045\,0\text{ kg}\right)\left(2.50\text{ m/s}\right)$$

or $v_{\text{thrower}} = \boxed{2.48\text{ m/s}}$

Now, consider the (catcher + ball), with velocity of v_{catcher} after the catch. From momentum conservation,

$$\left(60.0\text{ kg} + 0.045\,0\text{ kg}\right)v_{\text{catcher}} = \left(0.045\,0\text{ kg}\right)\left(30.0\text{ m/s}\right) + \left(60.0\text{ kg}\right)\left(0\right)$$

or

$$v_{\text{catcher}} = \boxed{2.25\times10^{-2}\text{ m/s}}$$

6.28 (a) B exerts a horizontal force on A.

(b) A exerts a force on B that is opposite in direction to the force B exerts on A.

(c) The force on A is equal in magnitude to the force on B, but is oppositely directed.

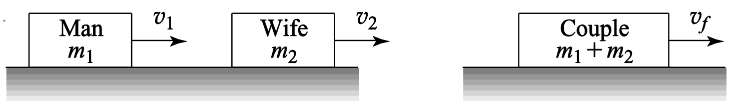
(d) Yes. The momentum of the system (the two skaters) is conserved because the net external force on the system is zero (neglecting friction).

(e) $\left(\Delta p_x\right)_{\text{system}} = \left(\Delta p_x\right)_A + \left(\Delta p_x\right)_B = 0 \quad\Rightarrow\quad m_A\left(v_A - 0\right) + m_B\left(v_B - 0\right) = 0$

$$v_A = -\left(\frac{m_B}{m_A}\right)v_B = -\left(\frac{\cancel{m_B}}{0.900\,\cancel{m_B}}\right)\left(2.00\text{ m/s}\right) = -2.22\text{ m/s}$$

$$\vec{\mathbf{v}}_{\text{A}} = \boxed{2.22\text{ m/s in the direction opposite to }\vec{\mathbf{v}}_B}$$

6.29 (a)



Before impact After impact

(b) The collision is best described as $\boxed{\text{perfectly inelastic}}$, because the skaters remain in contact after the collision.

(c) $\boxed{m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f}$ (d) $\boxed{v_f = \dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2}}$

(e) $$v_f = \frac{(70.0 \text{ kg})(8.00 \text{ m/s}) + (50.0 \text{ kg})(4.00 \text{ m/s})}{70.0 \text{ kg} + 50.0 \text{ kg}} = \boxed{6.33 \text{ m/s}}$$

6.30 Consider a system consisting of arrow and target from the instant just before impact until the instant after the arrow emerges from the target. No external horizontal forces act on the system, so total horizontal momentum must be conserved, or

$$(m_a v_a + m_t v_t)_f = (m_a v_a + m_t v_t)_i$$

Thus,

$$(v_a)_f = \frac{m_a (v_a)_i + m_t (v_t)_i - m_t (v_t)_f}{m_a}$$

$$= \frac{(22.5 \text{ g})(+35.0 \text{ m/s}) + (300 \text{ g})(-2.50 \text{ m/s}) - 0}{22.5 \text{ g}} = \boxed{1.67 \text{ m/s}}$$

6.31 When Gayle jumps on the sled, conservation of momentum gives

$$(50.0 \text{ kg} + 5.00 \text{ kg}) v_2 = (50.0 \text{ kg})(4.00 \text{ m/s}) + 0$$

or the speed of Gayle and the sled as they start down the hill is $\upsilon_2 = 3.64$ m/s.

After Gayle and the sled glide down 5.00 m, conservation of mechanical energy (taking $y = 0$ at the level of the top of the hill) gives

$$\frac{1}{2}(55.0 \text{ kg}) v_3^2 + (55.0 \text{ kg})(9.80 \text{ m/s}^2)(-5.00 \text{ m}) = \frac{1}{2}(55.0 \text{ kg})(3.64 \text{ m/s})^2 + 0$$

so Gayle's speed just before the brother hops on is $\upsilon_3 = 10.5$ m/s.

After her Brother jumps on, conservation of momentum yields

$$(55.0\text{ kg} + 30.0\text{ kg})v_4 = (55.0\text{ kg})(10.50\text{ m/s}) + 0$$

and the speed of Gayle, brother, and sled just after brother hops on is $\upsilon_4 = 6.82$ m/s.

After all slide an additional 10.0 m down (to a level 15.0 m below the level of the hilltop), conservation of mechanical energy from just after brother hops on to the end gives the final speed as

$$\frac{1}{2}\cancel{(85.0\text{ kg})}v_5^2 + \cancel{(85.0\text{ kg})}(9.80\text{ m/s}^2)(-15.0\text{ m}) = \frac{1}{2}\cancel{(85.0\text{ kg})}(6.82\text{ m/s})^2 + \cancel{(85.0\text{ kg})}(9.80\text{ m/s}^2)(-5.00\text{ m})$$

or $v_5 = \boxed{15.6\text{ m/s}}$

6.32 For ***each*** skater, the impulse–momentum theorem gives

$$|F_{av}| = \frac{|\Delta p|}{\Delta t} = \frac{m|\Delta v|}{\Delta t} = \frac{(75.0\text{ kg})(5.00\text{ m/s})}{0.100\text{ s}} = \boxed{3.75\times 10^3\text{ N}}$$

Since $F_{av} < 4500$ N, there are $\boxed{\text{no broken bones}}$.

6.33 (a) If M is the mass of a single car, conservation of momentum gives

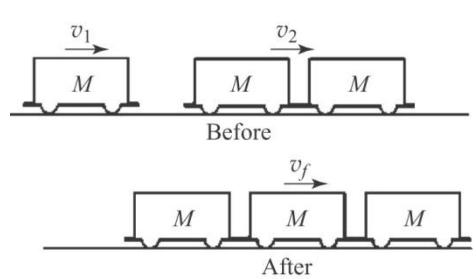
$$(3M)v_f = M(3.00\text{ m/s}) + (2M)(1.20\text{ m/s}), \text{ or } \quad v_f = \boxed{1.80\text{ m/s}}$$

(b) The kinetic energy lost is $KE_{lost} = KE_i - KE_f$, or

$$KE_{lost} = \frac{1}{2}M(3.00\text{ m/s})^2 + \frac{1}{2}(2M)(1.20\text{ m/s})^2 - \frac{1}{2}(3M)(1.80\text{ m/s})^2$$

With $M = 2.00\times 10^4$ kg, this yields $KE_{lost} = \boxed{2.16\times 10^4\text{ J}}$

6.34 (a) From conservation of momentum,



$$(3M)v_f = Mv_1 + (2M)v_2$$

or

$$\boxed{v_f = \frac{1}{3}(v_1 + 2v_2)}$$

(b) The kinetic energy before is

$$KE_i = \frac{1}{2}Mv_1^2 + \frac{1}{2}(2M)v_2^2 = \frac{M}{2}(v_1^2 + 2v_2^2)$$

After collision:

$$KE_f = \frac{1}{2}(3M)v_f^2 = \frac{3M}{2}\left[\frac{(v_1 + 2v_2)^2}{9}\right] = \frac{M}{6}(v_1^2 + 4v_1v_2 + 4v_2^2)$$

or

$$KE_f = \frac{M}{6}v_1^2 + \frac{2M}{3}v_1v_2 + \frac{2M}{3}v_2^2$$

The kinetic energy lost is

$$KE_i - KE_f = \left(\frac{1}{2} - \frac{1}{6}\right)Mv_1^2 + \left(1 - \frac{2}{3}\right)Mv_2^2 - \frac{2}{3}Mv_1v_2$$

or

$$\boxed{KE_i - KE_f = \frac{M}{3}(v_1^2 + v_2^2 - 2v_1v_2) = \frac{M}{3}(v_1 - v_2)^2}$$

6.35 (a) Because momentum is conserved even in a perfectly inelastic collision such as this, the ratio is $\boxed{p_f/p_i = 1}$.

(b) $p_f = p_i \quad \Rightarrow \quad (m_1 + m_2)v_f = m_1v_{1i} + m_2(0) \quad$ or $\quad v_f = \dfrac{m_1v_{1i}}{m_1 + m_2}$

$$KE_i = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2(0) = \frac{1}{2}m_1v_{1i}^2 \qquad \text{and} \qquad KE_f = \frac{1}{2}(m_1 + m_2)v_f^2$$

so $$\frac{KE_f}{KE_i} = \frac{(m_1 + m_2)v_f^2}{m_1 v_{1i}^2} = \frac{(m_1 + m_2)}{m_1 \cancel{v_{1i}^2}} \frac{m_1^2 \cancel{v_{1i}^2}}{(m_1 + m_2)^2} = \boxed{\frac{m_1}{m_1 + m_2}}$$

6.36 Let us apply conservation of energy to the block from the time just after the bullet has passed through until it reaches maximum height in order to find its speed V just after the collision.

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f \text{ becomes } \frac{1}{2}mV^2 + 0 = 0 + mgy_f$$

or

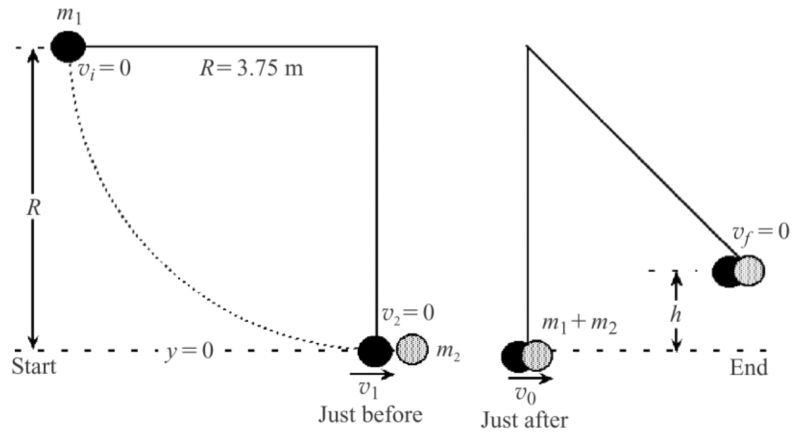
$$V = \sqrt{2gy_f} = \sqrt{2(9.80 \text{ m/s}^2)(0.120 \text{ m})} = 1.53 \text{ m/s}$$

Now use conservation of momentum from before until just after the collision in order to find the initial speed of the bullet, υ.

$$(7.0 \times 10^{-3} \text{ kg})v + 0 = (1.5 \text{ kg})(1.53 \text{ m/s}) + (7.0 \times 10^{-3} \text{ kg})(200 \text{ m/s})$$

from which $v = \boxed{5.3 \times 10^2 \text{ m/s}}$

6.37 The leftmost part of the sketch depicts the situation from when the actor starts from rest until just before he makes contact with his costar. Using conservation of energy over this period gives



$$\left(KE + PE\right)_1 = \left(KE + PE\right)_i$$

or

$$\frac{1}{2}m_1v_1^2 + 0 = 0 + mgR$$

so his speed just before impact is

$$v_1 = \sqrt{2gR} = \sqrt{2\left(9.80\ \text{m/s}^2\right)\left(3.75\ \text{m}\right)} = 8.57\ \text{m/s}$$

