ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The magnitude of the impulse is

$$I = \Delta p = p_f - p_i = mv_f - mv_i = m\left(v_f - v_i\right)$$

or

$$I = \left(0.450 \text{ kg}\right)\left[\left(12.8 \text{ m/s}\right) - \left(3.20 \text{ m/s}\right)\right] = 4.32 \text{ kg} \cdot \text{m/s}$$

making (b) the correct choice.

2. The impulse given to the ball is $I = F_{\text{av}}(\Delta t) = mv_f - mv_i = m\,(v_f - v_i)$. Choosing the direction of the final velocity of the ball as the positive direction, this gives

$$F_{\text{av}} = \frac{m\left(v_f - v_i\right)}{\Delta t} = \frac{\left(57.0 \times 10^{-3} \text{ kg}\right)\left[\left(+25.0 \text{ m/s}\right) - \left(-21.0 \text{ m/s}\right)\right]}{0.060 \text{ s}} = 43.7 \text{ kg} \cdot \text{m/s}^2 = 43.7 \text{ N}$$

and the correct choice is (c).

3. Assuming that the collision was head-on so that, after impact, the wreckage moves in original direction of the car's motion, conservation of momentum during the impact gives

$$\left(m_c + m_t\right)v_f = m_c v_{0c} + m_t v_{0t} = m_c v + m_t\left(0\right)$$

or

$$v_f = \left(\frac{m_c}{m_c + m_t}\right)v = \left(\frac{m}{m + 2m}\right)v = \frac{v}{3}$$

showing that (c) is the correct choice.

4. The mass in motion after the rice ball is added to the bowl is twice the original moving mass. Therefore, to conserve momentum, the speed of the (rice ball + bowl) after the event must be one half of the initial speed of the bowl (i.e., $\upsilon_f = \upsilon_i/2$). The final kinetic energy is then

$$KE_f = \frac{1}{2} \; m_{\text{ball}} + m_{\text{bowl}} \; v_f^2 = \frac{1}{2} \; 2m_{\text{bowl}} \; \left(\frac{v_i}{2}\right)^2 = \frac{1}{2}\left(\frac{1}{2} m_{\text{bowl}} v_i^2\right) = \frac{E}{2}$$

and the correct choice is (c).

5. Billiard balls all have the same mass and collisions between them may be considered to be elastic. The dual requirements of conservation of kinetic energy and conservation of momentum in a one-dimensional, elastic collision are summarized by the two relations:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad [1]$$

and

$$v_{1i} - v_{2i} = - \; v_{1f} - v_{2f} \qquad [2]$$

In this case, $m_1 = m_2$ and the masses cancel out of the first equation. Call the cue ball #1 and the red ball #2 so that $v_{1i} = -3v$, $v_{2i} = +v$, $v_{1f} = v_{\text{cue}}$, and $v_{2f} = v_{\text{red}}$. Then, the two equations become:

$$-3v + v = v_{\text{cue}} + v_{\text{red}} \quad \text{or} \quad v_{\text{cue}} + v_{\text{red}} = -2v \qquad [1]$$

and

$$-3v - v = - \; v_{\text{cue}} - v_{\text{red}} \quad \text{or} \quad v_{\text{cue}} - v_{\text{red}} = 4v \qquad [2]$$

Adding the final versions of these equations yields $2v_{\text{cue}} = 2v$, or $v_{\text{cue}} = v$. Substituting this result into either [1] or [2] above then yields $v_{\text{red}} = -3v$. Thus, the correct response for this question is (c).

6. We choose the original direction of motion of the cart as the positive direction. Then, $v_i = 6$ m/s and $v_f = -2$ m/s. The change in the momentum of the cart is

$$\Delta p = mv_f - mv_i = m \; v_f - v_i = 5\text{ kg} \;\; -2 \text{ m/s} - 6 \text{ m/s} = -40 \text{ kg} \cdot \text{m/s}$$

and choice (c) is the correct answer.

7. As in question 5 above, the requirements of conserving both momentum and kinetic energy in this one-dimensional, elastic collision are summarized by the equations

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad [1]$$

and

$$v_{1i} - v_{2i} = - \; v_{1f} - v_{2f} \qquad [2]$$

Choosing eastward as the positive direction, we have $m_1 = 0.10$ kg, $v_{1i} = +0.20$ m/s, $m_2 = 0.15$ kg, and $v_{2i} = 0$. The general equations then become

$$(0.10\text{ kg})(+0.20\text{ m/s}) + (0.15\text{ kg})(0) = (0.10\text{ kg})\, v_{1f} + (0.15\text{ kg})\, v_{2f}$$

or, after simplifying,

$$v_{1f} + 1.50 v_{2f} = 0.20\text{ m/s} \qquad [1]$$

and

$$0.20\text{ m/s} - 0 = -v_{1f} + v_{2f} \qquad \text{or} \qquad v_{2f} = v_{1f} + 0.20\text{ m/s} \qquad [2]$$

Substitute the final version of [2] into the final version of [1] to obtain $2.50 v_{1f} = -0.10$ m/s yielding

$$v_{1f} = -0.040\text{ m/s} \qquad \text{or} \qquad v_{1f} = 0.040\text{ m/s westward}$$

This means that (d) the correct choice for this question.

