PROBLEM SOLUTIONS

5.1 If the weights are to move at constant velocity, the net force on them must be zero. Thus, the force exerted on the weights is upward, parallel to the displacement, with magnitude 350 N. The work done by this force is

$$W = (F\cos\theta)s = \left[(350\text{ N})\cos 0°\right](2.00\text{ m}) = \boxed{700\text{ J}}$$

5.2 (a) We assume the object moved upward with constant speed, so the kinetic energy did not change. Then, the

work–energy theorem gives the work done on the object by the lifter as

$$W_{nc} = \Delta KE + \Delta PE = 0 + (mgy_f - mgy_i) = mg(\Delta y)\text{, or}$$

$$W_{nc} = (281.5\text{ kg})(9.80\text{ m/s}^2)\left[(17.1\text{ cm})(1\text{ m}/10^2\text{ cm})\right] = \boxed{472\text{ J}}$$

(b) If the object moved upward at constant speed, the net force acting on it was zero. Therefore, the magnitude of

the upward force applied by the lifter must have been equal to the weight of the object

$$F = mg = (281.5\text{ kg})(9.80\text{ m/s}^2) = 2.76\times 10^3\text{ N} = \boxed{2.76\text{ kN}}$$

5.3 Assuming the mass is lifted at constant velocity, the total upward force exerted by the two men equals the weight of the mass: $F_{\text{total}} = mg = (653.2\text{ kg})(9.80\text{ m/s}^2) = 6.40\times 10^3\text{ N}$. They exert this upward force through a total upward displacement of 96 inches or 8 feet (4 inches per lift for each of 24 lifts). Thus, the total work done during the upward movements of the 24 lifts is

$$W = (F\cos\theta)\Delta x = (F_{\text{total}}\cos 0°)\Delta x = (6.40\times 10^5\text{ N})(\cos 0°)\left[8\text{ ft}\left(\frac{1\text{ m}}{3.281\text{ ft}}\right)\right] = \boxed{2\times 10^4\text{ J}}$$

5.4 (a) The 35 N force applied by the shopper makes a 25° angle with the displacement of the cart (horizontal). The

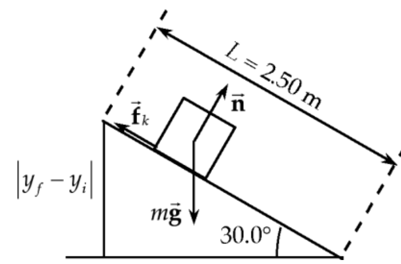
work done on the cart by the shopper is then

$$W_{\text{shopper}} = (F\cos\theta)\Delta x = (35\text{ N})(\cos 25°)(50.0\text{ m}) = \boxed{1.6\times 10^3\text{ J}}$$

(b) Since the speed of the cart is constant, $KE_f = KE_i$ and $W_{\text{net}} = \Delta KE = \boxed{0}$.

(c) Since the cart continues to move at constant speed, the net work done on the cart in the second aisle is again

zero. With both the net work and the work done by friction unchanged, the work done by the shopper $W_{\text{shopper}} = W_{\text{net}} - W_{\text{fric}}$ is also $\boxed{\text{unchanged}}$. However, the shopper now pushes horizontally on the cart, making $\boxed{F' = W_{\text{shopper}}/(\Delta x \cdot \cos 0°) = W_{\text{shopper}}/\Delta x \ \text{ smaller than before}}$ when the force was $F = W_{\text{shopper}}/\ \Delta x \cdot \cos 35°$



5.5 (a) The gravitational force acting on the object is

$$w = mg = \ 5.00 \text{ kg} \quad 9.80 \ \text{m/s}^2 \ = 49.0 \text{ N}$$

and the work done by this force is

$$W_g = -\Delta PE_g = -mg \ \ y_f - y_i \ = +w \ \ y_i - y_f$$

or

$$W_g = w \ \ L \sin 30.0° = \ 49.0 \text{ N} \quad 2.50 \text{ m} \ \sin 30.0° = \boxed{61.3 \text{ J}}$$

(b) The normal force exerted on the block by the incline is $n = mg\cos 30.0°$, so the friction force is

$$f_k = \mu_k n = \ 0.436 \quad 49.0 \text{ N} \ \cos 30.0° = 18.5 \text{ N}$$

This force is directed opposite to the displacement (that is θ= 180°), and the work it does is

$$W_f = \ f_k \cos\theta \ \ L = \left[\ 18.5 \text{ N} \ \cos 180°\right] \ 2.50 \text{ m} \ = \boxed{-46.3 \text{ J}}$$

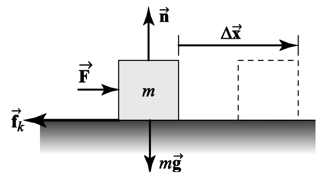
(c) Since the normal force is perpendicular to the displacement, the work done by the normal force is.

$$W_n = \ n\cos 90.0° \ L = \boxed{0}$$

(d) If a shorter ramp is used to increase the angle of inclination while maintaining the same vertical displacement $\left|y_f - y_i\right|$, the $\boxed{\text{work done by gravity will not change}}$, the $\boxed{\text{work done by the friction}}$

$\boxed{\text{force will decrease}}$ (because the normal force, and hence the friction force, will decrease and also because the ramp length L decreases), and the $\boxed{\text{work done by the normal force remains zero}}$ (because the normal force remains perpendicular to the displacement).

5.6 (a) $W_F = F\ \Delta x\ \cos\theta = \ 150\text{ N}\ \ 6.00\text{ m}\ \cos 0° = \boxed{900\text{ J}}$



(b) Since the crate moves at constant velocity, Thus, $a_x = a_y = 0.$

$$\Sigma F_x = 0 \ \Rightarrow\ f_k = F = 150\text{ N}$$

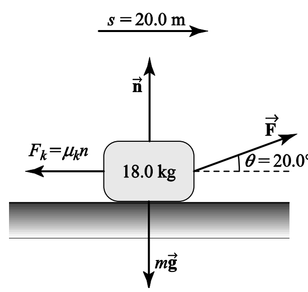
Also,

$$\Sigma F_y = 0 \ \Rightarrow\ n = mg = \ 40.0\text{ kg}\ \ 9.80\ \text{m/s}^2\ = 392\text{ N}$$

so

$$\mu_k = \frac{f_k}{n} = \frac{150\text{ N}}{392\text{ N}} = \boxed{0.383}$$

5.7 (a) $\Sigma F_y = F\sin\theta + n - mg = 0$



$$n = mg - F\sin\theta$$

$$\Sigma F_x = F\cos\theta - \mu_k n = 0$$

$$n = \frac{F\cos\theta}{\mu_k}$$

$$\therefore \quad mg - F\sin\theta = \frac{F\cos\theta}{\mu_k}$$

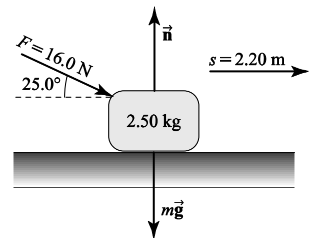
$$F = \frac{\mu_k mg}{\mu_k\sin\theta + \cos\theta} = \frac{0.500\ \ 18.0\text{ kg}\ \ 9.80\ \text{m/s}^2}{0.500\ \sin 20.0° + \cos 20.0°} = \boxed{79.4\text{ N}}$$

(b) $W_F = (F\cos\theta)s = [(79.4\text{ N})\cos 20.0°](20.0\text{ m}) = 1.49\times10^3\text{ J} = \boxed{1.49\text{ kJ}}$

(c) $f_k = F\cos\theta = 74.6\text{ N}$

$W_f = (f_k\cos\theta)s = [(74.6\text{ N})\cos 180°](20.0\text{ m}) = -1.49\times10^3\text{ J} = \boxed{-1.49\text{ kJ}}$

5.8 (a) $W_F = (F\cos\theta)s = [(16.0\text{ N})\cos 25.0°](2.20\text{ m})$



$W_F = \boxed{31.9\text{ J}}$

(b) $W_n = (n\cos 90°)s = \boxed{0}$

(c) $W_g = (mg\cos 90°)s = \boxed{0}$

(d) $W_{\text{net}} = W_F + W_n + W_g = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

5.9 (a) The work–energy theorem, $W_{\text{net}} = KE_f - KE_i$, gives

$5000\text{ J} = \frac{1}{2}(2.50\times10^3\text{ kg})v^2 - 0$, or $v = \boxed{2.00\text{ m/s}}$

(b) $W = (F\cos\theta)s = (F\cos 0°)(25.0\text{ m}) = 5000\text{ J}$, so $F = \boxed{200\text{ N}}$

5.10 Requiring that $KE_{\text{ping pong}} = KE_{\text{bowling}}$ with $KE = \frac{1}{2}mv^2$, we have

$\frac{1}{2}(2.45\times10^{-3}\text{ kg})v^2 = \frac{1}{2}(7.00\text{ kg})(3.00\text{ m/s})^2$

giving $v = \boxed{160\text{ m/s}}$.

5.11 (a) $KE_i = \frac{1}{2}mv_i^2$ where $v_i^2 = v_{0x}^2 + v_{0y}^2 = (6.00\text{ m/s})^2 + (-2.00\text{ m/s})^2 = 40.0\text{ m}^2/\text{s}^2$,

giving $KE_i = \frac{1}{2}(5.75\text{ kg})(40.0\text{ m}^2/\text{s}^2) = \boxed{115\text{ J}}$.

(b) $KE_f = \frac{1}{2}mv_f^2$ where $v_f^2 = v_x^2 + v_y^2 = (8.50\text{ m/s})^2 + (5.00\text{ m/s})^2 = 97.3\text{ m}^2/\text{s}^2$, so , $KE_f = \frac{1}{2}\left(5.75\text{ kg}\right)\left(97.3\text{ m}^2/\text{s}^2\right) = 280\text{ J}$, and the *change* in kinetic energy has been

$$KE_f - KE_i = 280\text{ J} - 115\text{ J} = \boxed{165\text{ J}}$$

5.12 (a) Since the applied force is horizontal, it is in the direction of the displacement giving $\theta = 0°$. The work done by this force is then

$$W_{F_0} = \left(F_0\cos\theta\right)\Delta x = \left(F_0\cos 0°\right)\Delta x = F_0\left(\Delta x\right)$$

and

$$F_0 = \frac{W_{F_0}}{\Delta x} = \frac{350\text{ J}}{12.0\text{ m}} = \boxed{29.2\text{ N}}$$

(b) Since the crate originally had zero acceleration, the original applied force was just enough to offset the retarding friction force. Therefore, when the applied force is increased, it has a magnitude greater than the friction force. This gives the crate a resultant force (and hence an acceleration) in the direction of motion, meaning the $\boxed{\text{speed of the crate will increase with time}}$.

(c) If the applied force is made smaller than F_0, the magnitude of the friction force will be greater than that of the applied force. This means the crate has a resultant force, and acceleration, in the direction of the friction force (opposite to the direction of motion).

$\boxed{\text{The crate will now slow down and come to rest}}$.

5.13 (a) We use the work–energy theorem to find the work.

$$W = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0 - \frac{1}{2}\left(70\text{ kg}\right)\left(4.0\text{ m/s}\right)^2 = \boxed{-5.6\times 10^2\text{ J}}$$

(b) $W = \left(F\cos\theta\right)s = \left(f_k\cos 180°\right)s = \left(-\mu_k n\right)s = \left(-\mu_k mg\right)s$,

so

$$s = -\frac{W}{\mu_k mg} = -\frac{\left(-5.6\times 10^2\text{ J}\right)}{\left(0.70\right)\left(70\text{ kg}\right)\left(9.80\text{ m/s}^2\right)} = \boxed{1.2\text{ m}}$$

5.14 At the top of the arc, $v_y = 0$, and $v_x = v_{0x} = v_0 \cos 30.0° = 34.6 \text{ m/s}$.

Therefore, $v^2 = v_x^2 + v_y^2 = (34.6 \text{ m/s})^2$, and

$$KE = \frac{1}{2} m v^2 = \frac{1}{2}(0.150 \text{ kg})(34.6 \text{ m/s})^2 = \boxed{90.0 \text{ J}}$$

5.15 (a) As the bullet penetrates the tree trunk, the only force doing work on it is the force of resistance exerted by the trunk. This force is directed opposite to the displacement, so the work done is $W_{\text{net}} = (f \cos 180°)\, \Delta x = KE_f - KE_i$, and the magnitude of the force is

$$f = \frac{KE_f - KE_i}{(\Delta x)\cos 180°} = \frac{0 - \frac{1}{2}(7.80 \times 10^{-3} \text{ kg})(575 \text{ m/s})^2}{-(5.50 \times 10^{-2} \text{ m})} = \boxed{2.34 \times 10^4 \text{ N}}$$

(b) If the friction force is constant, the bullet will have a constant acceleration and its average velocity while

stopping is $\bar{v} = (v_f + v_i)/2$. The time required to stop is then

$$\Delta t = \frac{\Delta x}{\bar{v}} = \frac{2(\Delta x)}{v_f + v_i} = \frac{2(5.50 \times 10^{-2} \text{ m})}{0 + 575 \text{ m/s}} = \boxed{1.91 \times 10^{-4} \text{ s}}$$

5.16 (a) $KE_A = \frac{1}{2} m v_A^2 = \frac{1}{2}(0.60 \text{ kg})(2.0 \text{ m/s})^2 = \boxed{1.2 \text{ J}}$

(b) $KE_B = \frac{1}{2} m v_B^2$, so

$$v_B = \sqrt{\frac{2(KE_B)}{m}} = \sqrt{\frac{2(7.5 \text{ J})}{0.60 \text{ kg}}} = \boxed{5.0 \text{ m/s}}$$

(c) $W_{\text{net}} = \Delta KE = KE_B - KE_A = (7.5 - 1.2) \text{ J} = \boxed{6.3 \text{ J}}$

5.17 $W_{\text{net}} = (F_{\text{road}} \cos \theta_1) s + (F_{\text{resist}} \cos \theta_2) s = \left[(1\,000 \text{ N})\cos 0°\right] s + \left[(950 \text{ N})\cos 180°\right] s$

$W_{\text{net}} = (1\,000 \text{ N} - 950 \text{ N})(20 \text{ m}) = 1.0 \times 10^3 \text{ J}$

Also, $W_{\text{net}} = KE_f - KE_i = \frac{1}{2}m\text{v}^2 - 0$, so

$$\text{v} = \sqrt{\frac{2\,W_{\text{net}}}{m}} = \sqrt{\frac{2\left(1.0 \times 10^3\ \text{J}\right)}{2000\ \text{kg}}} = \boxed{1.0\ \text{m/s}}$$

