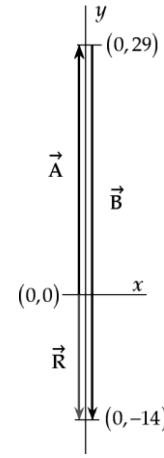
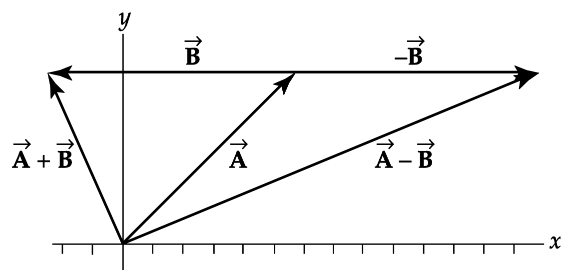
PROBLEM SOLUTIONS

3.1 We are given that $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. When two vectors are added graphically, the second vector is positioned with its tail at the tip of the first vector. The resultant then runs from the tail of the first vector to the tip of the second vector. In this case, vector $\vec{\mathbf{A}}$ will be positioned with its tail at the origin and its tip at the point (0, 29). The resultant is then drawn, starting at the origin (tail of first vector) and going 14 units in the negative y-direction to the point (0, –14). The second vector, $\vec{\mathbf{B}}$, must then start from the tip of $\vec{\mathbf{A}}$ at point (0, 29) and end on the tip of $\vec{\mathbf{R}}$ at point (0, –14) as shown in the sketch at the right. From this, it is seen that



$\boxed{\vec{\mathbf{B}} \text{ is 43 units in the negative } y\text{-direction}}$

3.2 (a) Using graphical methods, place the tail of vector $\vec{\mathbf{B}}$ at the head of vector $\vec{\mathbf{A}}$. The new vector $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ has a magnitude of $\boxed{6.1 \text{ units at } 113°}$ from the positive x-axis.

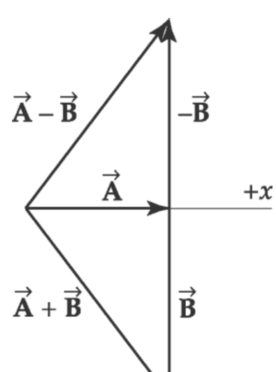


(b) The vector difference $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ is found by placing the negative of vector $\vec{\mathbf{B}}$ (a vector of the same magnitude as $\vec{\mathbf{B}}$, but opposite direction) at the head of vector $\vec{\mathbf{A}}$. The resultant vector $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ has magnitude $\boxed{15 \text{ units at } 23°}$ from the positive x-axis.

3.3 (a) In your vector diagram, place the tail of vector $\vec{\mathbf{B}}$ at the tip of vector $\vec{\mathbf{A}}$. The vector sum, $\vec{\mathbf{A}} + \vec{\mathbf{B}}$, is then found as shown in the vector diagram and should be

$\boxed{\vec{\mathbf{A}} + \vec{\mathbf{B}} = 5.0 \text{ units at } -53°}$

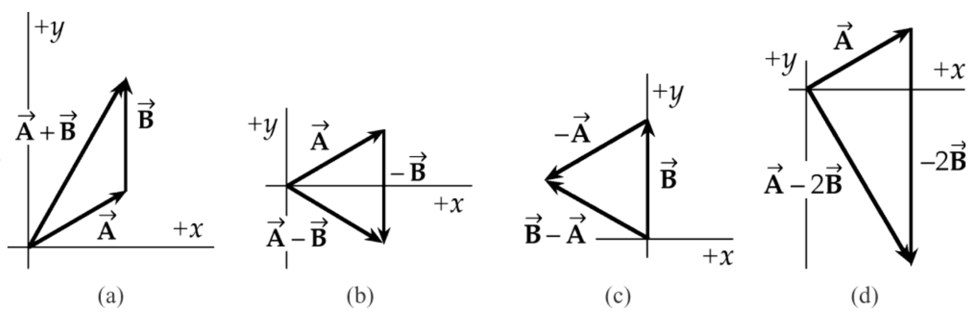
(b) To find the vector difference, form the vector $-\vec{\mathbf{B}}$ (same magnitude as $\vec{\mathbf{B}}$, opposite direction) and add it to vector



$\vec{\mathbf{A}}$ as shown in the diagram. You should find that

$\boxed{\vec{\mathbf{A}} - \vec{\mathbf{B}} = 5.0 \text{ units at} + 53°}$

3.4 Sketches of the scale drawings needed for parts (a) through (d) are given below. Following the sketches is a brief comment on each part with its answer.