Now, employing conservation of momentum from just before to just after impact gives

$$\left(m_1 + m_2\right)v_0 = m_1v_1 + m_2\left(0\right) \qquad \text{or} \qquad v_0 = \frac{m_1v_1}{m_1 + m_2} = \frac{\left(80.0\ \text{kg}\right)\left(8.57\ \text{m/s}\right)}{80.0\ \text{kg} + 55.0\ \text{kg}} = 5.08\ \text{m/s}$$

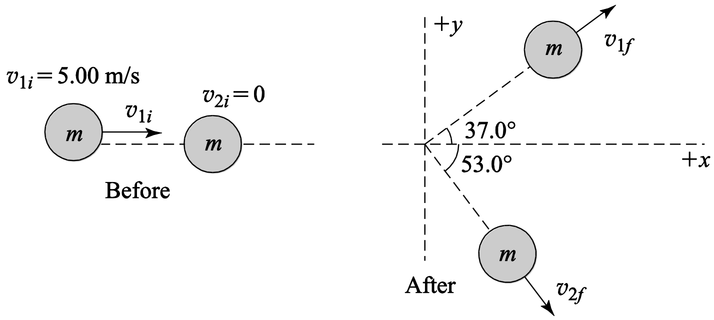
Finally, using conservation of energy from just after impact to the end yields

$$\left(KE + PE\right)_f = \left(KE + PE\right)_0 \qquad \text{or} \qquad 0 + \cancel{\left(m_1 + m_2\right)}gh = \frac{1}{2}\cancel{\left(m_1 + m_2\right)}v_0^2$$

and

$$h = \frac{v_0^2}{2g} = \frac{\left(5.08\ \text{m/s}\right)^2}{2\left(9.80\ \text{m/s}^2\right)} = \boxed{1.32\ \text{m}}$$

6.38



Consider the sketches above which show the situation just before and just after collision.

Conserving momentum in y-direction: $p_{yf} = p_{yi} \Rightarrow \cancel{m}v_{1f}\sin 37.0^\circ - \cancel{m}v_{2f}\sin 53.0^\circ = 0$, or

$$v_{2f} = \left(\frac{\sin 37.0^\circ}{\sin 53.0^\circ}\right)v_{1f} = 0.754v_{1f}$$

Now, conserving momentum in the x-direction:

$$p_{xf} = p_{xi} \Rightarrow \cancel{m}v_{1f}\cos 37.0^\circ + \cancel{m}v_{2f}\cos 53.0^\circ = \cancel{m}v_{1i} + 0$$

or

$$v_{1f}\cos 37.0^\circ + \left(0.754v_{1f}\right)\cos 53.0^\circ = v_{1i}$$

and

$$v_{1f} = \frac{v_{1i}}{\cos 37.0^\circ + (0.754)\cos 53.0^\circ} = \frac{5.00 \text{ m/s}}{\cos 37.0^\circ + (0.754)\cos 53.0^\circ} = \boxed{3.99 \text{ m/s}}$$

Then,

$$v_{2f} = 0.754v_{1f} = 0.754(3.99 \text{ m/s}) = \boxed{3.01 \text{ m/s}}$$

Now, we can verify that this collision was indeed an elastic collision:

$$KE_i = \frac{1}{2}mv_{1i}^2 = \frac{m}{2}(5.00 \text{ m/s})^2 = m\left(12.5 \text{ m}^2/\text{s}^2\right)$$

and

$$KE_f = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2 = \frac{m}{2}(3.99 \text{ m/s})^2 + \frac{m}{2}(3.01 \text{ m/s})^2 = m(12.5 \text{ m}^2/\text{s}^2)$$

so $KE_f = KE_i$, which is the criteria for an elastic collision.

6.39 Let M = mass of ball, m = mass of bullet, v = velocity of bullet, and V = the initial velocity of the ball-bullet combination. Then, using conservation of momentum from just before to just after collision gives

$$(M + m)V = mv + 0 \quad \text{or} \quad V = \left(\frac{m}{M + m}\right)v$$

Now, we use conservation of mechanical energy from just after the collision until the ball reaches maximum height to find

$$0 + (M + m)g\,h_{\text{max}} = \frac{1}{2}(M + m)V^2 + 0 \quad \text{or} \quad h_{\text{max}} = \frac{V^2}{2g} = \frac{1}{2g}\left(\frac{m}{M + m}\right)^2 v^2$$

With the data values provided, this becomes

$$h_{\text{max}} = \frac{1}{2(9.80 \text{ m/s}^2)}\left(\frac{0.030 \text{ kg}}{0.15 \text{ kg} + 0.030 \text{ kg}}\right)^2 (200 \text{ m/s})^2 = \boxed{57 \text{ m}}$$

6.40 First, we will find the horizontal speed, v_{0x}, of the block and embedded bullet just after impact. After this instant, the block-bullet combination is a projectile, and we find the time to reach the floor by use of $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, which becomes

$$-1.00 \text{ m} = 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)t^2, \text{ giving} \qquad t = 0.452 \text{ s}$$

Thus,

$$v_{0x} = \frac{\Delta x}{t} = \frac{2.00 \text{ m}}{0.452 \text{ s}} = 4.43 \text{ m/s}$$

Now use conservation of momentum for the collision, with v_b = speed of incoming bullet:

$$\left(8.00 \times 10^{-3}\ \text{kg}\right) v_b + 0 = \left(258 \times 10^{-3}\ \text{kg}\right)\left(4.43\ \text{m/s}\right)\text{, so}$$

$$v_b = \boxed{143\ \text{m/s}} \qquad \text{(about 320 mph)}$$

6.41 First, we use conservation of mechanical energy to find the speed of the block and embedded bullet just after impact:

$$\left(KE + PE_s\right)_f = \left(KE + PE_s\right)_i \text{ becomes } \frac{1}{2}\left(m + M\right)V^2 + 0 = 0 + \frac{1}{2}kx^2$$

and yields

$$V = \sqrt{\frac{kx^2}{m + M}} = \sqrt{\frac{\left(150\ \text{N/m}\right)\left(0.800\ \text{m}\right)^2}{\left(0.0120 + 0.100\right)\ \text{kg}}} = 29.3\ \text{m/s}$$

Now, employ conservation of momentum to find the speed of the bullet just before impact: $m\upsilon + M(0) = (m + M)V$, or

$$v = \left(\frac{m + M}{m}\right)V = \left(\frac{0.112\ \text{kg}}{0.0120\ \text{kg}}\right)\left(29.3\ \text{m/s}\right) = \boxed{273\ \text{m/s}}$$

6.42 (a) Conservation of momentum gives $m_T v_{fT} + m_c v_{fc} = m_T v_{iT} + m_c v_{ic}$, or

$$v_{fT} = \frac{m_T v_{iT} + m_c\left(v_{ic} - v_{fc}\right)}{m_T}$$

$$= \frac{\left(9\,000\ \text{kg}\right)\left(20.0\ \text{m/s}\right) + \left(1\,200\ \text{kg}\right)\left[\left(25.0 - 18.0\right)\ \text{m/s}\right]}{9\,000\ \text{kg}}$$

$$v_{fT} = \boxed{20.9\ \text{m/s}\ \ \text{East}}$$

(b) $$KE_{\text{lost}} = KE_i - KE_f = \left[\frac{1}{2}m_c v_{ic}^2 + \frac{1}{2}m_T v_{iT}^2\right] - \left[\frac{1}{2}m_c v_{fc}^2 + \frac{1}{2}m_T v_{fT}^2\right]$$

$$= \frac{1}{2}\left[m_c\left(v_{ic}^2 - v_{fc}^2\right) + m_T\left(v_{iT}^2 - v_{fT}^2\right)\right]$$

$$= \frac{1}{2}\left[\left(1\,200\ \text{kg}\right)\left(625 - 324\right)\left(\text{m}^2/\text{s}^2\right) + \left(9\,000\ \text{kg}\right)\left(400 - 438.2\right)\left(\text{m}^2/\text{s}^2\right)\right]$$

$$KE_{\text{lost}} = \boxed{8.68 \times 10^3 \text{ J, which becomes internal energy}}$$

Note: If 20.9 m/s were used to determine the energy lost instead of 20.9333 as the answer to Part (a), the answer would be very different. We have kept extra digits in all intermediate answers until the problem is complete.

6.43 (a) From conservation of momentum,

$$(5.00 \text{ g})v_{1f} + (10.0 \text{ g})v_{2f} = (5.00 \text{ g})(20.0 \text{ cm/s}) + 0$$

or

$$v_{1f} + 2v_{2f} = 20.0 \text{ cm/s} \qquad [1]$$

Also for an elastic, head-on, collision, we have $v_{1i} - v_{2f} = -\left(v_{1f} - v_{2i}\right)$,

which becomes 20.0 cm/s– 0 = $-v_{1f} + v_{2f}$, or

$$v_{2f} = v_{1f} + 20.0 \text{ cm/s} \qquad [2]$$

Substituting equation [2] into [1] yields $v_{1f} + 2\left(v_{1f} + 20.0 \text{ cm/s}\right) = 20.0 \text{ cm/s}$, or

$3v_{1f} = -20.0$ cm/s and $v_{1f} = \boxed{-6.67 \text{ cm/s}}$

Then [2] gives

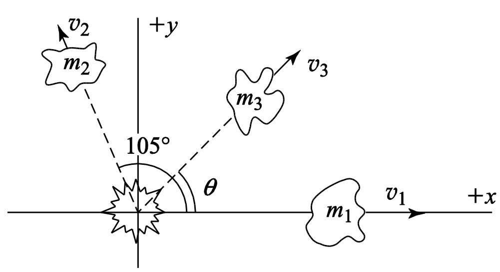
$$v_{2f} = -6.67 \text{ cm/s} + 20.0 \text{ cm/s} = \boxed{13.3 \text{ cm/s}}$$

(b) $KE_i = KE_{1i} + KE_{2i} = \frac{1}{2}\left(5.00 \times 10^{-3} \text{ kg}\right)\left(0.200 \text{ m/s}\right)^2 + 0 = 1.00 \times 10^{-4} \text{ J}$

$KE_{2f} = \frac{1}{2}m_2 v_{2f}^2 = \frac{1}{2}\left(10.0 \times 10^{-3} \text{ kg}\right)\left(13.3 \times 10^{-2} \text{ m/s}\right)^2 = 8.84 \times 10^{-5} \text{ J}$, so

$$\frac{KE_{2f}}{KE_i} = \frac{8.84 \times 10^{-5} \text{ J}}{1.00 \times 10^{-4} \text{ J}} = \boxed{0.884}$$