5.18 The initial kinetic energy of the sled is

$$KE_i = \frac{1}{2}m\text{v}_i^2 = \frac{1}{2}\left(10\ \text{kg}\right)\left(2.0\ \text{m/s}\right)^2 = 20\ \text{J}$$

and the friction force is $f_k = \mu_k n = \mu_k\, mg = \left(0.10\right)\left(98\ \text{N}\right) = 9.8\ \text{N}$.

$$W_{net} = \left(f_k \cos 180°\right) s = KE_f - KE_i \text{, so } s = \frac{0 - KE_i}{f_k \cos 180°} = \frac{-20\ \text{J}}{-9.8\ \text{N}} = \boxed{2.0\ \text{m}}$$

5.19 With only a conservative force acting on the falling ball,

$$\left(KE + PE_g\right)_i = \left(KE + PE_g\right)_f \qquad \text{or} \qquad \frac{1}{2}m\text{v}_i^2 + mgy_i = \frac{1}{2}m\text{v}_f^2 + mgy_h$$

Applying this to the motion of the ball gives $0 + mgy_i = \frac{1}{2}m\text{v}_f^2 + 0$

$$\text{or} \quad y_i = \frac{\text{v}_f^2}{2g} = \frac{\left(9.0\ \text{m/s}\right)^2}{2\left(9.80\ \text{m/s}^2\right)} = \boxed{4.1\ \text{m}}$$

5.20 (a) The force stretching the spring is the weight of the suspended object. Therefore, the force constant of the spring is

$$k = \left|\frac{F_g}{\Delta x}\right| = \frac{mg}{\left|\Delta x\right|} = \frac{\left(2.50\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{2.76 \times 10^{-2}\ \text{m}} = \boxed{8.88 \times 10^2\ \text{N/s}}$$

(b) If a 1.25-kg block replaces the original 2.50-kg suspended object, the force applied to the spring (weight of the suspended object) will be one-half the original stretching force. Since, for a spring obeying Hooke's law, the elongation is directly proportional to the stretching force, the amount the spring stretches now is

$$\Delta x_2 = \frac{1}{2}\left(\Delta x\right)_1 = \frac{1}{2}\left(2.76 \text{ cm}\right) = \boxed{1.38 \text{ cm}}$$

(c) The work an external agent must do *on* the initially unstretched spring to produce an elongation x_f is equal to

the potential energy stored in the spring at this elongation

$$W_{\substack{\text{done } on \\ \text{spring}}} = \left(PE_s\right)_f - \left(PE_s\right)_i = \frac{1}{2}kx_f^2 - 0 = \frac{1}{2}\left(8.88 \times 10^2 \text{ N/m}\right)\left(8.00 \times 10^{-2} \text{ m}\right)^2 = \boxed{2.84 \text{ J}}$$

5.21 The magnitude of the force a spring must exert on the 3.65-g object to give that object an acceleration of $a = 0.500g = 4.90 \text{ m/s}^2$ is given by Newton's second law as

$$F = ma = \left(3.65 \times 10^{-3} \text{ kg}\right)\left(4.90 \text{ m/s}^2\right) = 1.79 \times 10^{-2} \text{ N}$$

Then, by Newton's third law, this object exerts an oppositely directed force of equal magnitude in the on the spring. If this reaction force is to stretch the spring 0.350 cm, the required force constant of the spring is

$$k = \frac{F}{\Delta x} = \frac{1.79 \times 10^{-2} \text{ N}}{0.350 \text{ cm}} = \frac{1.79 \times 10^{-2} \text{ N}}{3.50 \times 10^{-3} \text{ m}} = \boxed{5.11 \text{ N/m}}$$

5.22 (a) While the athlete is in the air, the interacting objects are $\boxed{\text{the athlete and Earth}}$. They interact $\boxed{\text{through the gravitation force}}$ that one exerts on the other.

(b) If the athlete leaves the trampoline (at the $y = 0$ level) with an initial speed of, $v_i = 9.0 \text{ m/s}$ her initial kinetic energy is

$$KE_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\left(60.0 \text{ kg}\right)\left(9.0 \text{ m/s}\right)^2 = \boxed{2.4 \times 10^3 \text{ J}}$$

and her gravitational potential energy is $\left(PE_g\right)_i = mgy_i = mg\left(0\right) = \boxed{0 \text{ J}}$.

(c) When the athlete is at maximum height, she is momentarily at rest and $KE_f = \boxed{0 \text{ J}}$ Because the only force acting on the athlete during her flight is the conservative gravitation force, her total energy (kinetic plus potential) remains constant. Thus, the decrease in her kinetic energy as she goes from the launch point $(\text{where } PE_g = 0)$ to maximum height is matched by an equal size increase in the gravitational potential energy.

$$\Delta PE_g = -\Delta KE \quad \Rightarrow \quad PE_f - PE_i = -\left(KE_f - KE_i\right) \quad \text{or} \quad PE_f = PE_i + \left(KE_i - KE_f\right)$$

and

$$PE_f = 0 + \left(2.4 \times 10^3 \text{ J} - 0\right) = \boxed{2.4 \times 10^3 \text{ J}}$$

(d) The statement that the athlete's total energy is conserved is summarized by the equation $\Delta KE + \Delta PE = 0$ or $KE_2 + PE_2 = KE_1 + PE_1$. In terms of mass, speed, and height, this becomes $\boxed{\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_1^2 + mgy_1}$. Solving for the final height gives

$$y_2 = \frac{mgy_1 + \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2}{mg} \quad \text{or} \quad \boxed{y_2 = y_1 + \frac{v_1^2 - v_2^2}{2g}}$$

The given numeric values for this case are $y_1 = 0$, $v_1 = 9.0 \text{ m/s}$ (at the trampoline level) and $v_2 = 0$ $(\text{at maximum height})$. The maximum height attained is then

$$y_2 = y_1 + \frac{v_1^2 - v_2^2}{2g} = 0 + \frac{\left(9.0 \text{ m/s}\right)^2 - 0}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{4.1 \text{ m}}$$

(e) Solving the energy conservation equation given in part (d) for the final speed gives

$$v_2^2 = \frac{2}{m}\left(\frac{1}{2}mv_1^2 + mgy_1 - mgy_2\right) \quad \text{or} \quad \boxed{v_2 = \pm\sqrt{v_1^2 + 2g\left(y_1 - y_2\right)}}$$

With $y_1 = 0$, $v_1 = 9.0 \text{ m/s}$ and $y_2 = y_{max}/2 = \left(4.1 \text{ m}\right)/2$, the speed at half the maximum height is given as

$$v_2 = \pm\sqrt{\left(9.0 \text{ m/s}\right)^2 + 2\left(9.80 \text{ m/s}^2\right)\left(0 - \frac{4.1 \text{ m}}{2}\right)} = \boxed{\pm 6.4 \text{ m/s}}$$

5.23 The work the beam does on the pile driver is given by

$$W_{nc} = F\cos 180° \ \Delta x = -F \ 0.180 \text{ m}$$

Here, the factor $\cos 180°$ is included because the force F exerted on the driver by the beam is directed upward, but the $\Delta x = 18.0 \text{ cm} = 0.180 \text{ m}$ displacement undergone by the driver while in contact with the beam is directed downward.

From the work–energy theorem, this work can also be expressed as

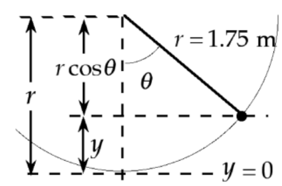
$$W_{nc} = KE_f - KE_i \ + \ PE_f - PE_i \ = \frac{1}{2}m \ v_f^2 - v_i^2 \ + mg \ y_f - y_i$$

Choosing $y = 0$ at the level where the pile driver first contacts the top of the beam, the driver starts from rest $v_i = 0$ at $y_i = +7.50 \text{ m}$ and comes to rest again at $v_f = 0$ at $y_f = -0.180 \text{ m}$. Therefore, we have

$$-F \ 0.180 \text{ m} \ = \frac{1}{2}m \ 0 - 0 \ + \ 2\,300 \text{ kg} \ \ 9.80 \text{ m/s}^2 \ \ -0.180 \text{ m} - 7.50 \text{ m}$$

yielding $F = \dfrac{-1.73 \times 10^5 \text{ J}}{-0.180 \text{ m}} = \boxed{9.62 \times 10^5 \text{ N directed upward}}$

5.24 When the child swings at the end of the ropes, she follows a circular path of radius $r = 1.75$ as shown at the right. If we choose $y = 0$ at the level of the child's lowest position on this path, then her y-coordinate when the rope is at θ angle with the vertical is $y = r - r\cos\theta = r \ 1 - \cos\theta$. Thus, her gravitational potential energy at this position is



$$PE_g = mgy = mgr \ 1 - \cos\theta \ = wr \ 1 - \cos\theta$$

(a) When the ropes are horizontal, $\theta = 90.0°$, and

$$PE_g = \ 3.50 \times 10^2 \text{ N} \ \ 1.75 \text{ m} \ \ 1 - \cos 90.0° = \boxed{+613 \text{ J}}$$

(b) When $\theta = 30.0°$, $PE_g = \ 3.50 \times 10^2 \text{ N} \ \ 1.75 \text{ m} \ \ 1 - \cos 30.0° = \boxed{+82.1 \text{ J}}$.

(c) At the bottom of the arc, $y = 0$, $\cos\theta = \cos 0° = 1$, and $PE_g = \boxed{0}$.

5.25 While the motorcycle is in the air, only the conservative gravitational force acts on cycle and rider. Thus, $\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$, which gives

$$\Delta y = y_f - y_i = \frac{v_i^2 - v_f^2}{2g} = \frac{35.0\ \text{m/s}^{\ 2} - 33.0\ \text{m/s}^{\ 2}}{2\ \ 9.80\ \text{m/s}^2} = \boxed{6.94\ \text{m}}$$

Note that this answer gives the maximum height of the cycle above the end of the ramp, which is an unknown distance above the ground.

5.26 (a) When equilibrium is reached, the total spring force supporting the load equals the weight of the load, or $F_{s,\,\text{total}} = F_{s,\,\text{leaf}} + F_{s,\,\text{helper}} = w_{\text{load}}$. Let k_ℓ and k_h represent the spring constants of the leaf spring and the helper spring, respectively. Then, if x_ℓ is the distance the leaf spring is compressed, the condition for equilibrium becomes

$$k_\ell x_\ell + k_h\ \ x_\ell - y_0\ \ = w_{\text{load}}$$

or

$$x_\ell = \frac{w_{\text{load}} + k_h y_0}{k_\ell + k_h} = \frac{5.00 \times 10^5\ \text{N} +\ \ 3.60 \times 10^5\ \ \text{N/m}\ \ \ 0.500\ \text{m}}{5.25 \times 10^5\ \ \text{N/m} + 3.60 \times 10^5\ \ \text{N/m}} = \boxed{0.768\ \text{m}}$$

(b) The work done compressing the springs equals the total elastic potential energy at equilibrium. Thus,

$$W = \tfrac{1}{2}k_\ell x_\ell^2 + \tfrac{1}{2}k_h\ \ x_\ell - 0.500\ \text{m}^{\ 2}$$

or

$$W = \frac{1}{2}\ \ 5.25 \times 10^5\ \ \text{N/m}\ \ \ 0.768\ \text{m}^{\ 2} + \tfrac{1}{2}\ \ 3.60 \times 10^5\ \ \text{N/m}\ \ \ 0.268\ \text{m}^{\ 2} = \boxed{1.68 \times 10^5\ \text{J}}$$

5.27 The total work done by the two bicep muscles as they contract is

$$W_{\text{biceps}} = 2F_{\text{av}}\Delta x = 2\ \ 800\ \text{N}\ \ \ 0.075\ \text{m}\ \ = \boxed{1.2 \times 10^2\ \text{J}}$$

The total work done on the body as it is lifted 40 cm during a chin-up is

$$W_{\text{chin-up}} = mgh =\ \ 75\ \text{kg}\ \ \ 9.80\ \text{m/s}^2\ \ \ 0.40\ \text{m}\ \ = \boxed{2.9 \times 10^2\ \text{J}}$$

Since $W_{\text{chin-up}} > W_{\text{biceps}}$, it is clear that $\boxed{\text{addition muscles must be involved}}$.

5.28 Applying $W_{nc} = KE + PE_{f} - KE + PE_{i}$ to the jump of the "original" flea gives

$$F_m d = 0 + mgy_f - 0 + 0 \qquad \text{or} \qquad y_f = \frac{F_m d}{mg}$$

where F_m is the force exerted by the muscle and d is the length of contraction.

If we scale the flea by a factor *f*, the muscle force increases by f^2 and the length of contraction increases by *f*. The mass, being proportional to the volume which increases by f^3, will also increase by f^3. Putting these factors into our expression for y_f gives

$$y_{f\,\substack{\text{super}\\\text{flea}}} = \frac{f^2 F_m \quad fd}{f^3 m \quad g} = \frac{F_m d}{mg} = y_f \approx \boxed{0.5\text{ m}}$$

so the "super flea" cannot jump any higher!

This analysis is used to argue that most animals should be able to jump to approximately the same height (~0.5 m). Data on mammals from elephants to shrews tend to support this.