(a) Drawing the vectors to scale and maintaining their respective directions yields a resultant of $\boxed{5.2 \text{ m at} + 60°}$.

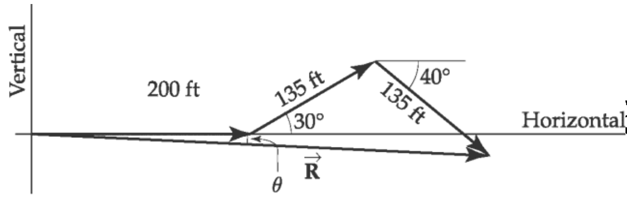
(b) Maintain the direction of $\vec{\mathbf{A}}$, but reverse the direction of $\vec{\mathbf{B}}$ to produce $-\vec{\mathbf{B}}$. The resultant is $\boxed{3.0 \text{ m at} - 30°}$.

(c) Maintain the direction of $\vec{\mathbf{B}}$, but reverse the direction of $\vec{\mathbf{A}}$ to produce $-\vec{\mathbf{A}}$. The resultant is $\boxed{3.0 \text{ m at} + 150°}$.

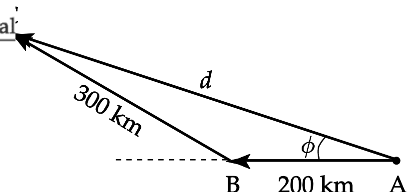
(d) Maintain the direction of $\vec{\mathbf{A}}$, reverse the direction of $\vec{\mathbf{B}}$ and double its magnitude, to produce $-2\vec{\mathbf{B}}$. The resultant is $\boxed{5.2 \text{ m at} - 60°}$.

3.5 Your sketch should be drawn to scale, similar to that pictured below. The length of $\vec{\mathbf{R}}$ and the angle θ can be measured to find, with use of your scale factor, the magnitude and direction of the resultant displacement. The result should be

3.6



approximately 421 ft at 3° below the horizontal



where $x = 200\text{ km} + \ 300\text{ km}\ \cos 30.0° = 460\text{ km}$

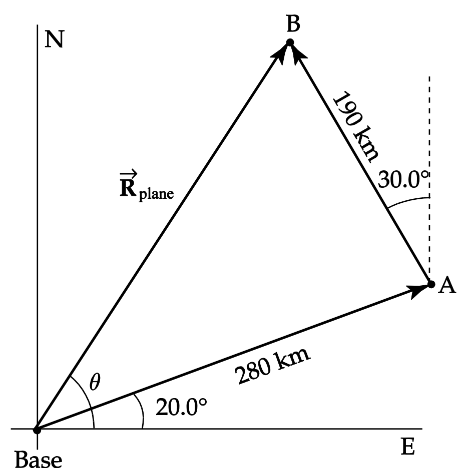
and $y = 0 + \ 300\text{ km}\ \sin 30.0° = 150\text{ km}$

$$\therefore \quad d = \sqrt{(460\text{ km})^2 + (150\text{ km})^2} = \boxed{484\text{ km}}$$

(b) $\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{150\text{ km}}{460\text{ km}}\right) = \boxed{18.1°\text{ N of W}}$

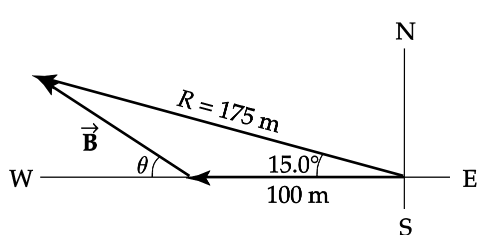
(c) Because of the curvature of the Earth, the plane doesn't travel along straight lines. .
Thus, the answer computed above is only approximately correct.