6.44 (a)



$m_1 = 48.0$ kg
$v_1 = 12.0$ m/s
$m_2 = 62.0$ kg
$v_2 = 15.0$ m/s
$m_3 = 112$ kg

(b) x-direction: $\Sigma p_{xf} = \Sigma p_{xi} \quad \Rightarrow \quad \boxed{m_1 v_1 \cos 0^\circ + m_2 v_2 \cos 105^\circ + m_3 v_3 \cos\theta = 0}$

y-direction: $\Sigma p_{yf} = \Sigma p_{yi} \quad \Rightarrow \quad \boxed{m_1 v_1 \sin 0^\circ + m_2 v_2 \sin 105^\circ + m_3 v_3 \sin\theta = 0}$

(c) $p_{1x} = m_1 v_1 \cos 0^\circ = (48.0 \text{ kg})(12.0 \text{ m/s})(1) = \boxed{576 \text{ kg}\cdot\text{m/s}}$

$p_{2x} = m_2 v_2 \cos 105^\circ = (62.0 \text{ kg})(15.0 \text{ m/s})(-0.259) = \boxed{-241 \text{ kg}\cdot\text{m/s}}$

(d) $p_{1y} = m_1 v_1 \sin 0^\circ = (48.0 \text{ kg})(12.0 \text{ m/s})(0) = \boxed{0}$

$p_{2y} = m_2 v_2 \sin 105^\circ = (62.0 \text{ kg})(15.0 \text{ m/s})(+0.966) = \boxed{898 \text{ kg}\cdot\text{m/s}}$

(e) x-direction: $\boxed{576 \text{ kg}\cdot\text{m/s} - 241 \text{ kg}\cdot\text{m/s} + (112 \text{ kg}) v_3 \cos\theta = 0}$

y-direction: $\boxed{0 + 898 \text{ kg}\cdot\text{m/s} + (112 \text{ kg}) v_3 \sin\theta = 0}$

(f) x-direction: $v_3 \cos\theta = \dfrac{-576 \text{ kg}\cdot\text{m/s} + 241 \text{ kg}\cdot\text{m/s}}{112 \text{ kg}}$ or $\boxed{v_3 \cos\theta = -2.99 \text{ m/s}}$

y-direction: $v_3 \sin\theta = \dfrac{-898 \text{ kg}\cdot\text{m/s}}{112 \text{ kg}}$ or $\boxed{v_3 \sin\theta = -8.02 \text{ m/s}}$

Then, squaring and adding these results, recognizing that $\cos^2\theta + \sin^2\theta = 1$, gives

$v_3^2\left(\cos^2\theta + \sin^2\theta\right) = (-2.99 \text{ m/s})^2 + (-8.02 \text{ m/s})^2$ and $v_3 = \sqrt{73.3 \text{ m}^2/\text{s}^2} = \boxed{8.56 \text{ m/s}}$

(g) $\dfrac{\cancel{v_3} \sin\theta}{\cancel{v_3} \cos\theta} = \tan\theta = \dfrac{-8.02 \text{ m/s}}{-2.99 \text{ m/s}} = 2.68$ so $\theta = \tan^{-1}(2.68) + 180^\circ = \boxed{250^\circ}$

Note that the factor of 180° was included in the last calculation because it was recognized that both the sine and cosine of angle θ were negative. This meant that θ had to be a third quadrant angle. Use of the inverse tangent function alone yields only the principle angles ($-90^\circ \le \theta \le +90^\circ$) that have the given value for the tangent function.

(h) Because the third fragment must have a momentum equal in magnitude and opposite direction to the resultant of the other two fragments momenta,

$\boxed{\text{all three pieces must travel in the same plane}}$.

6.45 Conservation of momentum gives

$$(25.0\text{ g})v_{1f} + (10.0\text{ g})v_{2f} = (25.0\text{ g})(20.0\text{ cm/s}) + (10.0\text{ g})(15.0\text{ cm/s})$$

or

$$2.50v_{1f} + v_{2f} = 65.0\text{ cm/s} \qquad [1]$$

For head-on, elastic collisions, we know that $v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$.

Thus,

$$20.0\text{ cm/s} - 15.0\text{ cm/s} = -v_{1f} + v_{2f} \quad \text{or} \quad v_{2f} = v_{1f} + 5.00\text{ cm/s} \qquad [2]$$

Substituting equation [2] into [1] yields 3.50 υ_{1f} = 60.0 cm/s, or $\upsilon_{1f} = \boxed{17.1\text{ cm/s}}$.

Equation [2] then gives υ_{2f} = 17.1 cm/s + 5.00 cm/s = 22.1 cm/s.

6.46 First, consider conservation of momentum and write

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

Since $m_1 = m_2$, this becomes

$$v_{1i} + v_{2i} = v_{1f} + v_{2f} \qquad [1]$$

For an elastic head-on collision, we also have $v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$, which may be written as

$$v_{1i} - v_{2i} = -v_{1f} + v_{2f} \qquad [2]$$

Adding Equations [1] and [2] yields

$$v_{2f} = v_{1i} \qquad [3]$$

Subtracting Equation [2] from [1] gives

$$v_{1f} = v_{2i} \qquad [4]$$

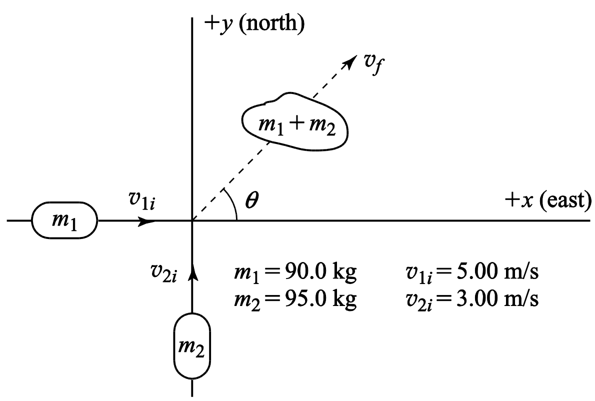
Equations [3] and [4] show us that, under the conditions of equal mass objects striking one another in a head-on, elastic collision, *the two objects simply exchange velocities*. Thus, we may write the results of the various collisions as

(a) $v_{1f} = \boxed{0}$, $v_{2f} = \boxed{1.50\ \text{m/s}}$

(b) $v_{1f} = \boxed{-1.00\ \text{m/s}}$, $v_{2f} = \boxed{1.50\ \text{m/s}}$

(c) $v_{1f} = \boxed{1.00\ \text{m/s}}$, $v_{2f} = \boxed{1.50\ \text{m/s}}$

6.47 (a) Over a the short time interval of the collision, external forces have no time to impart significant impulse to the players. $\boxed{\text{The two players move together after the tackle}}$, so the collision is completely inelastic.



(b) $p_{xf} = \Sigma p_{xi} \quad \Rightarrow \quad (m_1 + m_2) v_f \cos\theta = m_1 v_{1i} + 0$

or

$$v_f \cos\theta = \frac{m_1 v_{1i}}{(m_1 + m_2)} = \frac{(90.0\ \text{kg})(5.00\ \text{m/s})}{90.0\ \text{kg} + 95.0\ \text{kg}} \quad \text{and} \quad v_f \cos\theta = 2.43\ \text{m/s}$$

$$p_{yf} = \Sigma p_{yi} \quad \Rightarrow \quad (m_1 + m_2) v_f \sin\theta = 0 + m_2 v_{2i}$$

giving

$$v_f \sin\theta = \frac{m_2 v_{2i}}{(m_1 + m_2)} = \frac{(95.0\ \text{kg})(3.00\ \text{m/s})}{90.0\ \text{kg} + 95.0\ \text{kg}} \quad \text{and} \quad v_f \sin\theta = 1.54\ \text{m/s}$$

Therefore,

$$v_f^2\left(\sin^2\theta + \cos^2\theta\right) = v_f^2 = (1.54\ \text{m/s})^2 + (2.43\ \text{m/s})^2$$

and

$$v_f = \sqrt{8.28\ \text{m}^2/\text{s}^2} = 2.88\ \text{m/s}$$

Also,

$$\tan\theta = \frac{\cancel{v_f}\sin\theta}{\cancel{v_f}\cos\theta} = \frac{1.54\ \text{m/s}}{2.43\ \text{m/s}} = 0.633 \quad \text{and} \quad \theta = \tan^{-1}(0.633) = 32.3°$$

Thus,

$$\boxed{\vec{\mathbf{v}}_f = 2.88\ \text{m/s at } 32.3° \text{ north of east}}$$

(c) $$KE_{\text{lost}} = KE_i - KE_f = \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 - \frac{1}{2}(m_1 + m_2)v_f^2$$

$$= \frac{1}{2}\left[(90.0\ \text{kg})(5.00\ \text{m/s})^2 + (95.0\ \text{kg})(3.00\ \text{m/s})^2\right] - \frac{1}{2}(185\ \text{kg})(2.88\ \text{m/s})^2 = \boxed{785\ \text{J}}$$

The lost kinetic energy is transformed into other forms of energy, such as thermal energy and sound.

6.48 Consider conservation of momentum in the first event (twin A tossing the pack), taking the direction of the velocity given the backpack as positive. This yields

$$m_{\text{A}} v_{\text{A}f} + m_{\text{pack}} v_{\text{pack}} = (m_{\text{A}} + m_{\text{pack}})(0) = 0$$

or

$$v_{\text{A}f} = \frac{-m_{\text{pack}} v_{\text{pack}}}{m_{\text{A}}} = -\left(\frac{12.0\ \text{kg}}{55.0\ \text{kg}}\right)(+3.00\ \text{m/s}) = -0.655\ \text{m/s} \quad \text{and} \quad |v_{\text{A}f}| = \boxed{0.655\ \text{m/s}}$$

Conservation of momentum when twin B catches and holds onto the backpack yields

$$\left(m_B + m_{\text{pack}}\right) v_{\text{B}f} = m_{\text{B}}\left(0\right) + m_{\text{pack}} v_{\text{pack}}$$

or

$$v_{\text{B}f} = \frac{m_{\text{pack}} v_{\text{pack}}}{m_{\text{B}} + m_{\text{pack}}} = \frac{\left(12.0\ \text{kg}\right)\left(+3.00\ \text{m/s}\right)}{55.0\ \text{kg} + 12.0\ \text{kg}} = \boxed{0.537\ \text{m/s}}$$

6.49 Choose the $+x$-axis to be eastward and the $+y$-axis northward.

If v_i is the initial northward speed of the 3 000-kg car, conservation of momentum in the y direction gives

$$0 + \left(3\ 000\ \text{kg}\right) v_i = \left(3\ 000\ \text{kg} + 2\ 000\ \text{kg}\right)\left[\left(5.22\ \text{m/s}\right)\sin 40.0^\circ\right]$$

or

$$v_i = \boxed{5.59\ \text{m/s}}$$

Observe that knowledge of the initial speed of the 2 000-kg car was unnecessary for this solution.