5.29 (a) Taking *y* = 0, and hence $PE_g = mgy = 0$ at ground level, the initial total mechanical energy of the project tile is

$$E_{\text{total}\ i} = KE_i + PE_i = \frac{1}{2} m v_i^2 + mgy_i$$
$$= \frac{1}{2}\ 50.0\text{ kg}\quad 1.20 \times 10^2\ \text{m/s}^{\ 2} + 50.0\text{ kg}\quad 9.80\ \text{m/s}^2\quad 142\text{ m} = \boxed{4.30 \times 10^5\text{ J}}$$

(b) The work done on the projectile is equal to the change in its total mechanical energy.

$$W_{nc\ \text{rise}} = KE_f + PE_f - KE_i + PE_i = \frac{1}{2} m\ v_f^2 - v_i^2 + mg\ y_f - y_i$$
$$= \frac{1}{2}\ 50.0\text{ kg}\left[85.0\text{ m/s}^{\ 2} - 120\text{ m/s}^{\ 2}\right] + 50.0\text{ kg}\quad 9.80\ \text{m/s}^2\quad 427\text{ m} - 142\text{ m}$$
$$= \boxed{-3.97 \times 10^4\text{ J}}$$

(c) If, during the descent from the maximum height to the ground, air resistance does one and a half times as much work on the projectile as it did while the projectile was rising to the top of the arc, the total work done for the entire trip will be

$$W_{nc\ \text{total}} = W_{nc\ \text{rise}} + W_{nc\ \text{descent}} = W_{nc\ \text{rise}} + 1.50\ W_{nc\ \text{rise}}$$
$$= 2.50\ -3.97 \times 10^4\text{ J} = -9.93 \times 10^4\text{ J}$$

Then, applying the work–energy theorem to the entire flight of the projectile gives

$$W_{nc\ \text{total}} = (KE + PE)_{\substack{\text{just before}\\ \text{hitting ground}}} - (KE + PE)_{\text{at launch}} = \frac{1}{2}m\mathrm{v}_f^2 + mgy_f - \frac{1}{2}m\mathrm{v}_i^2 - mgy_i$$

and the speed of the projectile just before hitting the ground is

$$\mathrm{v}_f = \sqrt{\frac{2\left(W_{nc}\right)_{\text{total}}}{m} + \mathrm{v}_i^2 + 2g\left(y_i - y_f\right)}$$

$$= \sqrt{\frac{2\left(-9.93 \times 10^4\ \text{J}\right)}{50.0\ \text{kg}} + \left(120\ \text{m/s}\right)^2 + 2\left(9.80\ \text{m/s}^2\right)\left(142\ \text{m} - 0\right)} = \boxed{115\ \text{m/s}}$$

5.30 (a) The work done by the gravitational force equals the decrease in the gravitational potential energy, or

$$W_g = -\left(PE_f - PE_i\right) = PE_i - PE_f = mg\left(y_i - y_f\right) = \boxed{mgh}$$

(b) The change in kinetic energy is equal to the net work done on the projectile, which in the absence of air resistance is just that done by the gravitational force. Thus,

$$W_{\text{net}} = W_g = \Delta KE \quad \Rightarrow \quad \Delta KE = \boxed{mgh}$$

(c) $\Delta KE = KE_f - KE_i = mgh$ so $KE_f = KE_i + mgh = \boxed{\frac{1}{2}m\mathrm{v}_0^2 + mgh}$

(d) $\boxed{\text{No}}$. None of the calculations in Parts (a), (b), and (c) involve the initial angle.

5.31 (a) The system will consist of $\boxed{\text{the mass, the spring, and Earth}}$. The parts of this system interact via $\boxed{\text{the spring force, the gravitational force, and a normal force}}$.

(b) The points of interest are $\boxed{\text{where the mass is released from rest (at } x = 6.00\text{ cm)}}$ and $\boxed{\text{the equilibrium point, } x = 0}$.

(c) The energy stored in the spring is the elastic potential energy, $PE_s = \frac{1}{2}kx^2$.

At $x = 6.00$ cm,

$$PE_s = \frac{1}{2}\left(850\ \text{N/m}\right)\left(6.00 \times 10^{-2}\ \text{m}\right)^2 = \boxed{1.53\ \text{J}}$$

and at the equilibrium position , ($x_f = 0$)

$$PE_s = \frac{1}{2}k\left(0\right)^2 = \boxed{0}$$

(c) The only force doing work on the mass is the conservative spring force (the normal force and the gravitational force are both perpendicular to the motion). Thus, the total mechanical energy of the mass will be constant. Because we may choose $y = 0$, and hence $PE_g = 0$, at the level of the horizontal surface, the energy conservation equation becomes

$$\left(KE + PE_s\right)_f = \left(KE + PE_s\right)_i \quad \text{or} \quad \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2$$

and solving for the final speed gives

$$\boxed{v_f = \sqrt{v_i^2 + \left(\frac{k}{m}\right)\left(x_i^2 - x_f^2\right)}}$$

If the final position is the equilibrium position $\left(x_f = 0\right)$ and the object starts from rest $\left(v_i = 0\right)$ at, $x_i = 6.00\text{ cm}$ the final speed is

$$v_f = \sqrt{0 + \left(\frac{850\text{ N/m}}{1.00\text{ kg}}\right)\left[\left(6.00\times10^{-2}\text{ m}\right)^2 - 0\right]} = \sqrt{3.06\text{ m}^2/s^2} = \boxed{1.75\text{ m/s}}$$

(c) When the object is halfway between the release point and the equilibrium position, we have $v_i = 0,\ x_i = 6.00\text{ cm, and } x_f = 3.00\text{ cm}$, giving

$$v_f = \sqrt{0 + \left(\frac{850\text{ N/m}}{1.00\text{ kg}}\right)\left[\left(6.00\times10^{-2}\text{ m}\right)^2 - \left(3.00\times10^{-2}\text{ m}\right)^2\right]} = \boxed{1.51\text{ m/s}}$$

This is not half of the speed at equilibrium because the equation for final speed is $\boxed{\text{not a linear function of position}}$.

5.32 Using conservation of mechanical energy, we have

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + 0$$

or

$$y_f = \frac{v_i^2 - v_f^2}{2g} = \frac{(10\ \text{m/s})^2 - (1.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = \boxed{5.1\ \text{m}}$$

5.33 Since no nonconservative forces do work, we use conservation of mechanical energy, with the zero of potential energy selected at the level of the base of the hill. Then, $\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$ with $y_f = 0$ yields

$$y_i = \frac{v_f^2 - v_i^2}{2g} = \frac{(3.00\ \text{m/s})^2 - 0}{2(9.80\ \text{m/s}^2)} = \boxed{0.459\ \text{m}}$$

Note that this result is independent of the mass of the child and sled.

5.34 (a) The distance the spring is stretched is $x = \ell - \ell_0 = 41.5\ \text{cm} - 35.0\ \text{cm} = 6.5\ \text{cm}$. Since the object hangs in equilibrium on the end of the spring, the spring exerts an upward force of magnitude $F_s = mg$ N on the suspended object. Then, Hooke's law gives the spring constant as

$$k = \left|\frac{F_s}{x}\right| = \frac{mg}{\ell - \ell_0} = \frac{(7.50\ \text{kg})(9.80\ \text{m/s}^2)}{6.5 \times 10^{-2}\ \text{m}} = 1.1 \times 10^3\ \text{N/m} = \boxed{1.1\ \text{kN/m}}$$

(b) Consider one person (say the one holding the left end of the spring) to simply hold that end of the spring in

position while the person on the other end stretches it. The spring is then stretched a distance of $x = \ell - \ell_0 = \ell - 0.350\ \text{m}$ by a stretching force of magnitude $F_s = 190$. From Hooke's law, $|x| = |F_s|/k$, we have

$$\ell - 0.350\ \text{m} = \frac{190\ \text{N}}{1.1 \times 10^3\ \text{N/m}} = 0.17\ \text{m} \quad \text{and} \quad \ell = 0.17\ \text{m} + 0.350\ \text{m} = \boxed{0.52\ \text{m}}$$

5.35 (a) On a frictionless track, no external forces do work on the system consisting of the block and the spring as the

spring is being compressed. Thus, the total mechanical energy of the system is constant, or $KE_f + (PE_g)_f + (PE_s)_f = KE_i + (PE_g)_i + (PE_s)_i$. Because the track is horizontal, the gravitational potential energy when the mass comes to rest is the same as just before it made contact with the spring, or $(PE_g)_f = (PE_g)_i$. This gives

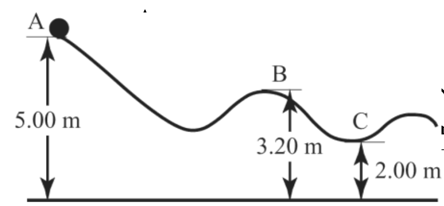
$$\frac{1}{2}m\mathrm{v}_f^2 + \frac{1}{2}kx_f^2 = \frac{1}{2}m\mathrm{v}_i^2 + \frac{1}{2}kx_i^2$$

Since $v_f = 0$ (the block comes to rest) and $x_i = 0$ (the spring is initially undistorted),

$$x_f = \mathrm{v}_i\sqrt{\frac{m}{k}} = 1.50 \text{ m/s} \sqrt{\frac{0.250 \text{ kg}}{4.60 \text{ N/m}}} = \boxed{0.350 \text{ m}}$$

(b) If the track was not frictionless, some of the original kinetic energy would be spent overcoming friction between the mass and track. This would mean that less energy would be stored as elastic potential energy in the spring when the mass came to rest. Therefore, the maximum compression of the spring $\boxed{\text{would be less}}$ in this case.

5.36 (a) From conservation of mechanical energy,



$$\frac{1}{2}m\mathrm{v}_B^2 + mgy_B = \frac{1}{2}m\mathrm{v}_A^2 + mgy_A$$

or

$$\mathrm{v}_B = \sqrt{\mathrm{v}_A^2 + 2g\ y_A - y_B}$$

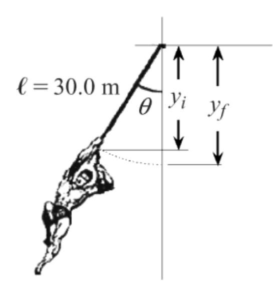
$$= \sqrt{0 + 2\ 9.80 \text{ m/s}^2\ \ 1.80 \text{ m}} = \boxed{5.94 \text{ m/s}}$$

Similarly,

$$\mathrm{v}_C = \sqrt{\mathrm{v}_A^2 + 2g\ y_A - y_C} = \sqrt{0 + 2g\ 5.00 \text{ m} - 2.00 \text{ m}} = \boxed{7.67 \text{ m/s}}$$

(b) $W_{g\ A\to C} = PE_{g\ A} - PE_{g\ C} = mg\ y_A - y_C = 49.0 \text{ N}\ \ 3.00 \text{ m} = \boxed{147 \text{ J}}$

5.37 (a) We choose the zero of potential energy at the level of the bottom of the arc. The initial height of Tarzan above this level is



$$y_i = 30.0 \text{ m}\ \ 1 - \cos 37.0° = 6.04 \text{ m}$$

Then, using conservation of mechanical energy,
we find

$$\frac{1}{2}mv_f^2 + 0 = \frac{1}{2}mv_i^2 + mgy_i$$

or

$$v_f = \sqrt{v_i^2 + 2gy_i} = \sqrt{0 + 2\left(9.80\ \text{m/s}^2\right)\left(6.04\ \text{m}\right)} = \boxed{10.9\ \text{m/s}}$$

(b) In this case, conservation of mechanical energy yields

$$v_f = \sqrt{v_i^2 + 2gy_i} = \sqrt{\left(4.00\ \text{m/s}\right)^2 + 2\left(9.80\ \text{m/s}^2\right)\left(6.04\ \text{m}\right)} = \boxed{11.6\ \text{m/s}}$$

5.38 At maximum height, $v_y = 0$ and $v_x = v_{0x} = \left(40\ \text{m/s}\right)\cos 60° = 20\ \text{m/s}$.

Thus, $v_f = \sqrt{v_x^2 + v_y^2} = 20\ \text{m/s}$. Choosing $PE_g = 0$ at the level of the launch point, conservation of mechanical energy gives $PE_f = KE_i - KE_f$, and the maximum height reached is

$$y_f = \frac{v_i^2 - v_f^2}{2g} = \frac{\left(40\ \text{m/s}\right)^2 - \left(20\ \text{m/s}\right)^2}{2\left(9.80\ \text{m/s}^2\right)} = \boxed{61\ \text{m}}$$

5.39 (a) Initially, all the energy is stored as elastic potential energy within the spring. When the gun is fired, and as
the projectile leaves the gun, most of the energy in the form of kinetic energy along with a small amount of gravitational potential energy. When the projectile comes to rest momentarily at its maximum height, all of the energy is in the form of gravitational potential energy.

(b) Use conservation of mechanical energy from when the projectile is at rest within the gun $\left(v_i = 0,\ y_i = 0,\ \text{and}\ x_i = -0.120\ \text{m}\right)$ until it reaches maximum height where $v_f = 0$, $y_f = y_{max} = 20.0\ \text{m}$, and $x_f = 0$ (the spring is relaxed after the gun is fired).

Then, $\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$ becomes

$$0 + mgy_{\max} + 0 = 0 + 0 + \frac{1}{2}kx_i^2$$

or

$$k = \frac{2mgy_{\max}}{x_i^2} = \frac{2\left(20.0\times10^{-3}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\left(20.0\text{ m}\right)}{\left(-0.120\text{ m}\right)^2} = \boxed{544\text{ N/m}}$$

(c) This time, we use conservation of mechanical energy from when the projectile is at rest within the gun $\left(\text{v}_i = 0,\ y_i = 0,\text{ and } x_i = -0.120\text{ m}\right)$ until it reaches the equilibrium position of the spring $\left(y_f = +0.120\text{ m, and } x_f = 0\right)$. This gives

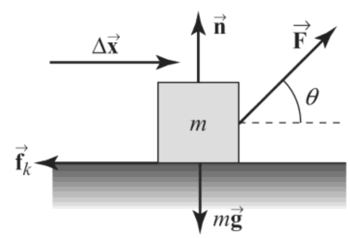
$$KE_f = \left(KE + PE_g + PE_s\right)_i - \left(PE_g + PE_s\right)_f \quad\text{or}\quad \frac{1}{2}m\text{v}_f^2 = \left(0 + 0 + \frac{1}{2}kx_i^2\right) - \left(mgy_f + 0\right)$$

$$\text{v}_f^2 = \left(\frac{k}{m}\right)x_i^2 - 2gy_f$$

$$= \left(\frac{544\text{ N/m}}{20.0\times10^{-3}\text{ kg}}\right)\left(-0.120\text{ m}\right)^2 - 2\left(9.80\text{ m/s}^2\right)\left(0.120\text{ m}\right)$$

yielding $\text{v}_f = \boxed{19.7\text{ m/s}}$

5.40 (a) $\Sigma F_y = 0 \Rightarrow n + F\sin\theta - mg = 0$, or $n = mg - F\sin\theta$.



The friction force is then

$$f_k = \mu_k n = \boxed{\mu_k\left(mg - F\sin\theta\right)}$$

(b) The work done by the applied force is

$$W_F = F\left|\Delta\vec{\mathbf{x}}\right|\cos\theta = \boxed{Fx\cos\theta}$$

and the work done by the friction force is $W_{f_k} = f_k\left|\Delta\vec{\mathbf{x}}\right|\cos\phi$ where ϕ is the angle between the direction of $\vec{\mathbf{f}}_k$ and $\Delta\vec{\mathbf{x}}$. Thus, $W_{f_k} = f_k x\cos 180° = \boxed{-\mu_k\left(mg - F\sin\theta\right)x}$.