8. First, consider the motion of the block, with embedded bullet, from just after impact until it comes to rest. During this time, the only force doing work on the block is the friction force between it and the horizontal surface. The work–energy theorem then gives

$$W_{\text{net}} = f_k \cos 180^\circ \;\; \Delta x = \frac{1}{2} m_{\text{total}} v_f^2 - \frac{1}{2} m_{\text{total}} v_i^2$$

Since $v_f = 0$ and $f_k = \mu_k n = \mu_k (m_{\text{total}} g)$, this becomes

$$-\mu_k \left(\cancel{m_{\text{total}}}\, g\right) \Delta x = 0 - \frac{1}{2} \cancel{m_{\text{total}}}\, v_i^2 \qquad \text{or} \qquad v_i = \sqrt{2 \mu_k g \;\; \Delta x}$$

and the speed of the (block with embedded bullet) just after impact is

$$v_i = \sqrt{2 \;\; 0.400 \;\;\; 9.80 \;\; \text{m/s}^2 \;\;\; 8.00 \text{ m} \;\;} = 7.92 \;\; \text{m/s}$$

Conservation of momentum from just before impact to just after gives $m_{\text{bullet}} v_0 = m_{\text{total}} v$, so the speed of the bullet before impact is

$$v_0 = \left(\frac{m_{\text{total}}}{m_{\text{bullet}}}\right) v_i = \left(\frac{0.200\ \text{kg} + 0.004\ \text{kg}}{0.004\ \text{kg}}\right) 7.92\ \text{m/s} = 404\ \text{m/s}$$

which is seen to be choice (d).

9. With the kinetic energy written as $KE = p^2/2m$, we solve for the magnitude of the momentum as $p = \sqrt{2m\ KE}$. The ratio of the final momentum of the rocket to its initial momentum is then given by

$$\frac{p_f}{p_i} = \frac{\sqrt{2m_f\ KE_f}}{\sqrt{2m_i\ KE_i}} = \sqrt{\left(\frac{m_f}{m_i}\right)\frac{KE_f}{KE_i}} = \sqrt{\left(\frac{1}{2}\right)8} = 2 \qquad \text{or} \qquad p_f = 2p_i$$

and the correct choice is (a).

10. The kinetic energy of a particle may be written as

$$KE = \frac{mv^2}{2} = \frac{m^2v^2}{2m} = \frac{(mv)^2}{2m} = \frac{p^2}{2m}.$$

The ratio of the kinetic energies of two particles is then

$$\frac{KE_2}{KE_1} = \frac{p_2^2/2m_2}{p_1^2/2m_1} = \left(\frac{p_2}{p_1}\right)^2\left(\frac{m_1}{m_2}\right)$$

We see that, if the magnitudes of the momenta are equal ($p_2 - p_1$), the kinetic energies will be equal only if the masses are also equal. The correct response is then (c).

11. Expressing the kinetic energy as $KE = p^2/2m$ (see questions 9 and 10), we see that the ratio of the magnitudes of the momenta of two particles is

$$\frac{p_2}{p_1} = \frac{\sqrt{2m_2\ KE_2}}{\sqrt{2m_1\ KE_1}} = \sqrt{\left(\frac{m_2}{m_1}\right)\frac{KE_2}{KE_1}}$$

Thus, we see that if the particles have equal kinetic energies $[(KE)_2 = (KE)_1]$, the magnitudes of their momenta are equal only if the masses are also equal. However, momentum is a *vector quantity* and we can say the two particles have equal momenta only it both the magnitudes and directions are equal, making choice (d) the correct answer.

12. Consider the sketches at the right. The leftmost sketch shows the rocket immediately after the engine is fired (while the rocket's velocity is still essentially zero). It has two forces acting on it, an upward thrust F exerted by the burnt fuel being ejected from the engine, and a downward force of gravity. These forces produce the upward acceleration of the rocket according to Newton's second law:

$$\Sigma F^{\upsilon} = F - F_g = Ma$$

Since $F_g = Mg$, the thrust exerted on the rocket by the ejected fuel is

$$F = F_g + Ma = M\,(a + g)$$

The rightmost part of the sketch shows a quantity of burnt fuel that was initially at rest within the rocket, but a very short time Δt later is moving downward at speed v. As this material is ejected, it exerts the upward thrust F on the rocket. By Newton's third law, the rocket exerts a downward force of equal magnitude on this burnt fuel. This force imparts an impulse $I = F(\Delta t) = \Delta p = \Delta m(\upsilon - 0)$ to the ejected material. Thus, the rate the rocket is burning and ejecting fuel must be

$$\frac{\Delta m}{\Delta t} = \frac{F}{v - 0} = \frac{M\ \ a + g}{v} = \frac{3.00 \times 10^5\ \text{kg}\left[\ 36.0 + 9.80\quad \text{m/s}^2\right]}{4.50 \times 10^3\ \ \text{m/s}} = 3.05 \times 10^3\ \ \text{kg/s}$$

and we see that choice (a) is the correct response.

Note: Failure to include the gravitational force in this analysis will lead some students to incorrectly select choice (b) as their answer.