6.50 We use conservation of momentum for both northward and eastward components.

For the eastward direction: $M\left(13.0\ \text{m/s}\right) = 2\,M\,V_f \cos 55.0^\circ$.

For the northward direction: $M\,v_{2i} = 2\,M\,V_f \sin 55.0^\circ$.

Divide the northward equation by the eastward equation to find

$$\frac{\cancel{M}v_{2i}}{\cancel{M}\left(13.0\ \text{m/s}\right)} = \frac{\cancel{2MV_f}\sin 55.0^\circ}{\cancel{2MV_f}\cos 55.0^\circ} \quad \text{or} \quad v_{2i} = \left(13.0\ \text{m/s}\right)\tan 55.0^\circ$$

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$$v_{2i} = \left[\left(13.0\ \text{m/s}\right)\left(\frac{2.237\ \text{mi/h}}{1\ \text{m/s}}\right)\right]\tan 55.0^\circ = \boxed{41.5\ \text{mi/h}}$$

Thus, the driver of the north bound car was untruthful.

6.51 Choose the x-axis to be along the original line of motion.

(a) From conservation of momentum in the x direction,

$$m(5.00 \text{ m/s}) + 0 = m(4.33 \text{ m/s})\cos 30.0^\circ + m v_{2f} \cos\theta$$

or

$$v_{2f} \cos\theta = 1.25 \text{ m/s} \qquad [1]$$

Conservation of momentum in the y direction gives

$$0 = m(4.33 \text{ m/s})\sin 30.0^\circ + m v_{2f} \sin\theta \text{ or } v_{2f} \sin\theta = -2.16 \text{ m/s} \qquad [2]$$

Dividing equation [2] by [1] gives

$$\tan\theta = \frac{-2.16}{1.25} = -1.73 \qquad \text{and } \theta = -60.0^\circ$$

Then, either [1] or [2] gives υ_{2f} = 2.50 m/s, so the final velocity of the second ball is

$$\vec{\mathbf{v}}_{2f} = \boxed{2.50 \text{ m/s at } -60.0^\circ}.$$

(b) $$KE_i = \frac{1}{2} m v_{1i}^2 + 0 = \frac{1}{2} m(5.00 \text{ m/s})^2 = m(12.5 \text{ m}^2/\text{s}^2)$$

$$KE_f = \frac{1}{2} m v_{1f}^2 + \frac{1}{2} m v_{2f}^2$$

$$= \frac{1}{2} m(4.33 \text{ m/s})^2 + \frac{1}{2} m(2.50 \text{ m/s})^2 = m(12.5 \text{ m}^2/\text{s}^2)$$

Since $KE_f = KE_i$, this is an $\boxed{\text{elastic collection}}$.

6.52 The recoil speed of the subject plus pallet after a heartbeat is

$$V = \frac{\Delta x}{\Delta t} = \frac{6.00 \times 10^{-5} \text{ m}}{0.160 \text{ s}} = 3.75 \times 10^{-4} \text{ m/s}$$

From conservation of momentum, $m\upsilon - MV = 0 + 0$, so the mass of blood leaving the heart is

$$m = M\left(\frac{V}{v}\right) = (54.0\text{ kg})\left(\frac{3.75\times10^{-4}\text{ m/s}}{0.500\text{ m/s}}\right) = 4.05\times10^{-2}\text{ kg} = \boxed{40.5\text{ g}}$$

6.53 Choose the positive direction to be the direction of the truck's initial velocity.

Apply conservation of momentum to find the velocity of the combined vehicles after collision:

$$(4\,000\text{ kg} + 800\text{ kg})V = (4\,000\text{ kg})(+8.00\text{ m/s}) + (800\text{ kg})(-8.00\text{ m/s})$$

which yields $V = +5.33$ m/s.

Use the impulse–momentum theorem, $I = F_{\text{av}}(\Delta t) = \Delta p = m(v_f - v_i)$, to find the magnitude of the average force exerted on each driver during the collision.

Truck Driver:

$$|F_{\text{av}}| = \frac{m\left|v_f - v_i\right|_{\text{truck}}}{\Delta t} = \frac{(80.0\text{ kg})|5.33\text{ m/s} - 8.00\text{ m/s}|}{0.120\text{ s}} = \boxed{1.78\times10^3\text{ N}}$$

Car Driver:

$$|F_{\text{av}}| = \frac{m\left|v_f - v_i\right|_{\text{car}}}{\Delta t} = \frac{(80.0\text{ kg})|5.33\text{ m/s} - (-8.00\text{ m/s})|}{0.120\text{ s}} = \boxed{8.89\times10^3\text{ N}}$$

6.54 First, we use conservation of mechanical energy to find the speed of m_1 at B just before collision. This gives $\frac{1}{2}m_1 v_1^2 + 0 = 0 + m_1 g h_i$,

or

$$v_1^2 = \sqrt{2gh_i} = \sqrt{2(9.80\text{ m/s}^2)(5.00\text{ m})} = 9.90\text{ m/s}$$

Next, we apply conservation of momentum and knowledge of elastic collisions to find the velocity of m_1 at B just after collision.

From conservation of momentum, with the second object initially at rest, we have

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + 0, \qquad \text{or} \qquad v_{2f} = \frac{m_1}{m_2}\left(v_{1i} - v_{1f}\right) \qquad [1]$$

For head-on elastic collisions, $v_{1i} - v_{2i} = -\left(v_{1f} - v_{2f}\right)$. Since $v_{2i} = 0$ in this case, this becomes $\upsilon_{2f} = \upsilon_{1f} + \upsilon_1$ and combining this with [1] above we obtain

$$v_{1f} + v_{1i} = \frac{m_1}{m_2}\left(v_{1i} - v_{1f}\right) \qquad \text{or} \qquad \left(m_1 + m_2\right)v_{1f} = \left(m_1 - m_2\right)v_{1i}$$

so

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} = \left(\frac{5.00 - 10.0}{5.00 + 10.0}\right)\left(9.90 \text{ m/s}\right) = -3.30 \text{ m/s}$$

Finally, use conservation of mechanical energy for m_1 after the collision to find the maximum rebound height. This gives $\left(KE + PE_g\right)_f = \left(KE + PE_g\right)_i$

or

$$0 + m_1 g h_{\max} = \frac{1}{2} m_1 v_{1f}^2 + 0 \qquad \text{and} \qquad h_{\max} = \frac{v_{1f}^2}{2g} = \frac{\left(-3.30 \text{ m/s}\right)^2}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{0.556 \text{ m}}$$

6.55 Note that the initial velocity of the target particle is zero (that is, $\upsilon_{2i} = 0$).

From conservation of momentum,

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + 0 \qquad [1]$$

For head-on elastic collisions, $\upsilon_{1i} - \upsilon_{2i} = -\left(\upsilon_{1f} - \upsilon_{2f}\right)$, and with $\upsilon_{2i} = 0$, this gives

$$v_{2f} = v_{1i} + v_{1f} \qquad [2]$$

Substituting equation [2] into [1] yields

$$m_1 v_{1f} + m_2\left(v_{1i} + v_{1f}\right) = m_1 v_{1i}$$

or

$$\left(m_1 + m_2\right) v_{1f} = \left(m_1 - m_2\right) v_{1i} \qquad \text{and} \qquad v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} \qquad [3]$$

Now, we substitute equation [3] into [2] to obtain

$$v_{2f} = v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} \qquad \text{or} \qquad v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} \qquad [4]$$

Equations [3] and [4] can now be used to answer both parts (a) and (b).

(a) If $m_1 = 2.0\text{ g}$, $m_2 = 1.0\text{ g}$, and $v_{1i} = 8.0\text{ m/s}$, then

$$v_{1f} = \boxed{\frac{8}{3}\text{ m/s}} \text{ and } v_{2f} = \boxed{\frac{32}{3}\text{ m/s}}$$

(b) If $m_1 = 2.0\text{ g}$, $m_2 = 10\text{ g}$, and $v_{1i} = 8.0\text{ m/s}$, we find

$$v_{1f} = \boxed{-\frac{16}{3}\text{ m/s}} \text{ and } v_{2f} = \boxed{\frac{8}{3}\text{ m/s}}$$

(c) The final kinetic energy of the 2.0 g particle in each case is

Case (a): $KE_{1f} = \frac{1}{2} m_1 v_{1f}^2 = \frac{1}{2}\left(2.0 \times 10^{-3}\text{ kg}\right)\left(\frac{8}{3}\text{ m/s}\right)^2 = 7.1 \times 10^{-3}\text{ J}$

Case (b): $KE_{1f} = \frac{1}{2} m_1 v_{1f}^2 = \frac{1}{2}\left(2.0 \times 10^{-3}\text{ kg}\right)\left(-\frac{16}{3}\text{ m/s}\right)^2 = 2.8 \times 10^{-2}\text{ J}$

Since the incident kinetic energy is the same in cases (a) and (b), we observe that $\boxed{\text{the incident particle loses more kinetic energy in case (a)}}$.

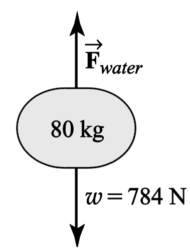
6.56 If the pendulum bob barely swings through a complete circle, it arrives at the top of the arc (having risen a vertical distance of 2 l) with essentially zero velocity.

From conservation of mechanical energy, we find the minimum velocity of the bob at the bottom of the arc as $\left(KE + PE_g\right)_{\text{bottom}} = \left(KE + PE_g\right)_{\text{top}}$, or $\frac{1}{2} M V^2 = 0 + M g \left(2\,\ell\right)$. This gives $V = 2\sqrt{g\,\ell}$ as the needed velocity of the bob just after the collision.

Conserving momentum through the collision then gives the minimum initial velocity of the bullet as

$$m\left(\frac{v}{2}\right) + M\left(2\sqrt{g\,\ell}\right) = mv + 0 \qquad \text{or} \qquad v = \boxed{\frac{4\,M}{m}\sqrt{g\,\ell}}$$

6.57 We first find the speed of the diver when he reaches the water by using $v_y^2 = v_0^2 + 2\,a_y\left(\Delta y\right)$. This becomes



$$v_y^2 = 0 + 2\left(-9.80\ \text{m/s}^2\right)\left(-3.0\ \text{m}\right), \text{ and yields } v_y = -\sqrt{59}\ \text{m/s}$$

The negative sign indicates the downward direction.