(c) The forces that do no work are those perpendicular to the direction of the displacement $\Delta\vec{\mathbf{x}}$.

These are $\boxed{\vec{\mathbf{n}},\ m\vec{\mathbf{g}},\ \text{and the vertical coponent of } \vec{\mathbf{F}}}$.

(d) For part (a): $n = mg - F\sin\theta = (2.00 \text{ kg})(9.80 \text{ m/s}^2) - (15.0 \text{ N})\sin 37.0° = 10.6 \text{ N}$

$f_k = \mu_k n = (0.400)(10.6 \text{ N}) = \boxed{4.23 \text{ N}}$

For part (b): $W_F = (F\cos\theta)x = (15.0 \text{ N})(4.00 \text{ m})\cos 37.0° = \boxed{47.9 \text{ J}}$

$W_{f_k} = (f_k \cos\phi)x = (4.23 \text{ N})(4.00 \text{ m})\cos 180° = \boxed{-16.9 \text{ J}}$

5.41 (a) When the child slides down a frictionless surface, the only nonconservative force acting on the child is the normal force. At each instant, this force is perpendicular to the motion and, hence, does no work. Thus, conservation of mechanical energy $\boxed{\text{can be used}}$ in this case.

(b) The equation for conservation of mechanical energy, $(KE + PE)_f = (KE + PE)_i$, for this situation is $\frac{1}{2}\cancel{m}v_f^2 + \cancel{m}gy_f = \frac{1}{2}\cancel{m}v_i^2 + \cancel{m}gy_i$. Notice that the mass of the child cancels out of the equation, so $\boxed{\text{the mass of the child is not a factor}}$ in the frictionless case.

(c) Observe that solving the energy conservation equation from above for the final speed gives $v_f = \sqrt{v_i^2 + 2g(y_i - y_f)}$ Since the child starts with the same initial speed ($v_i = 0$) and has the same change in altitude in both cases, $\boxed{v_f \text{ is the same}}$ in the two cases.

(d) $\boxed{\text{Work done by a non conservative force must be accounted for}}$ when friction is present. This is done by using the work–energy theorem rather than conservation of mechanical energy.

(e) From part (b), conservation of mechanical energy gives the final speed as

$$v_f = \sqrt{v_i^2 + 2g(y_i - y_f)} = \sqrt{0 + 2(9.80 \text{ m/s}^2)(12.0 \text{ m})} = \boxed{15.3 \text{ m/s}}$$

5.42 (a) $\boxed{\text{No}}$. The change in the kinetic energy of the plane is equal to the *net* work done by all forces doing work on

it. In this case, there are two such forces, the thrust due to the engine and a resistive force due to the air. Since the work done by the air resistance force is negative, the net work done (and hence the change in kinetic energy) is less than the positive work done by the engine thrust. Also, because the thrust from the engine and the air resistance force are nonconservative forces, $\boxed{\text{mechanical energy is not conserved in this case}}$.

(b) Since the plane is in level flight, $(PE_g)_f = (PE_g)_i$ and the work–energy theorem reduces to $W_{nc} = W_{\text{thrust}} + W_{\text{resistance}} = KE_f - KE_i$, or

$$F(\cos 0°)s + f(\cos 180°)s = \frac{1}{2}m\text{v}_f^2 - \frac{1}{2}m\text{v}_i^2$$

This gives

$$\text{v}_f = \sqrt{\text{v}_i^2 + \frac{2(F-f)s}{m}} = \sqrt{(60\ \text{m/s})^2 + \frac{2\left[(7.5-4.0)\times 10^4\ \text{N}\right](500\ \text{m})}{1.5\times 10^4\ \text{kg}}} = \boxed{77\ \text{m/s}}$$

5.43 We shall take $PE_g = 0$ at the lowest level reached by the diver under the water. The diver falls a total of 15 m, but the nonconservative force due to water resistance acts only during the last 5.0 m of fall. The work–energy theorem then gives

$$W_{nc} = (KE + PE_g)_f - (KE + PE_g)_i$$

or

$$F_{\text{av}}(\cos 180°)(5.0\ \text{m}) = (0+0) - \left[0 + (70\ \text{kg})(9.80\ \text{m/s}^2)(15\ \text{m})\right]$$

This gives the average resistance force as $F_{\text{av}} = 2.1\times 10^3\ \text{N} = \boxed{2.1\ \text{kN}}$.

5.44 (a) Choose $PE_g = 0$ at the level of the bottom of the arc. The child's initial vertical displacement from this level is

$$y_i = (2.00\ \text{m})(1 - \cos 30.0°) = 0.268\ \text{m}$$

In the absence of friction, we use conservation of mechanical energy as

$(KE + PE_g)_f = (KE + PE_g)_i$, or $\frac{1}{2}m\text{v}_f^2 + 0 = 0 + mgy_i$, which gives

$$v_f = \sqrt{2gy_i} = \sqrt{2\ \ 9.80\ \text{m/s}^2\ \ \ 0.268\ \text{m}} = \boxed{2.29\ \text{m/s}}$$

(b) With a nonconservative force present, we use

$$W_{nc} = \ KE + PE_g\ _f - \ KE + PE_g\ _i = \left(\frac{1}{2}mv_f^2 + 0\right) - \ 0 + mgy_i$$

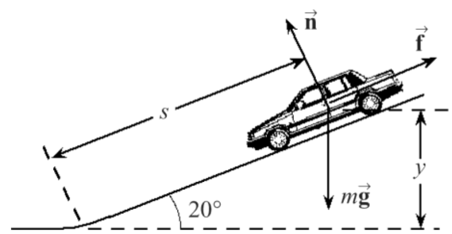
or

$$W_{nc} = m\left(\frac{v_f^2}{2} - gy_i\right)$$

$$= \ 25.0\ \text{kg}\ \left[\frac{\ 2.00\ \text{m/s}\ ^2}{2} - \ 9.80\ \text{m/s}^2\ \ \ 0.268\ \text{m}\ \right] = -15.7\ \text{J}$$

Thus, $\boxed{15.7\ \text{J}}$ of energy is spent overcoming friction.

5.45 Choose $PE_g = 0$ at the level of the bottom of the driveway.



Then $W_{nc} = \ KE + PE_g\ _f - \ KE + PE_g\ _i$ becomes

$$f\cos 180°\ \ s = \left[\frac{1}{2}mv_f^2 + 0\right] - \left[0 + mg\ \ s\sin 20°\ \right]$$

Solving for the final speed gives

$$v_f = \sqrt{2gs\ \ \sin 20° - \frac{2f\ s}{m}}$$

or

$$v_f = \sqrt{2\ \ 9.80\ \text{m/s}^2\ \ \ 5.0\ \text{m}\ \ \sin 20° - \frac{2\ \ 4.0\times 10^3\ \text{N}\ \ \ 5.0\ \text{m}}{2.10\times 10^3\ \text{kg}}} = \boxed{3.8\ \text{m/s}}$$

5.46 (a) $\boxed{\text{Yes}}$. Two forces, a conservative gravitational force and a nonconservative normal force, act on the child as

she goes down the slide. However, the normal force is perpendicular to the child's motion at each point on the path and does no work. In the absence of work done by nonconservative forces, mechanical energy is conserved.

(b) We choose the level of the pool to be the $y = 0$ (and hence, $PE_g = 0$) level. Then, when the child is at rest at the top of the slide, $PE_g = \boxed{mgh}$ and $KE = \boxed{0}$. Note that this gives the constant total mechanical energy of the child as. $E_{\text{total}} = KE + PE_g = mgh$ At the launching point (where $y = h/5$), we have $PE_g = mgy = \boxed{mgh/5}$ and $KE = E_{\text{total}} - PE_g = \boxed{4mgh/5}$. At the pool level, $PE_g = \boxed{0}$ and $KE = \boxed{mgh}$.

(c) At the launching point (i.e., where the child leaves the end of the slide),

$$KE = \frac{1}{2}m\text{v}_0^2 = \frac{4mgh}{5}$$

meaning that

$$\boxed{\text{v}_0 = \sqrt{\frac{8gh}{5}}}$$

(d) After the child leaves the slide and becomes a projectile, energy conservation gives $KE + PE_g = \frac{1}{2}m\text{v}^2 + mgy = E_{\text{total}} = mgh$ where $\text{v}^2 = \text{v}_x^2 + \text{v}_y^2$. Here, $\text{v}_x = \text{v}_{0x}$ is constant, but v_y varies with time. At maximum height, $y = y_{\text{max}}$ and $\text{v}_y = 0$, yielding

$$\frac{1}{2}m\ \text{v}_{0x}^2 + 0 + mgy_{\text{max}} = mgh \quad \text{and} \quad \boxed{y_{\text{max}} = h - \frac{\text{v}_{0x}^2}{2g}}$$

(e) If the child's launch angle leaving the slide is θ, then $v_{0x} = v_0 \cos\theta$. Substituting this into the result from part (d) and making use of the result from part (c) gives

$$y_{\text{max}} = h - \frac{\text{v}_0^2}{2g}\cos^2\theta = h - \frac{1}{2g}\left(\frac{8gh}{5}\right)\cos^2\theta \quad \text{or} \quad \boxed{y_{\text{max}} = h\left(1 - \frac{4}{5}\cos^2\theta\right)}$$

(f) $\boxed{\text{No}}$. If friction is present, mechanical energy would *not* be conserved, so her kinetic energy at all points after
leaving the top of the waterslide would be reduced when compared with the frictionless case. Consequently, her launch speed, maximum height reached, and final speed would be reduced as well.

5.47 Choose $PE_g = 0$ at the level of the base of the hill and let x represent the distance the skier moves along the horizontal portion before coming to rest. The normal force exerted on the skier by the snow while on the hill is $n_1 = mg\cos 10.5^\circ$ and, while on the horizontal portion, $n_2 = mg$.

Consider the entire trip, starting from rest at the top of the hill until the skier comes to rest on the horizontal portion. The work done by friction forces is

$$W_{nc} = \left[(f_k)_1 \cos 180^\circ \right](200\text{ m}) + \left[(f_k)_2 \cos 180^\circ \right] x$$

$$= -\mu_k(mg\cos 10.5^\circ)(200\text{ m}) - \mu_k(mg)x$$

Applying $W_{nc} = (KE + PE_g)_f - (KE + PE_g)_i$ to this complete trip gives

$$-\mu_k(mg\cos 10.5^\circ)(200\text{ m}) - \mu_k(mg)x = (0 + 0) - \left[0 + mg(200\text{ m})\sin 10.5^\circ\right]$$

or

$$x = \left(\frac{\sin 10.5^\circ}{\mu_k} - \cos 10.5^\circ \right)(200\text{ m})$$. If $\mu_k = 0.0750$, then $x = \boxed{289\text{ m}}$.

5.48 The normal force exerted on the sled by the track is $n = mg\cos\theta$ and the friction force is $f_k = \mu_k n = \mu_k\, mg\cos\theta$.

If s is the distance measured along the incline that the sled travels, applying $W_{nc} = (KE + PE_g)_f - (KE + PE_g)_i$ to the entire trip gives

$$\left[(\mu_k\, mg\cos\theta)\cos 180^\circ \right] s = \left[0 + mg(s\sin\theta)\right] - \left[\frac{1}{2}mv_i^2 + 0\right]$$

or

$$s = \frac{v_i^2}{2g(\sin\theta + \mu_k\cos\theta)} = \frac{(4.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)(\sin 20^\circ + 0.20\cos 20^\circ)} = \boxed{1.5\text{ m}}$$

5.49 (a) Consider the entire trip and apply $W_{nc} = (KE + PE_g)_f - (KE + PE_g)_i$ to obtain

$$(f_1\cos 180^\circ)d_1 + (f_2\cos 180^\circ)d_2 = \left(\frac{1}{2}mv_f^2 + 0\right) - (0 + mgy_i)$$

or

$$\text{v}_f = \sqrt{2\left(g\,y_i - \frac{f_1 d_1 + f_2 d_2}{m}\right)}$$

$$= \sqrt{2\left[\left(9.80\ \text{m/s}^2\right)\left(1\ 000\ \text{m}\right) - \frac{\left(50.0\ \text{N}\right)\left(800\ \text{m}\right) + \left(3\ 600\ \text{N}\right)\left(200\ \text{m}\right)}{80.0\ \text{kg}}\right]}$$

which yields $\text{v}_f = \boxed{24.5\ \text{m/s}}$.

(b) $\boxed{\text{Yes}}$, this is too fast for safety.

(c) Again, apply $W_{nc} = \left(KE + PE_g\right)_f - \left(KE + PE_g\right)_i$, now with d_2 considered to be a variable, $d_1 = 1\ 000\ \text{m} - d_2$, and $\text{v}_f = 5.00\ \text{m/s}$. This gives

$$f_1 \cos 180^\circ \left(1\ 000\ \text{m} - d_2\right) + f_2 \cos 180^\circ\, d_2 = \left(\frac{1}{2} m\text{v}_f^2 + 0\right) - \left(0 + mgy_i\right)$$

which reduces to $-\left(1\ 000\ \text{m}\right) f_1 + f_1 d_2 - f_2 d_2 = \frac{1}{2} m\text{v}_f^2 - mgy_i$. Therefore,

$$d_2 = \frac{mg\,y_i - \left(1\ 000\ \text{m}\right) f_1 - \frac{1}{2} m\text{v}_f^2}{f_2 - f_1}$$

$$= \frac{\left(784\ \text{N}\right)\left(1\ 000\ \text{m}\right) - \left(1\ 000\ \text{m}\right)\left(50.0\ \text{N}\right) - \frac{1}{2}\left(80.0\ \text{kg}\right)\left(5.00\ \text{m/s}\right)^2}{3\ 600\ \text{N} - 50.0\ \text{N}} = \boxed{206\ \text{m}}$$

(d) In reality, the air drag will depend on the skydiver's speed. It will be larger than her 784 N weight only after the chute is opened. It will be nearly equal to 784 N before she opens the chute and again before she touches down, whenever she moves near terminal speed.