3.7 Using a vector diagram, drawn to scale, like that shown at the right, the final displacement of the plane can be found to be $\vec{\mathbf{R}}_{\text{plane}} = 310\text{ km at } \theta = 57°\text{ N of E}$. The requested displacement of the base from point B is $-\vec{\mathbf{R}}_{\text{plane}}$, which has the same magnitude but the opposite direction. Thus, the answer is



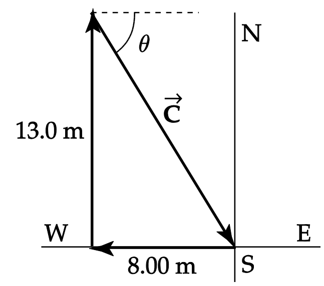
$$-\vec{\mathbf{R}}_{\text{plane}} = \boxed{310\text{ km at } \theta = 57°\text{ S of W}}$$

3.8 Your vector diagram should look like the one shown at the right. The initial displacement $\vec{\mathbf{A}} = 100\text{ m}$ due west and the resultant $\vec{\mathbf{R}} = 175\text{ m at } 15.0°\text{ N of W}$ are

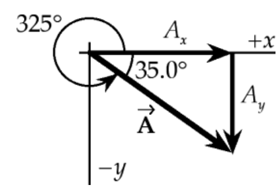


both known. In order to reach the end point of the run following the initial displacement, the jogger must follow the path shown as $\vec{\mathbf{B}}$. The length of $\vec{\mathbf{B}}$ and the angle θ can be measured. The results should be $\boxed{\text{83 m at 33° N of W}}$.

3.9 The displacement vectors $\vec{\mathbf{A}} = 8.00\text{ m westward}$ westward and $\vec{\mathbf{B}} = 13.0\text{ m north}$ north can be drawn to scale as at the right. The vector $\vec{\mathbf{C}}$ represents the displacement that the man in the maze must undergo to return to his starting point. The scale used to draw the sketch can be used to find $\vec{\mathbf{C}}$ to be $\boxed{\text{15 m at 58° S of E}}$.



3.10 The x- and y-components of vector $\vec{\mathbf{A}}$ are its projections on lines parallel to the x- and y-axis, respectively, as shown in the sketch. The magnitude of these components can be computed using the sine and cosine functions as shown below:

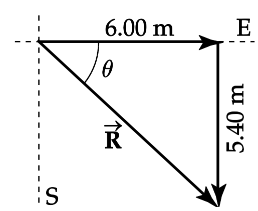


$$A_x = \left|\vec{\mathbf{A}}\right|\cos 325° = +\left|\vec{\mathbf{A}}\right|\cos 35° = \ 35.0 \ \cos 35° = \boxed{28.7\text{ units}}$$

and

$$A_y = \left|\vec{\mathbf{A}}\right|\sin 325° = -\left|\vec{\mathbf{A}}\right|\sin 35° = -\ 35.0 \ \sin 35° = \boxed{-20.1\text{ units}}$$

3.11 Using the vector diagram given at the right, we find



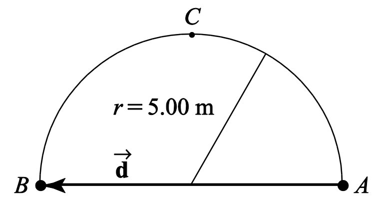
$$R = \sqrt{\ 6.00\text{ m}\ ^2 + \ 5.40\text{ m}\ ^2} = 8.07\text{ m}$$

and

$$\theta = \tan^{-1}\left(\frac{5.40\text{ m}}{6.00\text{ m}}\right) = \tan^{-1}\ 0.900\ = 42.0°$$

Thus, the required displacement is $\boxed{\text{8.07 m at 42.0°S of E}}$.

3.12 (a) The skater's displacement vector, $\vec{\mathbf{d}}$, extends in a straight line from her starting point A to the end point B. When she has coasted half way around a circular path as shown in the sketch at the right, the displacement vector coincides with the diameter of the circle and has magnitude



$$\left|\vec{\mathbf{d}}\right| = 2r = 2\;\;5.00\text{ m} \;\; = \boxed{10.0\text{ m}}$$

(b) The actual distance skated, s, is one half the circumference of the circular path of radius r. Thus,

$$s = \frac{1}{2}\;\;2\pi r \;\; = \pi\;\;5.00\text{ m} \;\; = \boxed{15.7\text{ m}}$$