Next, we use the impulse–momentum theorem to find the resistive force exerted by the water as the diver comes to rest.

$$I = F_{\text{net}}\left(\Delta t\right) = \Delta p = m\left(v_f - v_i\right) \quad \text{or} \quad \left(F_{\text{water}} - w\right)\Delta t = m\left(v_f - v_i\right)$$

and

$$\left(F_{\text{water}} - 784\ \text{N}\right)\left(2.0\ \text{s}\right) = \left(80\ \text{kg}\right)\left[0 - \left(-\sqrt{59}\ \text{m/s}\right)\right]$$

yielding

$$F_{\text{water}} = 784\ \text{N} + \left(\frac{80\sqrt{59}}{2.0}\right)\text{N} = \boxed{1.1\times 10^3\ \text{N}\ \left(\text{upward}\right)}$$

6.58 Use conservation of mechanical energy, $\left(KE + PE_g\right)_B = \left(KE + PE_g\right)_A$, to find the speed of the bead at point B just before it collides with the ball. This gives, $\frac{1}{2} m\, v_{1i}^2 + 0 = 0 + m\, g\, y_A$,

or

$$v_{1i} = \sqrt{2gy_A} = \sqrt{2\left(9.80\ \text{m/s}^2\right)\left(1.50\ \text{m}\right)} = 5.42\ \text{m/s}$$

Conservation of momentum during the collision gives

$$\left(0.400\ \text{kg}\right)v_{1f} + \left(0.600\ \text{kg}\right)v_{2f} = \left(0.400\ \text{kg}\right)\left(5.42\ \text{m/s}\right) + 0$$

or

$$v_{1f} + 1.50\,v_{2f} = 5.42\ \text{m/s} \qquad [1]$$

For a head-on elastic collision, we have $v_{1i} - v_{2i} = -\left(v_{1f} - v_{2f}\right)$, and with $v_{2i} = 0$, this becomes

$$v_{1f} = v_{2f} - v_{1i} \qquad \text{or} \qquad v_{1f} = v_{2f} - 5.42\ \text{m/s} \qquad [2]$$

Substitute equation [2] into [1] to find the speed of the ball just after collision as

$$v_{2f} - 5.42\ \text{m/s} + 1.50\,v_{2f} = 5.42\ \text{m/s} \quad \text{or} \qquad v_{2f} = \frac{2\left(5.42\ \text{m/s}\right)}{2.50} = 4.34\ \text{m/s}$$

Now, we use conservation of the mechanical energy of the ball after collision to find the maximum height the ball will reach. This gives

$$0 + m_{\text{ball}}\,g\,y_{\max} = \tfrac{1}{2}m_{\text{ball}}v_{2f}^2 + 0 \qquad \text{or} \qquad y_{\max} = \frac{v_{2f}^2}{2g} = \frac{\left(4.34\ \text{m/s}\right)^2}{2\left(9.80\ \text{m/s}^2\right)} = \boxed{0.961\ \text{m}}$$

6.59 From the instant it is released from rest, at 2.00 m above ground, until just before contact, the ball is a freely falling body with $a_y = -g$. Its speed just before impact is given by $v_y^2 = v_{0y}^2 + 2a_y\left(\Delta y\right)$ as

$$v_y = \sqrt{v_{0y}^2 + 2a_y\left(\Delta y\right)} = \sqrt{0 + 2\left(-9.80\ \text{m/s}^2\right)\left(-2.00\ \text{m}\right)} = 6.26\ \text{m/s}$$

and its velocity immediately prior to impact is $\vec{\mathbf{v}}_i = -6.26\ \text{m/s}$.

After the ball leaves the ground on the rebound (to a height of 1.40 m), it is again in free-fall and $v_y^2 = v_{0y}^2 + 2a_y\left(\Delta y\right)$ gives its rebound speed as

$$v_{0y} = \sqrt{v_y^2 - 2a_y(\Delta y)} = \sqrt{0 - 2(-9.80\ \text{m/s}^2)(+1.40\ \text{m})} = 5.24\ \text{m/s}$$

and its velocity immediately after impact is $\vec{\mathbf{v}}_f = +5.24\ \text{m/s}$.

The impulse–momentum theorem, $\vec{\mathbf{F}}_{\text{av}}(\Delta t) = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i$, then gives the average force acting on the ball during the impact as

$$\vec{\mathbf{F}}_{\text{av}} = \frac{m(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i)}{\Delta t} = \frac{(0.500\ \text{kg})[+5.24\ \text{m/s} - (-6.26\ \text{m/s})]}{0.080\ 0\ \text{s}} = +71.9\ \text{N} \text{ or } \boxed{71.9\ \text{N upward}}$$

6.60 The mass of the third fragment must be

$$m_3 = m_{\text{nucleus}} - m_1 - m_2 = (17 - 5.0 - 8.4) \times 10^{-27}\ \text{kg} = 3.6 \times 10^{-27}\ \text{kg}$$

Conserving momentum in both the x- and y-directions gives the following:

y-direction: $m_1 v_{1y} + m_2 v_{2y} + m_3 v_{3y} = 0$

or

$$v_{3y} = -\frac{m_1 v_{1y} + m_2 v_{2y}}{m_3} = -\frac{(5.0 \times 10^{-27}\ \text{kg})(6.0 \times 10^6\ \text{m/s}) + 0}{3.6 \times 10^{-27}\ \text{kg}} = -\frac{30}{3.6} \times 10^6\ \text{m/s}$$

x-direction: $m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x} = 0$

or

$$v_{3x} = -\frac{m_1 v_{1x} + m_2 v_{2x}}{m_3} = -\frac{0 + (8.4 \times 10^{-27}\ \text{kg})(4.0 \times 10^6\ \text{m/s})}{3.6 \times 10^{-27}\ \text{kg}} = -\frac{34}{3.6} \times 10^6\ \text{m/s}$$

and

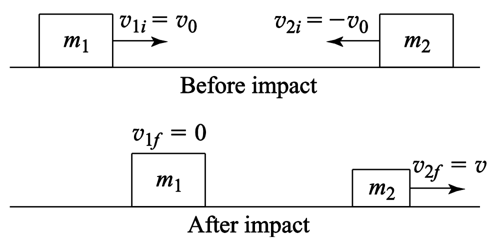
$$v_3 = \sqrt{v_{3x}^2 + v_{3y}^2} = \sqrt{\left(-(34/3.6) \times 10^6\ \text{m/s}\right)^2 + \left(-(30/3.6) \times 10^6\ \text{m/s}\right)^2} = 1.3 \times 10^6\ \text{m/s}$$

Also, $\theta = \tan^{-1}\left(\frac{v_{3y}}{v_{3x}}\right) + 180° = \tan^{-1}\left(\frac{30}{34}\right) + 180° = 2.2 \times 10^2 \text{ degrees} = 220°$

Therefore, $\boxed{\vec{\mathbf{v}}_3 = 1.3 \times 10^6 \text{ m/s at } 220° \text{ counterclockwise from the } +x\text{-axis}}$

Note that the factor of 180° was included in the calculation for θ because it was recognized that both v_{3x} and v_{3y} were negative. This meant that θ had to be a third quadrant angle. Use of the inverse tangent function alone yields only the principle angles ($-90° \le \theta \le +90°$) that have the given value for the tangent function.

6.61 The sketch at the right gives before and after views of the collision between these two objects. Since the collision is elastic, both kinetic energy and momentum must be conserved.



Conservation of Momentum:

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i}$$

$$m_1(0) + m_2 v = m_1 v_0 + m_2(-v_0)$$

or

$$v = \left(\frac{m_1}{m_2} - 1\right) v_0 \qquad [1]$$

Since this is an elastic collision, $v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$, and with the given velocities this becomes

$$v_0 - (-v_0) = -(0 - v) \qquad \text{or} \qquad v = 2v_0 \qquad [2]$$

(a) Substituting equation [2] into [1] gives

$$2\cancel{v_0} = \left(\frac{m_1}{m_2} - 1\right)\cancel{v_0} \qquad \text{or} \qquad \boxed{m_1/m_2 = 3}$$

(b) From equation [2] above, we have $\boxed{v/v_0 = 2}$.

6.62 (a) Let v_{1i} and v_{2i} be the velocities of m_1 and m_2 just before the collision. Then, using conservation of mechanical energy: $(KE + PE_g)_i = (KE + PE_g)_0$, or $\frac{1}{2} m v_i^2 + 0 = 0 + mgh_0$, gives

$$v_{1i} = -v_{2i} = \sqrt{2gh_0} = \sqrt{2\left(9.80\ \text{m/s}^2\right)\left(5.00\ \text{m}\right)} = 9.90\ \text{m/s}$$

and

$$v_{1i} = \boxed{+9.90\ \text{m/s}} \qquad \text{while} \qquad v_{2i} = \boxed{-9.90\ \text{m/s}}$$

(b) From conservation of momentum:

$$\left(2.00\ \text{g}\right)v_{1f} + \left(4.00\ \text{g}\right)v_{2f} = \left(2.00\ \text{g}\right)\left(9.90\ \text{m/s}\right) + \left(4.00\ \text{g}\right)\left(-9.90\ \text{m/s}\right).$$

or

$$v_{1f} + \left(2.00\right)v_{2f} = -9.90\ \text{m/s} \qquad [1]$$

For an elastic, head-on collision, $v_{1i} - v_{2i} = -\left(v_{1f} - v_{2f}\right)$, giving

$$+9.90\ \text{m/s} - \left(-9.90\ \text{m/s}\right) = -v_{1f} + v_{2f} \quad \text{or} \quad v_{2f} = v_{1f} + 19.8\ \text{m/s} \qquad [2]$$

Substituting equation [2] into [1] gives $v_{1f} + \left(2.00\right)\left(v_{1f} + 19.8\ \text{m/s}\right) = -9.90\ \text{m/s}$,

or

$$v_{1f} = \frac{-9.90\ \text{m/s} - 39.6\ \text{m/s}}{3.00} = \boxed{-16.5\ \text{m/s}}$$

Then, equation [2] yields $v_{2f} = -16.5\ \text{m/s} + 19.8\ \text{m/s} = \boxed{+3.30\ \text{m/s}}$.