5.50 (a) $W_{nc} = \Delta KE + \Delta PE$, but $\Delta KE = 0$ because the speed is constant. The skier rises a vertical distance of $\Delta y = \left(60\ \text{m}\right)\sin 30^\circ = 30\ \text{m}$. Thus,

$$W_{nc} = \left(70\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\left(30\ \text{m}\right) = 2.06 \times 10^4\ \text{J} = \boxed{21\ \text{kJ}}$$

(b) The time to travel 60 m at a constant speed of 2.0 m/s is 30 s. Thus, the required power input is

$$\mathcal{P} = \frac{W_{nc}}{\Delta t} = \frac{2.06 \times 10^4 \text{ J}}{30 \text{ s}} = 686 \text{ W}\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = \boxed{0.92 \text{ hp}}$$

5.51 As the piano is lifted at constant speed up to the apartment, the total work that must be done on it is

$$W_{nc} = \Delta KE + \Delta PE_g = 0 + mg\left(y_f - y_i\right) = \left(3.50 \times 10^3 \text{ N}\right)\left(25.0 \text{ m}\right) = 8.75 \times 10^4 \text{ J}$$

The three workmen (using a pulley system with an efficiency of 0.750) do work on the piano at a rate of

$$\mathcal{P}_{\text{net}} = 0.750\left(3\mathcal{P}_{\substack{\text{single}\\ \text{worker}}}\right) = 0.750\left[3\left(165 \text{ W}\right)\right] = 371 \text{ W} = 371 \text{ J/s}$$

so the time required to do the necessary work on the piano is

$$\Delta t = \frac{W_{nc}}{\mathcal{P}_{\text{net}}} = \frac{8.75 \times 10^4 \text{ J}}{371 \text{ J/s}} = \boxed{236 \text{ s}} = \left(236 \text{ s}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{3.93 \text{ min}}$$

5.52 Let ΔN be the number of steps taken in time Δt. We determine the number of steps per unit time by

$$\text{Power} = \frac{\text{work done}}{\Delta t} = \frac{\left(\text{work per step per unit mass}\right)\left(\text{mass}\right)\left(\#\,\text{steps}\right)}{\Delta t}$$

or

$$70 \text{ W} = \left(0.60 \ \frac{\text{J/step}}{\text{kg}}\right)\left(60 \text{ kg}\right)\left(\frac{\Delta N}{\Delta t}\right), \text{ giving } \frac{\Delta N}{\Delta t} = 1.9 \text{ steps/s}$$

The running speed is then

$$v_{\text{av}} = \frac{\Delta x}{\Delta t} = \left(\frac{\Delta N}{\Delta t}\right)\left(\text{distance traveled per step}\right) = \left(1.9 \ \frac{\text{step}}{\text{s}}\right)\left(1.5 \ \frac{\text{m}}{\text{step}}\right) = \boxed{2.9 \text{ m/s}}$$

5.53 Assuming a level track, $PE_f = PE_i$, and the work done on the train is

$$W_{nc} = \left(KE + PE\right)_f - \left(KE + PE\right)_i$$

$$= \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{1}{2}\left(0.875 \text{ kg}\right)\left[\left(0.620 \text{ m/s}\right)^2 - 0\right] = 0.168 \text{ J}$$

The power delivered by the motor is then

$$\mathcal{P} = \frac{W_{nc}}{\Delta t} = \frac{0.168 \text{ J}}{21.0 \times 10^{-3} \text{ s}} = \boxed{8.01 \text{ W}}$$

5.54 When the car moves at constant speed on a level roadway, the power used to overcome the total frictional force equals the power input from the engine, or $\mathcal{P}_{\text{output}} = f_{\text{total}} v = \mathcal{P}_{\text{input}}$. This gives

$$f_{\text{total}} = \frac{\mathcal{P}_{\text{input}}}{v} = \frac{175 \text{ hp}}{29 \text{ m/s}}\left(\frac{746 \text{ W}}{1 \text{ hp}}\right) = \boxed{4.5 \times 10^{5} \text{ N}}$$

5.55 The work done on the older car is

$$(W_{\text{net}})_{\text{old}} = (KE_f - KE_i)_{\text{old}} = \frac{1}{2}mv^2 - 0 = \frac{1}{2}mv^2$$

The work done on the newer car is

$$(W_{\text{net}})_{\text{new}} = (KE_f - KE_i)_{\text{new}} = \frac{1}{2}m(2v)^2 - 0 = 4\left(\frac{1}{2}mv^2\right) = 4(W_{\text{net}})_{\text{old}}$$

and the power input to this car is

$$\mathcal{P}_{\text{new}} = \frac{(W_{\text{net}})_{\text{new}}}{\Delta t} = \frac{4(W_{\text{net}})_{\text{old}}}{\Delta t} = 4\mathcal{P}_{\text{old}}$$

or $\boxed{\text{the power of the newer car is 4 times that of the older car}}$.

5.56 Neglecting any variation of gravity with altitude, the work required to lift a 3.20×10^{7} kg load at constant speed to an altitude of $\Delta y = 1.75$ km is

$$W = \Delta PE_g = mg(\Delta y) = (3.20 \times 10^{7} \text{ kg})(9.80 \text{ m/s}^2)(1.75 \times 10^{3} \text{ m}) = 5.49 \times 10^{11} \text{ J}$$

The time required to do this work using a $\mathcal{P} = 2.70 \text{ kW} = 2.70 \times 10^{3}$ J/s pump is

$$\Delta t = \frac{W}{\mathcal{P}} = \frac{5.49 \times 10^{11} \text{ J}}{2.70 \times 10^{3} \text{ J/s}} = \boxed{2.03 \times 10^{8} \text{ s}} = (2.03 \times 10^{8} \text{ s})\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right) = \boxed{5.64 \times 10^{4} \text{ h}}$$

5.57 (a) The acceleration of the car is

$$a = \frac{v - v_0}{t} = \frac{18.0 \text{ m/s} - 0}{12.0 \text{ s}} = 1.50 \text{ m/s}^2$$

Thus, the constant

forward force due to the engine is found from $\Sigma F = F_{\text{engine}} - F_{\text{air}} = ma$ as

$$F_{\text{engine}} = F_{\text{air}} + ma = 400 \text{ N} + \left(1.50 \times 10^3 \text{ kg}\right)\left(1.50 \text{ m/s}^2\right) = 2.65 \times 10^3 \text{ N}$$

The average velocity of the car during this interval is $v_{\text{av}} = \left(v + v_0\right)/2 = 9.00 \text{ m/s}$, so the average power input from the engine during this time is

$$\mathcal{P}_{\text{av}} = F_{\text{engine}} v_{\text{av}} = \left(2.65 \times 10^3 \text{ N}\right)\left(9.00 \text{ m/s}\right)\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = \boxed{32.0 \text{ hp}}$$

(b) At $t = 12.0$ s, the instantaneous velocity of the car is $v = 18.0 \text{ m/s}$ and the instantaneous power input from the engine is

$$\mathcal{P} = F_{\text{engine}} v = \left(2.65 \times 10^3 \text{ N}\right)\left(18.0 \text{ m/s}\right)\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = \boxed{63.9 \text{ hp}}$$

5.58 (a) The acceleration of the elevator during the first 3.00 s is

$$a = \frac{v - v_0}{t} = \frac{1.75 \text{ m/s} - 0}{3.00 \text{ s}} = 0.583 \text{ m/s}^2$$

so $F_{net} = F_{motor} - mg = ma$ gives the force exerted by the motor as

$$F_{\text{motor}} = m\left(a + g\right) = \left(650 \text{ kg}\right)\left[\left(0.583 + 9.80\right) \text{ m/s}^2\right] = 6.75 \times 10^3 \text{ N}$$

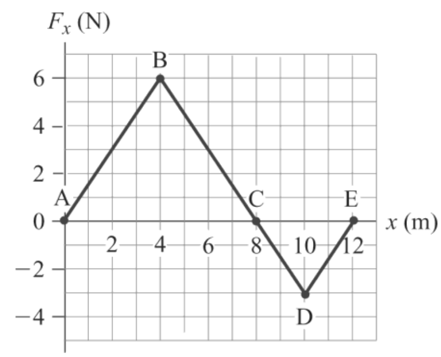
The average velocity of the elevator during this interval is $v_{\text{av}} = \left(v + v_0\right)/2 = 0.875 \text{ m/s}$ so the average power input from the motor during this time is

$$\mathcal{P}_{av} = F_{motor}\, \text{v}_{av} = 6.75 \times 10^3 \text{ N} \quad 0.875 \text{ m/s} \left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = \boxed{7.92 \text{ hp}}$$

(b) When the elevator moves upward with a constant speed of $\text{v} = 1.75 \text{ m/s}$, the upward force exerted by the motor is $F_{motor} = mg$ and the instantaneous power input from the motor is

$$\mathcal{P} = mg\,\text{v} = 650 \text{ kg} \quad 9.80 \text{ m/s}^2 \quad 1.75 \text{ m/s} \left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = \boxed{14.9 \text{ hp}}$$

5.59 The work done on the particle by the force F as the particle moves from $x = x_i$ to $x = x_f$ is the area under the curve from x_i to x_f.



(a) For $x = 0$ to $x = 8.00 \text{ m}$,

$$W = \text{area of triangle } ABC = \frac{1}{2}\overline{AC} \times \text{altitude}$$

$$W_{0\to 8} = \frac{1}{2} \quad 8.00 \text{ m} \quad 6.00 \text{ N} = \boxed{24.0 \text{ J}}$$

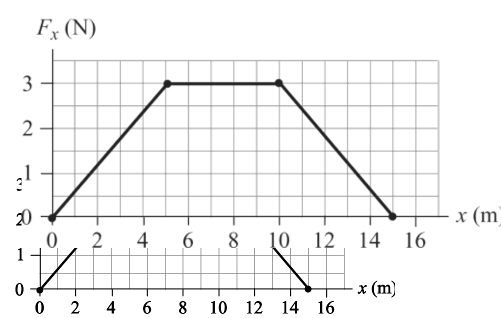
(b) For to $x = 10.0 \text{ m}$,

$$W_{8\to 10} = \text{area of triangle } CDE = \frac{1}{2}\overline{CE} \times \text{altitude}$$

$$= \frac{1}{2} \quad 2.00 \text{ m} \quad -3.00 \text{ N} = \boxed{-3.00 \text{ J}}$$

(c) $W_{0\to 10} = W_{0\to 8} + W_{8\to 10} = 24.0 \text{ J} + \quad -3.00 \text{ J} = \boxed{21.0 \text{ J}}$

5.60 The work done by a force equals the area under the force versus displacement curve.



(a) For the region $0 \le x \le 5.00 \text{ m}$,

$$W_{0 \text{ to } 5} = \frac{3.00 \text{ N} \quad 5.00 \text{ m}}{2} = \boxed{7.50 \text{ J}}$$

(b) For the region $5.00 \text{ m} \le x \le 10.0 \text{ m}$,

$$W_{5\text{ to }10} = 3.00\text{ N} \quad 5.00\text{ m} = \boxed{15.0\text{ J}}$$

(c) For the region $10.0\text{ m} \le x \le 15.0\text{ m}$, $W_{10\text{ to }15} = \dfrac{3.00\text{ N} \quad 5.00\text{ m}}{2} = \boxed{7.50\text{ J}}$

(d) $KE\big|_{x=x_f} - KE\big|_{x=0} = W_{0\text{ to }x_f}$ = area under F vs. x curve from $x = 0$ m to $x = x_f$, or

$$\frac{1}{2}m\text{v}_f^2 = \frac{1}{2}m\text{v}_0^2 + W_{0\text{ to }x_f}$$

giving

$$\text{v}_f = \sqrt{\text{v}_0^2 + \left(\frac{2}{m}\right)W_{0\text{ to }x_f}}$$

For $x_f = 5.00$ m:

$$\text{v}_f = \sqrt{\text{v}_0^2 + \left(\frac{2}{m}\right)W_{0\text{ to }5}} = \sqrt{0.500\text{ m/s}^{\;2} + \left(\frac{2}{3.00\text{ kg}}\right)\;7.50\text{ J}} = \boxed{2.29\text{ m/s}}$$

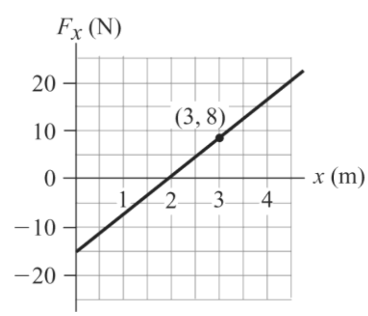
For $x_f = 15.0$ m:

$$\text{v}_f = \sqrt{\text{v}_0^2 + \left(\frac{2}{m}\right)W_{0\text{ to }15}} = \sqrt{\text{v}_0^2 + \left(\frac{2}{m}\right)\;W_{0\text{ to }5} + W_{5\text{ to }10} + W_{10\text{ to }15}}$$

or

$$\text{v}_f = \sqrt{0.500\text{ m/s}^{\;2} + \left(\frac{2}{3.00\text{ kg}}\right)\;7.50\text{ J} + 15.0\text{ J} + 7.50\text{ J}} = \boxed{4.50\text{ m/s}}$$

5.61 (a) $F_x = 8x - 16$ N See the graph at the right.



(b) The net work done is the total area under the graph from $x = 0$ to $x = 3.00$ m. This consists of two triangular shapes, one below the axis (negative area) and one above the axis (positive). The net work is then

$$W_{net} = \frac{2.00\text{ m} \quad -16.0\text{ N}}{2} + \frac{1.00\text{ m} \quad 8.00\text{ N}}{2}$$

$$= \boxed{-12.0\text{ J}}$$

5.62 (b) Solving part (b) first, we recognize that the egg has constant acceleration $a_y = -g$ as it falls 32.0 m from rest before contacting the pad. Taking upward as positive, its velocity just before contacting the pad is given by $v^2 = v_0^2 + 2a_y \; \Delta y$ as

$$v_1 = -\sqrt{0 + 2 \;\; -9.80\text{ m/s}^2 \quad -32.0\text{ m}} = -25.0\text{ m/s}$$

The average acceleration as the egg comes to rest in 9.20 ms after contacting the pad is

$$a_{av} = \frac{v_f - v_1}{\Delta t} = \frac{0 - \;\; -25.0\text{ m/s}}{9.20 \times 10^{-3}\text{ s}} = +\left(\frac{25.0}{9.20}\right) \times 10^3\text{ m/s}^2$$

and the average net force acting on the egg during this time is

$$F_{net \;\; av} = ma_{av} = \;\; 75.0 \times 10^{-3}\text{ kg} \;\; \left(\frac{25.0}{9.20}\right) \times 10^3\text{ m/s}^2 = +204\text{ N} = \boxed{204\text{ N upward}}$$

(a) The egg has a downward displacement of magnitude $|\Delta y|$ as the upward force $F_{net \;\; av}$ brings the egg to rest. The net work done on the egg in this process is

$$W_{net} = \left[\;\; F_{net \;\; av} \cos 180°\right]|\Delta y| = KE_f - KE_1 = 0 - \frac{1}{2}mv_1^2$$

so

$$|\Delta y| = \frac{-mv_1^2}{2 \;\; F_{net \;\; av} \cos 180°} = \frac{75.0 \times 10^{-3}\text{ kg} \quad 25.0\text{ m/s}^{\;\;2}}{2 \;\; 204\text{ N}} = 0.115\text{ m} = \boxed{11.5\text{ cm}}$$

5.63 The person's mass is

$$m = \frac{w}{g} = \frac{700 \text{ N}}{9.80 \text{ m/s}^2} = 71.4 \text{ kg}$$

The net upward force acting on the body is $F_{net} = 2\ 355 \text{ N} - 700 \text{ N} = 10.0 \text{ N}$. The final upward velocity can then be calculated from the work–energy theorem as

$$W_{net} = KE_f - KE_i = \frac{1}{2}m\text{v}^2 - \frac{1}{2}m\text{v}_i^2$$

or

$$F_{net}\cos\theta\ s = \left[10.0 \text{ N} \cos 0°\right] 0.250 \text{ m} = \frac{1}{2} 71.4 \text{ kg}\ \text{v}^2 - 0$$

which gives $\text{v} = \boxed{0.265 \text{ m/s upward}}$.