(c) Applying conservation of energy to each block after the collision gives

$$\frac{1}{2}m\left(0\right)^2 + mgh_{\text{max}} = \frac{1}{2}mv_f^2 + mg\left(0\right) \quad \text{or} \quad h_{\text{max}} = \frac{v_f^2}{2g}$$

Thus,

$$h_{1f} = \frac{v_{1f}^2}{2g} = \frac{\left(-16.5\ \text{m/s}\right)^2}{2\left(9.80\ \text{m/s}^2\right)} = \boxed{13.9\ \text{m}}$$

and

$$h_{2f} = \frac{v_{2f}^2}{2g} = \frac{(3.30 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{0.556 \text{ m}}$$

6.63 (a) Use conservation of mechanical energy to find the speed m_1 of just before collision. Taking $y = 0$ at the tabletop level, this gives $\frac{1}{2} m_1 v_{1i}^2 + mg(0) = \frac{1}{2} m_1 (0) + m_1 g h_1$, or

$$v_{1i} = \sqrt{2 g h_1} = \sqrt{2(9.80 \text{ m/s}^2)(2.50 \text{ m})} = 7.00 \text{ m/s}$$

Apply conservation of momentum from just before to just after the collision:

$$(0.500 \text{ kg}) v_{1f} + (1.00 \text{ kg}) v_{2f} = (0.500 \text{ kg})(7.00 \text{ m/s}) + 0$$

or

$$v_{1f} + 2 v_{2f} = 7.00 \text{ m/s} \qquad [1]$$

For a head-on elastic collision, $v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$, and with $v_{2i} = 0$, this becomes

$$v_{2f} = v_{1f} + v_{1i} \qquad \text{or} \qquad v_{2f} = v_{1f} + 7.00 \text{ m/s} \qquad [2]$$

Substituting equation [2] into [1] yields

$$v_{1f} + 2(v_{1f} + 7.00 \text{ m/s}) = 7.00 \text{ m/s} \qquad \text{and} \qquad v_{1f} = \frac{-7.00 \text{ m/s}}{3} = \boxed{-2.33 \text{ m/s}}$$

Then, from equation [2], $v_{2f} = -2.33 \text{ m/s} + 7.00 \text{ m/s} = \boxed{4.67 \text{ m/s}}$.

(b) Apply conservation of mechanical energy to m_1 after the collision to find the rebound height of this object

$$\frac{1}{2} m_1 (0) + m_1 g h_1' = \frac{1}{2} m_1 v_{1f}^2 + mg(0) \qquad \text{or} \qquad h_1' = \frac{v_{1f}^2}{2g} = \frac{(-2.33 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{0.277 \text{ m}}$$

(c) From $\Delta y = v_{0y} t + \frac{1}{2} a_y t^2$, with $v_{0y} = 0$, the time for m_2 to reach the floor after it flies horizontally off the table is

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-2.00\ \text{m})}{-9.80\ \text{m/s}^2}} = 0.639\ \text{s}$$

During this time it travels a horizontal distance

$$\Delta x = v_{0x}t = (4.67\ \text{m/s})(0.639\ \text{s}) = \boxed{2.98\ \text{m}}$$

(d) After the 0.500 kg mass comes back down the incline, it flies off the table with a horizontal velocity of 2.33 m/s. The time of the flight to the floor is 0.639 s as found above and the horizontal distance traveled is

$$\Delta x = v_{0x}t = (2.33\ \text{m/s})(0.639\ \text{s}) = \boxed{1.49\ \text{m}}$$

6.64 Conservation of the x-component of momentum gives

$$(3m)v_{2x} + 0 = -mv_0 + (3m)v_0 \qquad \text{or} \qquad v_{2x} = \frac{2}{3}v_0 \qquad [1]$$

Likewise, conservation of the y-component of momentum gives

$$-m\,v_{1y} + (3m)v_{2y} = 0 \qquad \text{and} \qquad v_{1y} = 3\,v_{2y} \qquad [2]$$

Since the collision is elastic, $(KE)_f = (KE)_i$, or

$$\frac{1}{2}m\,v_{1y}^2 + \frac{1}{2}(3m)\left(v_{2x}^2 + v_{2y}^2\right) = \frac{1}{2}m\,v_0^2 + \frac{1}{2}(3m)\,v_0^2$$

which reduces to

$$v_{1y}^2 + 3\left(v_{2x}^2 + v_{2y}^2\right) = 4\,v_0^2 \qquad [3]$$

Substituting equations [1] and [2] into [3] yields

$$9\,v_{2y}^2 + 3\left(\frac{4}{9}v_0^2 + v_{2y}^2\right) = 4\,v_0^2 \qquad \text{or} \qquad v_{2y} = v_0\,\frac{\sqrt{2}}{3}$$

(a) From equation [2], the particle of mass m has final speed $v_{1y} = 3\,v_{2y} = \boxed{v_0\sqrt{2}}$ and the particle of mass $3m$ moves at

$$v_2 = \sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{\frac{4}{9}v_0^2 + \frac{2}{9}v_0^2} = \boxed{v_0\sqrt{\frac{2}{3}}}$$

(b) $$\theta = \tan^{-1}\left(\frac{v_{2y}}{v_{2x}}\right) = \tan^{-1}\left(\frac{v_0\sqrt{2}/3}{2v_0/3}\right) = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) = \boxed{35.3°}$$

6.65 (a) The momentum of the system is initially zero and remains constant throughout the motion. Therefore, when m_1 leaves the wedge, we must have $m_2 v_{\text{wedge}} + m_1 v_{\text{block}} = 0$, or

$$v_{\text{wedge}} = -\left(\frac{m_1}{m_2}\right)v_{\text{block}} = -\left(\frac{0.500}{3.00}\right)(4.00\ \text{m/s}) = \boxed{-0.667\ \text{m/s}}$$

(b) Using conservation of energy as the block slides down the wedge, we have

$\left(KE + PE_g\right)_i = \left(KE + PE_g\right)_f$, or

$$0 + m_1 gh = \frac{1}{2}m_1 v_{\text{block}}^2 + \frac{1}{2}m_2 v_{\text{wedge}}^2 + 0$$

Thus,

$$h = \frac{1}{2g}\left[v_{\text{block}}^2 + \left(\frac{m_2}{m_1}\right)v_{\text{wedge}}^2\right]$$

$$= \frac{1}{19.6\ \text{m/s}^2}\left[(4.00\ \text{m/s})^2 + \left(\frac{3.00}{0.500}\right)(-0.667\ \text{m/s})^2\right] = \boxed{0.952\ \text{m}}$$

6.66 Choose the positive x-axis in the direction of the initial velocity of the cue ball. Let v_{ci} be the initial speed of the cue ball, v_{cf} be the final speed of the cue ball, v_{Tf} be the final speed of the target, and θ be the angle the target's final velocity makes with the x-axis.

Conservation of momentum in the x-direction, recognizing that all billiard balls have the same mass, gives

$$m\,v_{Tf}\cos\theta + m\,v_{cf}\cos 30.0° = 0 + mv_{ci} \quad \text{or} \quad v_{Tf}\cos\theta = v_{ci} - v_{cf}\cos 30.0° \qquad [1]$$

To conserve momentum in the y-direction, recognize that the y-components of the final velocities of the target and cue balls must have opposite signs. Thus, if the cue ball scatters at 30.0° *below* the x-axis, the target ball must scatter at angle θ above the x-axis. The conservation equation for momentum in the y-direction is:

$$m\,v_{Tf}\sin\theta - m\,v_{cf}\sin 30.0^\circ = 0 + 0 \qquad \text{or} \qquad v_{Tf}\sin\theta = v_{cf}\sin 30.0^\circ \qquad [2]$$

Since this is an elastic collision, kinetic energy is conserved, giving

$$\frac{1}{2}m\,v_{Tf}^2 + \frac{1}{2}m\,v_{cf}^2 = \frac{1}{2}m\,v_{ci}^2 \qquad \text{or} \qquad v_{Tf}^2 = v_{ci}^2 - v_{cf}^2 \qquad [3]$$

(b) To solve, square equations [1] and [2] and add the results to obtain

$$v_{Tf}^2\left(\cos^2\theta + \sin^2\theta\right) = v_{ci}^2 - 2\,v_{ci}v_{cf}\cos 30.0^\circ + v_{cf}^2\left(\cos^2 30.0^\circ + \sin^2 30.0^\circ\right)$$

or $v_{Tf}^2 = v_{ci}^2 - 2\,v_{ci}v_{cf}\cos 30.0^\circ + v_{cf}^2$

Now, substitute this result into equation [3] to get

$$\cancel{v_{ci}^2} - 2\,v_{ci}v_{cf}\cos 30.0^\circ + v_{cf}^2 = \cancel{v_{ci}^2} - v_{cf}^2 \quad \text{or} \qquad 2v_{cf}\left(v_{cf} - v_{ci}\cos 30.0^\circ\right) = 0$$

Since $v_{cf} \neq 0$, it is necessary that $v_{cf} = v_{ci}\cos 30.0^\circ = (4.00\ \text{m/s})\cos 30.0^\circ = \boxed{3.46\ \text{m/s}}$.

Then, equation [3] yields $v_{Tf} = \sqrt{v_{ci}^2 - v_{cf}^2}$, or

$$v_{Tf} = \sqrt{(4.00\ \text{m/s})^2 - (3.46\ \text{m/s})^2} = \boxed{2.00\ \text{m/s}}$$

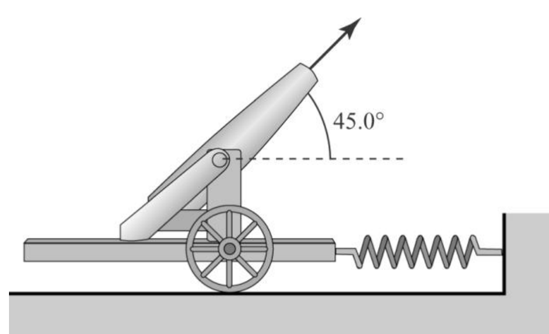
(a) With the results found above, equation [2] gives

$$\sin\theta = \left(\frac{v_{cf}}{v_{Tf}}\right)\sin 30.0^\circ = \left(\frac{3.46\ \text{m/s}}{2.00\ \text{m/s}}\right)\sin 30.0^\circ = 0.866\text{, or } \theta = 60.0^\circ$$

Thus, the angle between the velocity vectors after collision is

$$\phi = 60.0^\circ + 30.0^\circ = \boxed{90.0^\circ}$$

6.67 (a) Use conservation of the horizontal component of momentum from just before to just after the cannon firing.



$$\left(\Sigma p_x\right)_f = \left(\Sigma p_x\right)_i \text{ gives}$$

$$m_{\text{shell}}\left(v_{\text{shell}}\cos 45.0^\circ\right) + m_{\text{cannon}}v_{\text{recoil}} = 0\text{,}$$

or

$$v_{\text{recoil}} = -\left(\frac{m_{\text{shell}}}{m_{\text{cannon}}}\right) v_{\text{shell}} \cos 45.0°$$

$$= -\left(\frac{200 \text{ kg}}{5\ 000 \text{ kg}}\right)(125 \text{ m/s}) \cos 45.0° = \boxed{-3.54 \text{ m/s}}$$

(b) Use conservation of mechanical energy for the cannon-spring system from right after the cannon is fired to the instant when the cannon comes to rest.

$$\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$$

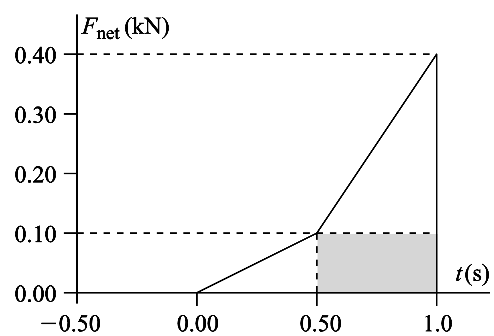
$$0 + 0 + \frac{1}{2} k x_{\text{max}}^2 = \frac{1}{2} m_{\text{cannon}} v_{\text{recoil}}^2 + 0 + 0$$

$$x_{\text{max}} = \sqrt{\frac{m_{\text{cannon}} v_{\text{recoil}}^2}{k}} = \sqrt{\frac{(5\ 000 \text{ kg})(-3.54 \text{ m/s})^2}{2.00 \times 10^4 \text{ N/m}}} = \boxed{1.77 \text{ m}}$$

(c) $|F_{\text{max}}| = k\, x_{\text{max}} = \left(2.00 \times 10^4 \text{ N/m}\right)(1.77 \text{ m}) = \boxed{3.54 \times 10^4 \text{ N}}$

(d) No. The rail exerts a vertical external force (the normal force) on the cannon and prevents it from recoiling vertically. Momentum is not conserved in the vertical direction. The spring does not have time to stretch during the cannon firing. Thus, no external horizontal force is exerted on the system (cannon plus shell) from just before to just after firing. Momentum is conserved in the horizontal direction during this interval.