5.64 Taking $y = 0$ at ground level, and using conservation of energy from when the boy starts from rest $(\text{v}_i = 0)$ at the top of the slide $(y_i = H)$ to the instant he leaves the lower end $(y_f = h)$ of the frictionless slide with a horizontal velocity $(\text{v}_{0x} = \text{v}_f,\ \text{v}_{0y} = 0)$, yields

$$\frac{1}{2}m\text{v}_f^2 + mgh = 0 + mgH \qquad \text{or} \qquad \text{v}_f^2 = 2g\ H - h \qquad [1]$$

Considering his flight as a projectile after leaving the end of the slide, $\Delta y = \text{v}_{0y}t + \frac{1}{2}a_y t^2$ gives the time to drop distance h to the ground as

$$-h = 0 + \frac{1}{2}\ -g\ t^2 \qquad \text{or} \qquad t = \sqrt{\frac{2h}{g}}$$

The horizontal distance traveled (at constant horizontal velocity) during this time is d, so

$$d = \text{v}_{0x}t = \text{v}_f\sqrt{\frac{2h}{g}} \qquad \text{and} \qquad \text{v}_f = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$$

Substituting this result into Equation [1] above gives

$$\frac{gd^2}{2h} = 2g \;\; H - h \qquad \text{or} \qquad \boxed{H = h + \frac{d^2}{4h}}$$

5.65 (a) If $y = 0$ at point B, then $y_A = \;\; 35.0 \text{ m} \;\; \sin 50.0° = 26.8 \text{ m}$ and $y_B = 0$. Thus,

$$PE_A = mgy_A = \;\; 1.50 \times 10^3 \text{ kg} \;\; 9.80 \text{ m/s}^2 \;\; 26.8 \text{ m} \;\; = \boxed{3.94 \times 10^5 \text{ J}}$$

$$PE_B = mgy_B = \boxed{0} \text{ and } \Delta PE_{A \to B} = PE_B - PE_A = 0 - 3.94 \times 10^5 \text{ J} = \boxed{-3.94 \times 10^5 \text{ J}}$$

(b) If $y = 0$ at point C, then $y_A = \;\; 50.0 \text{ m} \;\; \sin 50.0° = 38.3 \text{ m}$ and

$y_B = \;\; 15.0 \text{ m} \;\; \sin 50.0° = 11.5 \text{ m}$. In this case,

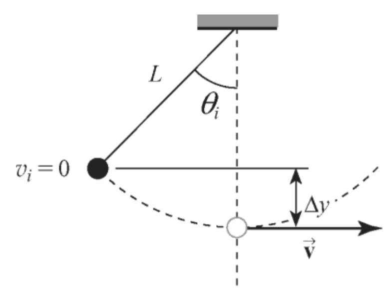
$$PE_A = mgy_A = \;\; 1.50 \times 10^3 \text{ kg} \;\; 9.80 \text{ m/s}^2 \;\; 38.3 \text{ m} \;\; = \boxed{5.63 \times 10^5 \text{ J}}$$

$$PE_B = mgy_B = \;\; 1.50 \times 10^3 \text{ kg} \;\; 9.80 \text{ m/s}^2 \;\; 11.5 \text{ m} \;\; = \boxed{1.69 \times 10^5 \text{ J}}$$

and

$$\Delta PE_{A \to B} = PE_B - PE_A = 1.69 \times 10^5 \text{ J} - 5.63 \times 10^5 \text{ J} = \boxed{-3.94 \times 10^5 \text{ J}}$$

5.66 The support string always lies along a radius line of the circular path followed by the bob. This means that the tension force in the string is always perpendicular to the motion of the bob and does no work. Thus, mechanical energy is conserved and (taking $y = 0$ at the point of support) this gives



$$\tfrac{1}{2} m v^2 = \tfrac{1}{2} m v_i^2 + mg \;\; y_i - y_f \;\; = 0 + mg\left[-L\cos\theta_i - \;\; -L \;\;\right]$$

or

$$v = \sqrt{2gL \;\; 1 - \cos\theta_i} = \sqrt{2 \;\; 9.8 \text{ m/s}^2 \;\; 2.0 \text{ m} \;\; 1 - \cos 25°}$$

$v = \boxed{1.9 \text{ m/s}}$

5.67 (a) The equivalent spring constant of the bow is given $F = kx$ by as

$$k = \frac{F_f}{x_f} = \frac{230 \text{ N}}{0.400 \text{ m}} = \boxed{575 \text{ N/m}}$$

(b) From the work–energy theorem applied to this situation,

$$W_{nc} = \left(KE + PE_g + PE_s\right)_f - \left(KE + PE_g + PE_s\right)_i = \left(0 + 0 + \frac{1}{2}kx_f^2\right) - \left(0 + 0 + 0\right)$$

The work done pulling the bow is then

$$W_{nc} = \frac{1}{2}kx_f^2 = \frac{1}{2}\left(575 \text{ N/m}\right)\left(0.400 \text{ m}\right)^2 = \boxed{46.0 \text{ J}}$$

5.68 Choose $PE_g = 0$ at the level where the block comes to rest against the spring. Then, in the absence of work done by nonconservative forces, the conservation of mechanical energy gives

$$\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$$

or

$$0 + 0 + \frac{1}{2}kx_f^2 = 0 + mgL\sin\theta + 0$$

Thus,

$$x_f = \sqrt{\frac{2mgL\sin\theta}{k}} = \sqrt{\frac{2\left(12.0 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)\left(3.00 \text{ m}\right)\sin 35.0^\circ}{3.00\times 10^4 \text{ N/m}}} = \boxed{0.116 \text{ m}}$$

5.69 (a) From $v^2 = v_0^2 + 2a_y\left(\Delta y\right)$, we find the speed just before touching the ground as

$$v = \sqrt{0 + 2\left(9.80 \text{ m/s}^2\right)\left(1.0 \text{ m}\right)} = \boxed{4.4 \text{ m/s}}$$

(b) Choose $PE_g = 0$ at the level where the feet come to rest. Then

$W_{nc} = \left(KE + PE_g\right)_f - \left(KE + PE_g\right)_i$ becomes

$$\left(F_{av} \cos 180°\right) s = \left(0 + 0\right) - \left(\frac{1}{2} m v_i^2 + mg\,s\right)$$

or

$$F_{av} = \frac{m v_i^2}{2s} + mg = \frac{\left(75 \text{ kg}\right)\left(4.4 \text{ m/s}\right)^2}{2\left(5.0 \times 10^{-3} \text{ m}\right)} + \left(75 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right) = \boxed{1.5 \times 10^5 \text{ N}}$$

5.70 From the work–energy theorem,

$$W_{nc} = \left(KE + PE_g + PE_s\right)_f - \left(KE + PE_g + PE_s\right)_i$$

we have

$$\left(f_k \cos 180°\right) s = \left(\frac{1}{2} m v_f^2 + 0 + 0\right) - \left(0 + 0 + \frac{1}{2} k x_i^2\right)$$

or

$$v_f = \sqrt{\frac{k x_i^2 - 2 f_k s}{m}} = \sqrt{\frac{(8.0 \text{ N/m})(5.0 \times 10^{-2} \text{ m})^2 - 2(0.032 \text{ N})(0.15 \text{ m})}{5.3 \times 10^{-3} \text{ kg}}} = \boxed{1.4 \text{ m/s}}$$

5.71 (a) The two masses will pass when both are at $y_f = 2.00$ m above the floor. From conservation of energy, $\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$

$$\frac{1}{2}\left(m_1 + m_2\right) v_f^2 + \left(m_1 + m_2\right) g y_f + 0 = 0 + m_1 g y_{1i} + 0$$

or

$$v_f = \sqrt{\frac{2\, m_1 g y_{1i}}{m_1 + m_2} - 2\, g y_f}$$

$$= \sqrt{\frac{2\left(5.00 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)\left(4.00 \text{ m}\right)}{8.00 \text{ kg}} - 2\left(9.80 \text{ m/s}^2\right)\left(2.00 \text{ m}\right)}$$

This yields the passing speed as $v_f = \boxed{3.13 \text{ m/s}}$.

(b) When $m_1 = 5.00 \text{ kg}$ reaches the floor, is above the floor, $m_2 = 3.00 \text{ kg}$ IS $y_{2f} = 4.00 \text{ m}$ above the floor.

Thus, $\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$ becomes

$$\frac{1}{2}\left(m_1 + m_2\right) v_f^2 + m_2 g y_{2f} + 0 = 0 + m_1 g y_{1i} + 0$$

or

$$v_f = \sqrt{\frac{2g\left(m_1 y_{1i} - m_2 y_{2f}\right)}{m_1 + m_2}}$$

Thus,

$$v_f = \sqrt{\frac{2\left(9.80 \text{ m/s}^2\right)\left[\left(5.00 \text{ kg}\right)\left(4.00 \text{ m}\right) - \left(3.00 \text{ kg}\right)\left(4.00 \text{ m}\right)\right]}{8.00 \text{ kg}}} = \boxed{4.43 \text{ m/s}}$$

(c) When the 5.00-kg hits the floor, the string goes slack and the 3.00-kg becomes a projectile launched straight upward with initial speed $v_{0y} = 4.43 \text{ m/s}$. At the top of its arc, $v_y = 0$ and $v_y^2 = v_{0y}^2 + 2a_y\left(\Delta y\right)$ gives

$$\Delta y = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (4.43\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = \boxed{1.00\ \text{m}}$$

5.72 The normal force the incline exerts on block A is $n_A = (m_A g)\cos 37°$, and the friction force is $f_k = \mu_k n_A = \mu_k m_A g \cos 37°$. The vertical distance block A rises is $\Delta y_A = (20\ \text{m})\sin 37° = 12\ \text{m}$, while the vertical displacement of block B is $\Delta y_B = -20\ \text{m}$.

We find the common final speed of the two blocks by use of

$$W_{nc} = (KE + PE_g)_f - (KE + PE_g)_i = \Delta KE + \Delta PE_g$$

This gives $-(\mu_k m_A g \cos 37°)s = \left[\frac{1}{2}(m_A + m_B)v_f^2 - 0\right] + \left[m_A g(\Delta y_A) + m_B g(\Delta y_B)\right]$, or

$$v_f^2 = \frac{2g\left[-m_B(\Delta y_B) - m_A(\Delta y_A) - (\mu_k m_A \cos 37°)s\right]}{m_A + m_B}$$

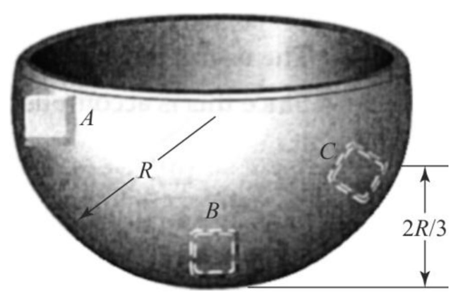
$$= \frac{2(9.80\ \text{m/s}^2)\left[-(100\ \text{kg})(-20\ \text{m}) - (50\ \text{kg})(12\ \text{m}) - 0.25(50\ \text{kg})(20\ \text{m})\cos 37°\right]}{150\ \text{kg}}$$

which yields $v_f^2 = 157\ \text{m}^2/\text{s}^2$.

The change in the kinetic energy of block A is then

$$\Delta KE_A = \frac{1}{2}m_A v_f^2 - 0 = \frac{1}{2}(50\ \text{kg})(157\ \text{m}^2/\text{s}^2) = 3.9 \times 10^3\ \text{J} = \boxed{3.9\ \text{kJ}}$$

5.73 Since the bowl is smooth (that is, frictionless), mechanical energy is conserved or



$$(KE + PE)_f = (KE + PE)_i$$

Also, if we choose $y = 0$ (and hence, $PE_g = 0$) at the lowest point in the bowl, then

$y_A = +R,\ y_B = 0,$ and $y_C = 2R/3$.

(a) $\quad PE_{g\ A} = mgy_A = mgR$, or

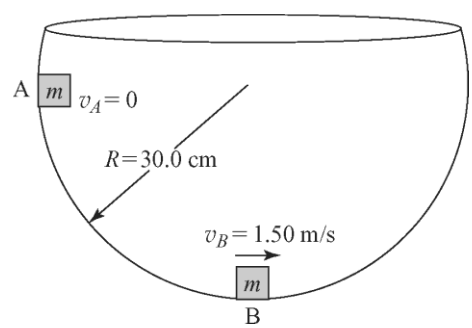
$$PE_{g\ A} = 0.200\text{ kg}\quad 9.80\ \text{m/s}^2\quad 0.300\text{ m} = \boxed{0.588\text{ J}}$$

(b) $\quad KE_B = KE_A + PE_A - PE_B = 0 + mgy_A - mgy_B = 0.588\text{ J} - 0 = \boxed{0.588\text{ J}}$

(c) $\quad KE_B = \frac{1}{2}m\text{v}_B^2 \quad\Rightarrow\quad \text{v}_B = \sqrt{\dfrac{2\ KE_B}{m}} = \sqrt{\dfrac{2\ \ 0.588\text{ J}}{0.200\text{ kg}}} = \boxed{2.42\ \text{m/s}}$

(d) $\quad PE_{g\ C} = mgy_C = 0.200\text{ kg}\quad 9.80\ \text{m/s}^2\ \left[\dfrac{2\ \ 0.300\text{ m}}{3}\right] = \boxed{0.392\text{ J}}$

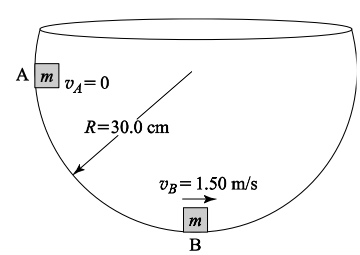
$$KE_C = KE_B + PE_B - PE_C = 0.588\text{ J} + 0 - 0.392\text{ J} = \boxed{0.196\text{ J}}$$



5.74 (a) $\quad KE_B = \dfrac{1}{2}m\text{v}_B^2 = \dfrac{1}{2}\ 0.200\text{ kg}\quad 1.50\ \text{m/s}^{\ 2} = \boxed{0.225\text{ J}}$

(b) The change in the altitude of the particle as it goes from A to B is $y_A - y_B = R$, where $R = 0.300$ m is the radius of the bowl. Therefore, the work–energy theorem gives

$$\begin{aligned} W_{nc} &= KE_B - KE_A + PE_B - PE_A \\ &= KE_B - 0 + mg\ y_B - y_A = KE_B + mg\ -R \end{aligned}$$



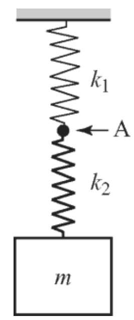
or

$$W_{nc} = 0.225\text{ J} + 0.200\text{ kg}\quad 9.80\ \text{m/s}^2\quad -0.300\text{ m} = -0.363\text{ J}$$

The loss of mechanical energy as a result of friction is then .0.363 J.

(c) No. Because the normal force, and hence the friction force, vary with the position of the particle on its path,
it is not possible to use the result from part (b) to determine the coefficient of friction without using calculus.

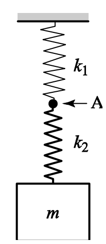
5.75 (a) Consider the sketch at the right. When the mass $m = 1.50$ kg is in equilibrium, the upward spring force exerted on it by the lower spring (i.e., the tension in this spring) must equal the weight of the object, or $F_{S2} = mg$. Hooke's law then gives the elongation of this spring as



$$x_2 = \frac{F_{S2}}{k_2} = \frac{mg}{k_2}$$

Now, consider point A where the two springs join. Because this point is in equilibrium, the upward spring force exerted on A by the upper spring must have the same magnitude as the downward spring force exerted on A by the lower spring (that is, the tensions in the two springs must be equal).

The elongation of the upper spring must be



$$x_1 = \frac{F_{S1}}{k_1} = \frac{F_{S2}}{k_1} = \frac{mg}{k_1}$$

and the total elongation of the spring system is

$$x = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$= 1.50 \text{ kg} \quad 9.80 \text{ m/s}^2 \left(\frac{1}{1.20 \times 10^3 \text{ N/m}} + \frac{1}{1.80 \times 10^3 \text{ N/m}}\right) = \boxed{2.04 \times 10^{-2} \text{ m}}$$

(b) The spring system exerts an upward spring force of $F_S = mg$ on the suspended object and undergoes an elongation of x. The effective spring constant is then

$$k_{\text{effective}} = \frac{F_S}{x} = \frac{mg}{x} = \frac{1.50 \text{ kg} \quad 9.80 \text{ m/s}^2}{2.04 \times 10^{-2} \text{ m}} = \boxed{7.20 \times 10^2 \text{ N/m}}$$

5.76 Refer to the sketch given in the solution of Problem 5.75.

(a) Because the object of mass m is in equilibrium, the tension in the lower spring, F_{S2} must equal the weight of the object. Therefore, from Hooke's law, the elongation of the lower spring is

$$x_2 = \frac{F_{S2}}{k_2} = \frac{mg}{k_2}$$

From the fact that the point where the springs join (A) is in equilibrium, we conclude that the tensions in the two springs must be equal, $F_{S1} = F_{S2} = mg$. The elongation of the upper spring is then

$$x_1 = \frac{F_{S1}}{k_1} = \frac{mg}{k_1}$$

and the total elongation of the spring system is

$$\boxed{x = x_1 + x_2 = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$$

(b) The two spring system undergoes a total elongation of x and exerts an upward spring force $F_S = mg$ on the
suspended mass. The effective spring constant of the two springs in series is then

$$k_{\text{effective}} = \frac{F_S}{x} = \frac{mg}{x} = \frac{mg}{mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)} = \boxed{\left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}}$$

5.77 (a) The person walking uses $E_w = (220 \text{ kcal})(4\,186 \text{ J/1 kcal} = 9.21 \times 10^5$ J of energy while going 3.00 miles. The quantity of gasoline which could furnish this much energy is

$$V_1 = \frac{9.21 \times 10^5 \text{ J}}{1.30 \times 10^8 \text{ J/gal}} = 7.08 \times 10^{-3} \text{ gal}$$

This means that the walker's fuel economy in equivalent miles per gallon is

$$\textit{fuel economy} = \frac{3.00 \text{ mi}}{7.08 \times 10^{-3} \text{ gal}} = \boxed{423 \text{ mi/gal}}$$

(b) In 1 hour, the bicyclist travels 10.0 miles and uses

$$E_B = 400 \text{ kcal} \left(\frac{4\,186 \text{ J}}{1 \text{ kcal}}\right) = 1.67 \times 10^6 \text{ J}$$

which is equal to the energy available in

$$V_2 = \frac{1.67 \times 10^6 \text{ J}}{1.30 \times 10^8 \text{ J/gal}} = 1.29 \times 10^{-2} \text{ gal}$$

of gasoline. Thus, the equivalent fuel economy for the bicyclist is

$$\frac{10.0 \text{ mi}}{1.29 \times 10^{-2} \text{ gal}} = \boxed{776 \text{ mi/gal}}$$

5.78 When 1 pound (454 grams) of fat is metabolized, the energy released is $E = (454 \text{ g})(9.00 \text{ kcal/g}) = 4.09 \times 10^3$ kcal. Of this, 20.0% goes into mechanical energy (climbing stairs in this case). Thus, the mechanical energy generated by metabolizing 1 pound of fat is

$$E_m = 0.200 \quad 4.09 \times 10^3 \text{ kcal} \quad = 817 \text{ kcal}$$

Each time the student climbs the stairs, she raises her body a vertical distance of $\Delta y = \quad 80 \text{ steps} \quad 0.150 \text{ m/step} \quad = 12.0 \text{ m}$. The mechanical energy required to do this is $\Delta PE_g = mg \quad \Delta y \quad$, or

$$\Delta PE_g = \quad 50.0 \text{ kg} \quad 9.80 \text{ m/s}^2 \quad 12.0 \text{ m} \quad = \quad 5.88 \times 10^3 \text{ J} \left(\frac{1 \text{ kcal}}{4186 \text{ J}}\right) = 1.40 \text{ kcal}$$

(a) The number of times the student must climb the stairs to metabolize 1 pound of fat is

$$N = \frac{E_m}{\Delta PE_g} = \frac{817\text{ kcal}}{1.40\text{ kcal/trip}} = \boxed{582\text{ trips}}$$

It would be more practical for her to reduce food intake.

(c) The useful work done each time the student climbs the stairs is $W = \Delta PE_g = 5.88 \times 10^3\text{ J}$. Since this is accomplished in 65.0 s, the average power output is

$$\mathcal{P}_{av} = \frac{W}{t} = \frac{5.88 \times 10^3\text{ J}}{65.0\text{ s}} = \boxed{90.5\text{ W}} = 90.5\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{0.121\text{ hp}}$$

5.79 (a) Use conservation of mechanical energy, $(KE + PE_g)_f = (KE + PE_g)_i$, from the start to the end of the track to find the speed of the skier as he leaves the track. This gives $\frac{1}{2}mv^2 + mgy_f = 0 + mgy_i$, or

$$\upsilon = \sqrt{2g(y_i - y_f} = \sqrt{2(9.80\text{ m/s}^2)(4.00\text{ m})} = \boxed{28.0\text{ m/s}}$$

(b) At the top of the parabolic arc the skier follows after leaving the track, and $v_y = 0$. Thus, $v_x = 28.0\text{ m/s}\ \cos 45.0° = 19.8\text{ m/s}$. Thus, $v_{top} = \sqrt{v_x^2 + v_y^2} = 19.8\text{ m/s}$ Applying conservation of mechanical energy from the end of the track to the top of the arc gives $\frac{1}{2}m\ 19.8\text{ m/s}^{\ 2} + mg\,y_{max} = \frac{1}{2}m\ 28.0\text{ m/s}^{\ 2} + mg\ 10.0\text{ m}$, or

$$y_{max} = 10.0\text{ m} + \frac{28.0\text{ m/s}^{\ 2} - 19.8\text{ m/s}^{\ 2}}{2\ 9.80\text{ m/s}^2} = \boxed{30.0\text{ m}}$$

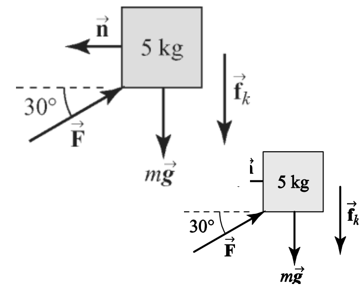
(c) Using $\Delta y = v_{0y}t + \frac{1}{2}a_yt^2$ for the flight from the end of the track to the ground gives

$$-10.0\text{ m} = \left[\ 28.0\text{ m/s}\ \sin 45.0°\right]t + \frac{1}{2}\ -9.80\text{ m/s}^2\ t^2$$

The positive solution of this equation gives the total time of flight as $t = 4.49$ s. During this time, the skier has a horizontal displacement of

$$\Delta x = v_{0x}t = \left[\ 28.0\ \text{m/s}\ \cos 45.0°\right]\ 4.49\ \text{s}\ = \boxed{89.0\ \text{m}}$$

5.80 First, determine the magnitude of the applied force by considering a free-body diagram of the block. Since the block moves with constant velocity, $\Sigma F_x = \Sigma F_y = 0$.



From $\Sigma F_x = 0$, we see that $n = F\cos 30°$.

Thus, $f_k = \mu_k n = \mu_k F\cos 30°$, and $\Sigma F_y = 0$ becomes

$$F\sin 30° = mg + \mu_k F\cos 30°$$

or

$$F = \frac{mg}{\sin 30° - \mu_k \cos 30°} = \frac{5.0\ \text{kg}\quad 9.80\ \text{m/s}^2}{\sin 30° -\ \ 0.30\ \ \cos 30°} = 2.0\times 10^2\ \text{N}$$

(a) The applied force makes a 60° angle with the displacement up the wall. Therefore,

$$W_F = F\cos 60°\ \ s = \left[\ 2.0\times 10^2\ \text{N}\ \cos 60°\right]\ 3.0\ \text{m}\ = \boxed{3.1\times 10^2\ \text{J}}$$

(b) $W_g = mg\cos 180°\ \ s = \ 49\ \text{N}\quad -1.0\quad 3.0\ \text{m}\ = \boxed{-1.5\times 10^2\ \text{J}}$

(c) $W_n = n\cos 90°\, s = \boxed{0}$

(d) $PE_g = mg\ \Delta y = \ 49\ \text{N}\quad 3.0\ \text{m}\ = \boxed{1.5\times 10^2\ \text{J}}$

5.81 We choose $PE_g = 0$ at the level where the spring is relaxed ($x = 0$), or at the level of position B.

(a) At position A, $KE = 0$ and the total energy of the system is given by

$$E = \ 0 + PE_g + PE_s\ _A = mgx_1 + \frac{1}{2}k\,x_1^2\text{, or}$$

$$E = (25.0 \text{ kg})(9.80 \text{ m/s}^2)(-0.100 \text{ m}) + \frac{1}{2}(2.50 \times 10^4 \text{ N/m})(-0.100 \text{ m})^2 = \boxed{101 \text{ J}}$$

(b) In position C, and the spring is uncompressed, so $PE_s = 0$. Hence,

$$E = (0 + PE_g + 0)_C = mg\,x_2$$

or

$$x_2 = \frac{E}{mg} = \frac{101 \text{ J}}{(25.0 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{0.410 \text{ m}}$$

(c) At position B, $PE_g = PE_s = 0$ and $E = (KE + 0 + 0)_B = \frac{1}{2} m v_B^2$

Therefore,

$$v_B = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(101 \text{ J})}{25.0 \text{ kg}}} = \boxed{2.84 \text{ m/s}}$$

(b) Where the velocity (and hence the kinetic energy) is a maximum, the acceleration is $a_y = (\Sigma F_y)/m = 0$ (at this point, an upward force due to the spring exactly balances the downward force of gravity). Thus, taking upward as Positive, $\Sigma F_y = -kx - mg = 0$

$$x = -\frac{mg}{k} = -\frac{245 \text{ kg}}{2.50 \times 10^4 \text{ N/m}} = -9.80 \times 10^{-3} \text{ m} = \boxed{-9.80 \text{ mm}}$$

(e) From the total energy, $E = KE + PE_g + PE_s = \frac{1}{2} m v^2 + mgx + \frac{1}{2} kx^2$, we find

$$v = \sqrt{\frac{2E}{m} - 2gx - \frac{k}{m}x^2}$$

Where the speed, and hence kinetic energy is a maximum (that is, at $x = -9.80$ mm), this gives

$$v_{max} = \sqrt{\frac{2(101 \text{ J})}{25.0 \text{ kg}} - 2(9.80 \text{ m/s}^2)(-9.80 \times 10^{-3} \text{ m}) - \left(\frac{2.50 \times 10^4 \text{ N/m}}{25.0 \text{ kg}}\right)(-9.80 \times 10^{-3} \text{ m})^2}$$

or

$$v_{max} = \boxed{2.85 \text{ m/s}}$$

5.82 When the hummingbird is hovering, the magnitude of the average upward force exerted by the air on the wings (and hence the average downward force the wings exert on the air) must be $F_{av} = mg$ where mg is the weight of the bird. Thus, if the wings move downward distance d during a wing stroke, the work done each beat of the wings is

$$W_{beat} = F_{av}d = mgd = \left(3.0 \times 10^{-3} \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)\left(3.5 \times 10^{-2} \text{ m}\right) = 1.0 \times 10^{-3} \text{ J}$$

In 1 minute, the number of beats of the wings that occur is

$$N = \left(80 \text{ beats/s}\right)\left(60 \text{ s/min}\right) = 4.8 \times 10^3 \text{ beats/min}$$

so the total work preformed in 1 minute is

$$W_{total} = NW_{beat}\left(1 \text{ min}\right) = \left(4.8 \times 10^3 \frac{\text{beats}}{\text{min}}\right)\left(1.0 \times 10^{-3} \frac{\text{J}}{\text{beat}}\right)\left(1 \text{ min}\right) = \boxed{4.9 \text{ J}}$$

5.83 Choose $PE_g = 0$ at the level of the river. Then $y_i = 36.0$ m, $y_f = 4.00$, the jumper falls 32.0 m, and the cord stretches 7.00 m. Between the balloon and the level where the diver stops momentarily, $\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$ gives

$$0 + \left(700 \text{ N}\right)\left(4.00 \text{ m}\right) + \frac{1}{2}k\left(7.00 \text{ m}\right)^2 = 0 + \left(700 \text{ N}\right)\left(36.0 \text{ m}\right) + 0$$

or

$$k = \boxed{914 \text{ N/m}}$$

5.84 If a projectile is launched, in the absence of air resistance, with speed v_0 at angle θ above the horizontal, the time required to return to the original level is found from $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ as $0 = \left(v_0 \sin\theta\right)t - (g/2)t^2$, which gives $t = (2v_0 \sin\theta)/g$. The range is the horizontal displacement occurring in this time.