6.68 Observe from Figure P6.68, the platform exerts a 0.60-kN to support the weight of the standing athlete prior to $t = 0.00$ s. From this, we determine the mass of the athlete:



$$m = \frac{w}{g} = \frac{0.60 \text{ kN}}{g} = \frac{600 \text{ N}}{9.8 \text{ m/s}^2} = 61 \text{ kg}$$

For the interval $t = 0.00$ s to $t = 1.0$ s, we subtract the 0.60-kN used to counterbalance the weight to get the net upward force exerted on the athlete by the platform during the jump. The result is shown in the force versus time graph at the right. The net impulse imparted to the athlete is given by the area under this graph. Note that this area can be broken into two triangular areas plus a rectangular area.

The net upward impulse is then

$$I = \frac{1}{2}(0.50\text{ s})(100\text{ N}) + \frac{1}{2}(0.50\text{ s})(300\text{ N}) + (0.50\text{ N})(100\text{ N}) = 150\text{ N}\cdot\text{s}$$

The upward velocity v_i of the athlete as he lifts off of the platform (at $t = 1.0\text{ s}$) is found from

$$I = \Delta p = mv_i - mv_0 = mv_i - 0 \quad\Rightarrow\quad v_i = \frac{I}{m} = \frac{150\text{ N}\cdot\text{s}}{61\text{ kg}} = 2.5\text{ m/s}$$

The height of the jump can then be found from $v_f^2 = v_i^2 + 2a_y\Delta y$ (with $v_f = 0$) to be

$$\Delta y = \frac{v_f^2 - v_i^2}{2a_y} = \frac{0 - (2.5\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = \boxed{0.31\text{ m}}$$

6.69 Let particle 1 be the neutron and particle 2 be the carbon nucleus. Then, we are given that $m_2 = 12\,m_1$.

(a) From conservation of momentum $m_2v_{2f} + m_1v_{1f} = m_1v_{1i} + 0$.

Since $m_2 = 12\,m_1$, this reduces to

$$12v_{2f} + v_{1f} = v_{1i} \qquad [1]$$

For a head-on elastic collision

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

Since $v_{2i} = 0$, this becomes

$$v_{2f} = v_{1i} + v_{1f} \qquad [2]$$

Substitute equation [2] into [1] to obtain $12(v_{1i} + v_{1f}) + v_{1f} = v_{1i}$, or

$$13v_{1f} = -11\,v_{1i} \quad \text{and} \quad v_{1f} = -\frac{11}{13}v_{1i}$$

Then, equation [2] yields

$$v_{2f} = \frac{2}{13}v_{1i}$$

The initial kinetic energy of the neutron is $KE_{1i} = \frac{1}{2} m_1 v_{1i}^2$, and the final kinetic energy of the carbon nucleus is

$$KE_{2f} = \frac{1}{2} m_2 v_{2f}^2 = \frac{1}{2}(12\, m_1)\left(\frac{4}{169} v_{1i}^2\right) = \frac{48}{169}\left(\frac{1}{2} m_1 v_{1i}^2\right) = \frac{48}{169} KE_{1i}$$

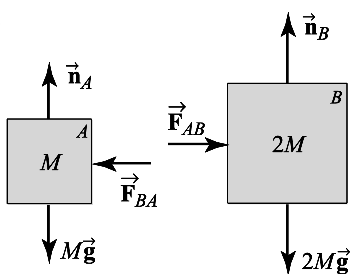
The fraction of kinetic energy transferred is $\dfrac{KE_{2f}}{KE_{1i}} = \dfrac{48}{169} = \boxed{0.28}$.

(b) If $KE_{1i} = 1.6 \times 10^{-13}$ J, then

$$KE_{2f} = \frac{48}{169} KE_{1i} = \frac{48}{169}\left(1.6 \times 10^{-13} \text{ J}\right) = \boxed{4.5 \times 10^{-14} \text{ J}}$$

The remaining energy $1.6 \times 10^{-13} \text{ J} - 4.5 \times 10^{-14} \text{ J} = \boxed{1.1 \times 10^{-13} \text{ J}}$ stays with the neutron.

6.70 (a)



(b) From Newton's third law, the force $\vec{\mathbf{F}}_{BA}$ exerted by B on A is at each instant equal in magnitude and opposite in direction to the force $\vec{\mathbf{F}}_{AB}$ exerted by A on B.

(c) There are no horizontal external forces acting on system C which consists of both blocks. The forces $\vec{\mathbf{F}}_{BA}$ and $\vec{\mathbf{F}}_{AB}$ are internal forces exerted on one part of system C by another part of system C.

Thus,

$$\Sigma\vec{\mathbf{F}}_{\text{external}} = \frac{\Delta\vec{\mathbf{p}}_C}{\Delta t} = 0 \quad \Rightarrow \quad \boxed{\Delta\vec{\mathbf{p}}_C = 0}$$

This gives

$$\left(\vec{\mathbf{p}}_C\right)_f = \left(\vec{\mathbf{p}}_C\right)_i = \left(\vec{\mathbf{p}}_A\right)_i + \left(\vec{\mathbf{p}}_B\right)_i \quad \text{or} \quad \left(M + 2M\right)V = M\left(+v\right) + 0$$

so the velocity of the combined blocks after collision is $V = +v/3$.

The change in momentum of A is then

$$\Delta\vec{\mathbf{p}}_A = \left(\vec{\mathbf{p}}_A\right)_f - \left(\vec{\mathbf{p}}_A\right)_i = MV - Mv = M\left(\frac{v}{3} - v\right) = \boxed{-2Mv/3}$$

and the change in momentum for B is:

$$\Delta\vec{\mathbf{p}}_B = \left(\vec{\mathbf{p}}_B\right)_f - \left(\vec{\mathbf{p}}_B\right)_i = 2MV - 0 = 2M\left(\frac{+v}{3}\right) = \boxed{+2Mv/3}$$

(d) $$\Delta KE = \left(KE_C\right)_f - \left[\left(KE_A\right) + \left(KE_B\right)\right]_i = \tfrac{1}{2}\left(3M\right)\left(\frac{v}{3}\right)^2 - \left[\tfrac{1}{2}Mv^2 + 0\right] = -\tfrac{1}{3}Mv^2$$

Thus, $\boxed{\text{kinetic energy is not conserved in this inelastic collision}}$.

6.71 (a) The owner's $\boxed{\text{claim should be denied}}$. Immediately prior to impact, the total momentum of the two-car system had a northward component and an eastward component. Thus, after impact, the wreckage moved in a northeasterly direction and could not possibly have damaged the owner's property on the southeast corner.

(b) From conservation of momentum:

$$\left(p_x\right)_{\text{after}} = \left(p_x\right)_{\text{before}} \quad \Rightarrow \quad \left(m_1 + m_2\right)v_x = m_1\left(v_{1i}\right)_x + m_2\left(v_{2i}\right)_x,$$

or

$$v_x = \frac{m_1\left(v_{1i}\right)_x + m_2\left(v_{2i}\right)_x}{m_1 + m_2} = \frac{\left(1\,300\text{ kg}\right)\left(30.0\text{ km/h}\right)+0}{1\,300\text{ kg} + 1\,100\text{ kg}} = 16.3\text{ km/h}$$

$$\left(p_y\right)_{\text{after}} = \left(p_y\right)_{\text{before}} \quad \Rightarrow \quad \left(m_1 + m_2\right)v_y = m_1\left(v_{1i}\right)_y + m_2\left(v_{2i}\right)_y$$

or

$$v_y = \frac{m_1\left(v_{1i}\right)_y + m_2\left(v_{2i}\right)_y}{m_1 + m_2} = \frac{0 + \left(1\,100\text{ kg}\right)\left(20.0\text{ km/h}\right)}{1\,300\text{ kg} + 1\,100\text{ kg}} = 9.17\text{ km/h}$$

Thus, the velocity of the wreckage immediately after impact is

$$v = \sqrt{v_x^2 + v_y^2} = 18.7 \text{ km/h} \qquad \text{and} \qquad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}(0.564) = 29.4°$$

or $\vec{\mathbf{v}} = \boxed{18.7 \text{ km/h at } 29.4° \text{ north of east, consistent with part (a)}}$

6.72 Ignoring the force of gravity during the brief collision time, we use the conservation of momentum to obtain:

$$(0.45 \text{ kg})v_{bf} + (60 \text{ kg})v_{pf} = (0.45 \text{ kg})(-25 \text{ m/s}) + (60 \text{ kg})(4.0 \text{ m/s})$$

or

$$v_{pf} = 3.8 \text{ m/s} - \left(7.5 \times 10^{-3}\right)v_{bf} \qquad [1]$$

Also, elastic collision $\Rightarrow$ $v_{bf} - v_{pf} = -\left(v_{bi} - v_{pi}\right) = -\left(-25 \text{ m/s} - 4.0 \text{ m/s}\right)$, or

$$v_{bf} = 29 \text{ m/s} + v_{pf} \qquad [2]$$

Substituting equation [1] into [2] yields

$$v_{bf} = \frac{29 \text{ m/s} + 3.8 \text{ m/s}}{1 + 7.5 \times 10^{-3}} = \boxed{33 \text{ m/s}}$$

The average acceleration of the ball during the collision is

$$a_{\text{av}} = \frac{v_{bf} - v_{bi}}{\Delta t} = \frac{33 \text{ m/s} - (-25 \text{ m/s})}{20 \times 10^{-3} \text{ s}} = \boxed{2.9 \times 10^3 \text{ m/s}^2}.$$

6.73 (a) The speed v_i of both balls just before the basketball reaches the ground may be found from $v_y^2 = v_{0y}^2 + 2a_y\Delta y$ as

$$v_i = \sqrt{v_{0y}^2 + 2a_y\Delta y} = \sqrt{0 + 2\left(-9.80 \text{ m/s}^2\right)(-1.20 \text{ m/s})} = \boxed{4.85 \text{ m/s}}$$

(b) Immediately after the basketball rebounds from the floor, it and the tennis ball meet in an elastic collision. The velocities of the two balls just before collision are:

For the tennis ball: $v_{1i} = -v_i = -4.85 \text{ m/s}$

For the basketball: $v_{2i} = +v_i = +4.85 \text{ m/s}$

We determine the velocity of the tennis ball immediately after this elastic collision as follows:

Momentum conservation gives

$$(57.0\ \text{g})v_{1f} + (590\ \text{g})v_{2f} = (57.0\ \text{g})(-4.85\ \text{m/s}) + (590\ \text{g})(+4.85\ \text{m/s})$$

which reduces to

$$v_{2f} = 4.38\ \text{m/s} - (9.66 \times 10^{-2})v_{1f} \qquad [1]$$

Elastic Collision $\Rightarrow \quad v_{1i} - v_{2i} = -(v_{1f} - v_{2f}) \qquad$ or $\qquad v_{1f} = v_{2f} - v_{1i} + v_{2i}$

so

$$v_{1f} = v_{2f} - (-4.85\ \text{m/s}) + 4.85\ \text{m/s} \qquad \text{and} \qquad v_{1f} = v_{2f} + 9.70\ \text{m/s} \qquad [2]$$

Substituting equation [1] into [2] gives $(1 + 9.66 \times 10^{-2})\, v_{1f} = 4.38\ \text{m/s} + 9.70\ \text{m/s}$, or

$$v_{1f} = +12.84\ \text{m/s}$$

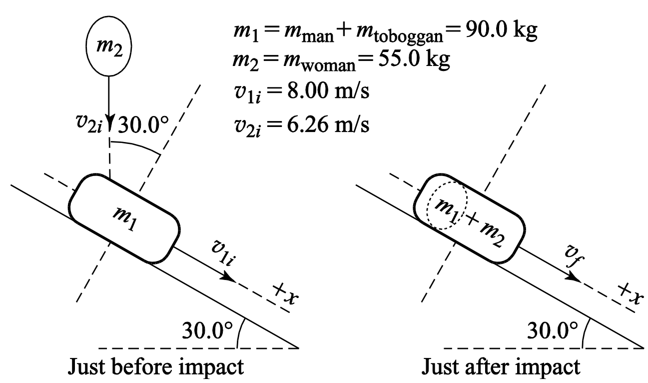
The vertical displacement of the tennis ball during its rebound is given by $v_y^2 = v_{0y}^2 + 2a_y\Delta y$ as

$$\Delta y = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (12.84\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = \boxed{8.36\ \text{m}}$$

6.74 The woman starts from rest $(v_{0y} = 0)$ and drops freely with $a_y = -g$ for 2.00 m before the impact with the toboggan. Then, $v_{2i}^2 = v_{0y}^2 + 2a_y(\Delta y)$ gives her speed just before impact as

$$v_{2i} = \sqrt{v_{0y}^2 + 2a_y(\Delta y)} = \sqrt{0 + 2(-9.80\ \text{m/s}^2)(-2.00\ \text{m})} = 6.26\ \text{m/s}$$

The sketches at the right show the situation just before and just after the woman's impact with the toboggan. Since no external forces impart any significant impulse directed parallel to the incline (+x-direction) to the system consisting of man, woman, and toboggan during the very brief duration of the impact, we will consider the total momentum parallel to the incline to be conserved. That is,



$$(m_1 + m_2)v_f = m_1 v_{1i} + m_2(v_{2i})_x = m_1 v_{1i} + m_2 v_{2i}\sin 30.0°$$

or the speed of the system *immediately* after impact is

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}\sin 30.0^\circ}{m_1 + m_2} = \frac{(90.0\text{ kg})(8.00\text{ m/s}) + (55.0\text{ kg})(6.26\text{ m/s})\sin 30.0^\circ}{90.0\text{ kg} + 55.0\text{ kg}} = \boxed{6.15\text{ m/s}}$$

6.75 First consider the motion of the block and embedded bullet from immediately after impact until the block comes to rest after sliding distance d across the horizontal table. During this time, a kinetic friction force $f_k = \mu_k n = \mu_k (M + m) g$, directed opposite to the motion, acts on the block. The net work done on the (block plus bullet) during this time is

$$W_{\text{net}} = (f_k \cos 180^\circ) d = KE_f - KE_i = 0 - \frac{1}{2}(M + m)V^2$$

so the speed, V, of the block and embedded bullet immediately after impact is

$$V = \sqrt{\frac{-2 f_k d}{-(M + m)}} = \sqrt{\frac{2\mu_k \cancel{(M + m)} gd}{\cancel{M + m}}} = \sqrt{2\mu_k gd}$$

Now, make use of conservation of momentum from just before to just after impact to obtain

$$p_{xi} = p_{xf} \qquad \Rightarrow \qquad mv_0 = (M + m)V = (M + m)\sqrt{2\mu_k gd}$$

and the initial velocity of the bullet was

$$\boxed{v_0 = \left(\frac{M + m}{m}\right)\sqrt{2\mu_k gd}}$$

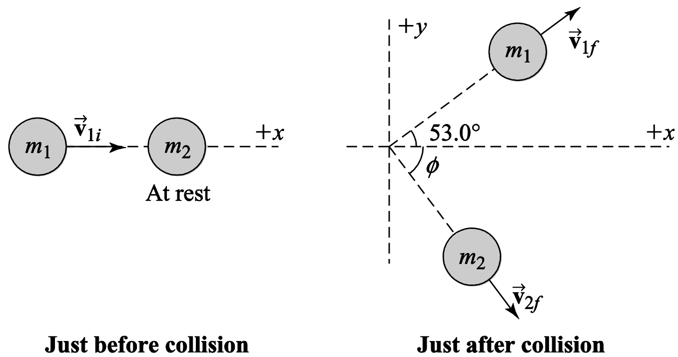
6.76 (a) Apply conservation of momentum in the vertical direction to the squid-water system from the instant before to the instant after the water is ejected. This gives

$$m_s v_s + m_w v_w = (m_s + m_w)(0) \qquad \text{or} \qquad v_s = -\left(\frac{m_w}{m_s}\right)v_w = -\left(\frac{0.30\text{ kg}}{0.85\text{ kg}}\right)(-20\text{ m/s}) = \boxed{7.1\text{ m/s}}$$

(b) Apply conservation of mechanical energy to the squid from the instant after the water is ejected until the squid reaches maximum height to find:

$$0 + m_s g y_f = \tfrac{1}{2} m_s v_s^2 + m g y_i \qquad \text{or} \qquad \Delta y = y_f - y_i = \frac{v_s^2}{2g} = \frac{(7.1 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = \boxed{2.6 \text{ m}}$$

6.77 (a)



The situations just before and just after the collision are shown above. Conserving momentum in both the *x*- and *y*-directions gives

$$(p_y)_f = (p_y)_i \;\Rightarrow\; m_1 v_{1f} \sin 53^\circ - m_2 v_{2f} \sin\phi = 0 \qquad \text{or} \qquad m_2 v_{2f} \sin\phi = m_1 v_{1f} \sin 53^\circ \qquad [1]$$

$$(p_x)_f = (p_x)_i \;\Rightarrow\; m_1 v_{1f} \cos 53^\circ + m_2 v_{2f} \cos\phi = m_1 v_{1i} + 0\,,$$

or

$$m_2 v_{2f} \cos\phi = m_1 v_{1i} - m_1 v_{1f} \cos 53^\circ \qquad [2]$$

Dividing equation [1] by [2] yields

$$\tan\phi = \frac{v_{1f} \sin 53^\circ}{v_{1i} - v_{1f}\cos 53^\circ} = \frac{(1.0 \text{ m/s})\sin 53^\circ}{(2.0 \text{ m/s}) - (1.0 \text{ m/s})\cos 53^\circ} = 0.57 \qquad \text{or} \qquad \boxed{\phi = 30^\circ}$$

Equation [1] then gives

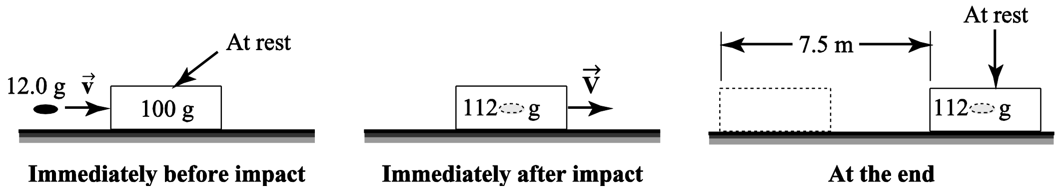
$$v_{2f} = \frac{m_1 v_{1f} \sin 53^\circ}{m_2 \sin\phi} = \frac{(0.20 \text{ kg})(1.0 \text{ m/s})\sin 53^\circ}{(0.30 \text{ kg})\sin 30^\circ} = \boxed{1.1 \text{ m/s}}$$

(b) The fraction of the incident kinetic energy lost in this collision is

$$\frac{|\Delta KE|}{KE_i} = \frac{KE_i - KE_f}{KE_i} = 1 - \frac{KE_f}{KE_i} = 1 - \frac{\tfrac{1}{2}(0.20 \text{ kg})(1.0 \text{ m/s})^2 + \tfrac{1}{2}(0.30 \text{ kg})(1.1 \text{ m/s})^2}{\tfrac{1}{2}(0.20 \text{ kg})(2.0 \text{ m/s})^2}$$

$$\frac{|\Delta KE|}{KE_i} = \boxed{0.30} \qquad \text{or} \qquad \boxed{30\%}$$

6.78



Using the work–energy theorem from immediately after impact to the end gives

$$W_{\text{net}} = f_k\left(\cos 180°\right)s = KE_{\text{end}} - KE_{\text{after}}$$

or

$$-\left[\mu_k\left(M+m\right)g\right]s = 0 - \frac{1}{2}\left(M+m\right)V^2 \quad \text{and} \qquad V = \sqrt{2\,\mu_k\,g\,s}$$

Then, using conservation of momentum from immediately before to immediately after impact gives $mv + 0 = \left(M+m\right)V$, or

$$v = \left(\frac{M+m}{m}\right)V = \left(\frac{M+m}{m}\right)\sqrt{2\,\mu_k\,g\,s} = \left(\frac{112\text{ g}}{12.0\text{ g}}\right)\sqrt{2\left(0.650\right)\left(9.80\text{ m/s}^2\right)\left(7.5\text{ m}\right)}$$

$$v = \boxed{91\text{ m/s}}$$