Thus,

$$R = v_{0x}\, t = v_0 \cos\theta \left(\frac{2 v_0 \sin\theta}{g}\right) = \frac{v_0^2 \; 2\sin\theta\cos\theta}{g} = \frac{v_0^2 \sin\; 2\theta}{g}$$

Maximum range occurs when $\theta = 45^{\circ}$, giving or $R_{max} = v_0^2/g$ .or $v_0^2 = g\,R_{max}$ The minimum kinetic energy required to reach a given maximum range is then

$$KE = \frac{1}{2} m v_0^2 = \frac{1}{2} mg\, R_{max}$$

(a) The minimum kinetic energy needed in the record throw of each object is

Javelin: $KE = \frac{1}{2}\; 0.80\text{ kg}\quad 9.80\ \text{m/s}^2\quad 98\text{ m} = \boxed{3.8\times 10^2\ \text{J}}$

Discus: $KE = \frac{1}{2}\; 2.0\text{ kg}\quad 9.80\ \text{m/s}^2\quad 74\text{ m} = \boxed{7.3\times 10^2\ \text{J}}$

Shot: $KE = \frac{1}{2}\; 7.2\text{ kg}\quad 9.80\ \text{m/s}^2\quad 23\text{ m} = \boxed{8.1\times 10^2\ \text{J}}$

(b) The average force exerted on an object during launch, when it starts from rest and is given the kinetic energy found above, is computed from as $W_{net} = F_{av}\, s = \Delta KE$ as $F_{av} = \; KE - 0 \;/s$ Thus, the required force for each object is

Javelin: $F_{av} = \dfrac{3.8\times 10^2\ \text{J}}{2.00\text{ m}} = \boxed{1.9\times 10^2\ \text{N}}$

Discus: $F_{av} = \dfrac{7.3\times 10^2\ \text{J}}{2.00\text{ m}} = \boxed{3.6\times 10^2\ \text{N}}$

Shot: $F_{av} = \dfrac{8.1\times 10^2\ \text{J}}{2.00\text{ m}} = \boxed{4.1\times 10^2\ \text{N}}$

(c) $\boxed{\text{Yes}}$. If the muscles are capable of exerting 4.1×10^2 N on an object and giving that object a kinetic energy of 8.1×10^2 J, as in the case of the shot, those same muscles should be able to give the javelin a launch speed of

$$v_0 = \sqrt{\frac{2\,KE}{m}} = \sqrt{\frac{2\left(8.1 \times 10^2 \text{ J}\right)}{0.80 \text{ kg}}} = 45 \text{ m/s}$$

with a resulting range of

$$R_{\max} = \frac{v_0^2}{g} = \frac{\left(45 \text{ m/s}\right)^2}{9.80 \text{ m/s}^2} = 2.1 \times 10^2 \text{ m}$$

Since this far exceeds the record range for the javelin, one must conclude that air resistance plays a very significant role in these events.

5.85 From the work–energy theorem, $W_{\text{net}} = KE_f - KE_i$. Since the package moves with constant velocity, $KE_f = KE_i$ giving $W_{\text{net}} = \boxed{0}$.

Note that the above result can also be obtained by the following reasoning:
Since the object has zero acceleration, the net (or resultant) force acting on it must be zero. The net work done is $W_{\text{net}} = F_{\text{net}}d = \boxed{0}$.

The work done by the conservative gravitational force is

$$W_{\text{grav}} = -\Delta PE_g = -mg\left(y_f - y_i\right) = -mg\left(d\sin\theta\right)$$

or

$$W_{\text{grav}} = -\left(50 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)\left(340 \text{ m}\right)\sin 7.0° = \boxed{-2.0 \times 10^4 \text{ J}}p$$

The normal force is perpendicular to the displacement. The work it does is

$$W_{\text{normal}} = nd\cos 90° = \boxed{0}$$

Since the package moves up the incline at constant speed, the net force parallel to the incline is zero. Thus, $\Sigma F_{\|} = 0 \;\Rightarrow\; f_s - mg\sin\theta = 0$, or $f_s = mg\sin\theta$.

The work done by the friction force in moving the package distance d up the incline is

$$W_{\text{friction}} = f_k d = \left(mg\sin\theta\right)d = \left[\left(50 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)\sin 7.0°\right]\left(340 \text{ m}\right) = \boxed{2.0 \times 10^4 \text{ J}}$$

5.86 Each 5.00-m length of the cord will stretch 1.50 m when the tension in the cord equals the weight of the jumper (that is, when $F_s = w = mg$). Thus, the elongation in a cord of original length L when $F_s = w$ will be

$$x = \left(\frac{L}{5.00\text{ m}}\right)(1.50\text{ m}) = 0.300L$$

and the force constant for the cord of length L is

$$k = \frac{F_s}{x} = \frac{w}{0.300L}$$

(a) In the bungee-jump from the balloon, the daredevil drops $y_i - y_f = 55.0\text{ m}$.

The stretch of the cord at the start of the jump is $x_i = 0$, and that at the lowest point is $x_f = 55.0\text{ m} - L$. Since $KE_i = KE_f = 0$ for the fall, conservation of mechanical energy gives

$$0 + (PE_g)_f + (PE_s)_f = 0 + (PE_g)_i + (PE_s)_i \quad\Rightarrow\quad \frac{1}{2}k\left(x_f^2 - x_i^2\right) = mg\left(y_i - y_f\right)$$

giving

$$\frac{1}{2}\left(\frac{\cancel{mg}}{0.300L}\right)(55.0\text{ m} - L)^2 = \cancel{mg}(55.0\text{ m}) \text{ and } (55.0\text{ m} - L)^2 = (33.0\text{ m})L$$

which reduces to

$$(55.0\text{ m})^2 - (110\text{ m})L + L^2 = (33.0\text{ m})L$$

or

$$L^2 - (143\text{ m})L + (55.0\text{ m})^2 = 0$$

and has solutions of

$$L = \frac{-(-143\text{ m}) \pm \sqrt{(-143\text{ m})^2 - 4(1)(55.0\text{ m})^2}}{2(1)}$$

This yields

$$L = \frac{143 \text{ m} \pm 91.4 \text{ m}}{2} \text{ and } L = 117 \text{ m} \quad \text{or} \quad L = 25.8 \text{ m}$$

Only the $L = \boxed{25.8 \text{ m}}$ solution is physically acceptable!

(b) During the jump, $\Sigma F_y = ma_y \Rightarrow kx - mg = ma_y$, or $\left(\frac{\cancel{m}g}{0.300L}\right)x - \cancel{m}g = \cancel{m}a_y$

Thus,

$$a_y = \left(\frac{x}{0.300L} - 1\right)g$$

which has maximum value at $x = x_{max} = 55.0 \text{ m} - L = 29.2 \text{ m}$.

$$a_y{}_{max} = \left[\frac{29.2 \text{ m}}{0.300\ 25.8 \text{ m}} - 1\right]g = \boxed{2.77g} = \boxed{27.1 \text{ m/s}^2}$$

5.87 (a) While the car moves at constant speed, the tension in the cable is $F = mg \sin \theta$, and the power input is $F = mg\sin\theta$, $\mathcal{P} = F\text{v} = mg\text{v}\sin\theta$, or

$$\mathcal{P} = 950 \text{ kg} \quad 9.80 \text{ m/s}^2 \quad 2.20 \text{ m/s} \ \sin 30.0° = 1.02 \times 10^4 \text{ W} = \boxed{10.2 \text{ kW}}$$

(b) While the car is accelerating, the tension in the cable is

$$F_a = mg\sin\theta + ma = m\left(g\sin\theta + \frac{\Delta \text{v}}{\Delta t}\right)$$

$$= 950 \text{ kg} \left[9.80 \text{ m/s}^2 \ \sin 30.0° + \frac{2.20 \text{ m/s} - 0}{12.0 \text{ s}}\right] = 4.83 \times 10^3 \text{ N}$$

Maximum power input occurs the last instant of the acceleration phase. Thus,

$$\mathcal{P}_{max} = F_a\text{v}_{max} = 4.83 \times 10^3 \text{ N} \quad 2.20 \text{ m/s} = \boxed{10.6 \text{ kW}}$$

(c) The work done by the motor in moving the car up the frictionless track is

$$W_{nc} = (KE + PE)_f - (KE + PE)_i = (KE_f + (PE_g)_f) - 0 = \frac{1}{2}mv_f^2 + mg(L\sin\theta)$$

or

$$W_{nc} = (950 \text{ kg})\left[\frac{1}{2}(2.20 \text{ m/s})^2 + (9.80 \text{ m/s}^2)(1\,250 \text{ m})\sin 30.0°\right] = \boxed{5.82 \times 10^6 \text{ J}}$$

5.88 (a) Since the tension in the string is always perpendicular to the motion of the object, the string does no work on

the object. Then, mechanical energy is conserved:

$$(KE + PE_g)_f = (KE + PE_g)_i$$

Choosing $PE_g = 0$ at the level where the string attaches to the cart, this gives

$$0 + mg(-L\cos\theta) = \frac{1}{2}mv_0^2 + mg(-L)$$

or

$$\boxed{v_0 = \sqrt{2gL(1 - \cos\theta)}}$$

(b) If $L = 1.20$ m and $\theta = 35.0°$, the result of part (a) gives

$$v_0 = \sqrt{2(9.80 \text{ m/s}^2)(1.20 \text{ m})(1 - \cos 35.0°)} = \boxed{2.06 \text{ m/s}}$$

5.90 (a) Realize that, with the specified arrangement of springs, each spring supports one-fourth the weight of the load (shelf plus trays). Thus, adding the weight ($w = mg$) of one tray to the load increases the tension in each spring by $\Delta F = mg/4$. If this increase in tension causes an additional elongation in each spring equal to the thickness of a tray, the upper surface of the stack of trays stays at a fixed level above the floor as trays are added to or removed from the stack.

(b) If the thickness of a single tray is t, the force constant each spring should have to allow the fixed-level tray dispenser to work properly is

$$k = \frac{\Delta F}{\Delta x} = \frac{mg/4}{t} = \frac{mg}{4t}$$

or

$$k = \frac{(0.580\text{ kg})(9.80\text{ m/s}^2)}{4(0.450 \times 10^{-2}\text{ m})} = \boxed{316\text{ N/m}}$$

The $\boxed{\text{length and width of a tray}}$ are unneeded pieces of data.

5.91 When the cyclist travels at constant speed, the magnitude of the forward static friction force on the drive wheel equals that of the retarding air resistance force. Hence, the friction force is proportional to the square of the speed, and her power output may be written as

$$\mathcal{P} = f_s v = (kv^2)v = kv^3$$

where k is a proportionality constant.

If the heart rate R is proportional to the power output, then $R = k'\mathcal{P} = k'(kv^3) = k'kv^3$ where k' is also a proportionality constant.

The ratio of the heart rate R_2 at speed v_2 to the rate R_1 at speed v_1 is then

$$\frac{R_2}{R_1} = \frac{k'kv_2^3}{k'kv_1^3} = \left(\frac{v_2}{v_1}\right)^3$$

giving

$$v_2 = v_1\left(\frac{R_2}{R_1}\right)^{1/3}$$

Thus, if $R = 90.0\text{ beats/min}$ at $v = 22.0\text{ km/h}$, the speed at which the rate would be is 136 beats/min is

$$v = (22.0\ \text{km/h})\left(\frac{136\ \text{beats/min}}{90.0\ \text{beats/min}}\right)^{1/3} = \boxed{25.2\ \text{km/h}}$$

and the speed at which the rate would be 166 beats/min

$$v = (22.0\ \text{km/h})\left(\frac{166\ \text{beats/min}}{90.0\ \text{beats/min}}\right)^{1/3} = \boxed{27.0\ \text{km/h}}$$

5.92 (a) The needle has maximum speed during the interval between when the spring returns to normal length and the needle tip first contacts the skin. During this interval, the kinetic energy of the needle equals the original elastic potential energy of the spring, or $\frac{1}{2}mv_{max}^2 = \frac{1}{2}kx_i^2$. This gives

$$v_{max} = x_i\sqrt{\frac{k}{m}} = (8.10\times10^{-2}\ \text{m})\sqrt{\frac{375\ \text{N/m}}{5.60\times10^{-3}\ \text{kg}}} = \boxed{21.0\ \text{m/s}}$$

(b) If F_1 is the force the needle must overcome as it penetrates a thickness x_1 of skin and soft tissue while F_2 is the force overcame while penetrating thickness x_2 of organ material, application of the work–energy theorem from the instant before skin contact until the instant before hitting the stop gives

$$W_{net} = -F_1x_1 - F_2x_2 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_{max}^2$$

or

$$v_f = \sqrt{v_{max}^2 - \frac{2(F_1x_1 + F_2x_2)}{m}}$$

$$v_f = \sqrt{(21.0\ \text{m/s})^2 - \frac{2\left[(7.60\ \text{N})(2.40\times10^{-2}\ \text{m}) + (9.20\ \text{N})(3.50\times10^{-2}\ \text{m})\right]}{5.60\times10^{-3}\ \text{kg}}} = \boxed{16.1\ \text{m/s}}$$