(c) When the skater skates all the way around the circular path, her end point, B, coincides with the start point, A. Thus, the displacement vector has zero length, or $\left|\vec{\mathbf{d}}\right| = \boxed{0}$

3.13 (a) Her net x (east–west) displacement is $-3.00 + 0 + 6.00 = +3.00\text{ blocks}$, while her net y (north–south) displacement is $0 + 4.00 + 0 = +4.00\text{ blocks}$. The magnitude of the resultant displacement is

$$R = \sqrt{\;\Sigma x\;^2 + \;\Sigma y\;^2} = \sqrt{\;3.00\;^2 + \;4.00\;^2} = 5.00\text{ blocks}$$

and the angle the resultant makes with the x-axis (eastward direction) is

$$\theta = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right) = \tan^{-1}\left(\frac{4.00}{3.00}\right) = \tan^{-1}\;\;1.33\;\; = 53.1°$$

The resultant displacement is then $\boxed{5.00\text{ blocks at }53.1°\text{N of E}}$.

(b) The total distance traveled is $3.00 + 4.00 + 6.00 = \boxed{13.0\text{ blocks}}$.

3.14 (a) The resultant displacement is $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}}$, where $\vec{\mathbf{A}} = 75.0\text{ m}$, due north, $\vec{\mathbf{B}} = 250\text{ m}$, due east, $\vec{\mathbf{C}} = 125\text{ m at }30.0°\text{ north of east}$, and $\vec{\mathbf{D}} = 150\text{ m}$ due south. Choosing east as the positive x-direction and north as the positive y-direction, we find the components of the resultant to be

$$R_x = A_x + B_x + C_x + D_x = 0 + 250\text{ m} + \;\;125\text{ m}\;\;\cos 30.0° + 0 = 358\text{ m}$$

and

$$R_y = A_y + B_y + C_y + D_y = 75.0\text{ m} + 0 + (125\text{ m})\sin 30.0° + (-150\text{ m}) = -12.5\text{ m}$$

The magnitude and direction of the resultant are then

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(358\text{ m})^2 + (-12.5\text{ m})^2} = 358\text{ m}$$

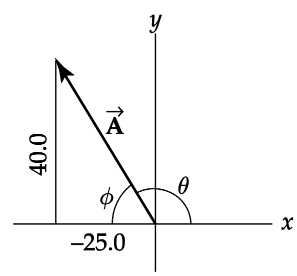
and

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-12.5\text{ m}}{358\text{ m}}\right) = -2.00°$$

Thus, $\vec{\mathbf{R}} = \boxed{358\text{ m at }2.00°\text{ south of east}}$.

(b) Because of the commutative property of vector addition, the net displacement is the same regardless of the order in which the individual displacements are executed.

3.15 $A_x = -25.0$ $\qquad A_y = 40.0$



$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0)^2 + (40.0)^2} = 47.2\text{ units}$$

From the triangle, we find that

$$\phi = \tan^{-1}\left(\frac{A_y}{|A_x|}\right) = \tan^{-1}\left(\frac{40.0}{25.0}\right) = 58.0°\text{, so } \theta = 180° - \phi = 122°$$

Thus, $\boxed{\vec{\mathbf{A}} = 47.2\text{ units at }122°\text{ counterclockwise from }+x\text{-axis}}$.

3.16 Let $\vec{\mathbf{A}}$ be the vector corresponding to the 10.0-yd run, $\vec{\mathbf{B}}$ to the 15.0-yd run, and $\vec{\mathbf{C}}$ to the 50.0-yd pass. Also, we choose a coordinate system with the $+y$ direction downfield, and the $+x$ direction toward the sideline to which the player runs.

The components of the vectors are then

$$A_x = 0 \qquad A_y = -10.0\text{ yds}$$

$$B_x = 15.0 \text{ yds} \qquad B_y = 0$$

$$C_x = 0 \qquad C_y = +50.0 \text{ yds}$$

From these, $R_x = \Sigma x = 15.0 \text{ yds}$, and $R_y = \Sigma y = 40.0 \text{ yds}$, and

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(15.0 \text{ yds})^2 + (40.0 \text{ yds})^2} = \boxed{42.7 \text{ yards}}$$

3.17 After 3.00 h moving at 41.0 km/h, the hurricane is 123 km at 60.0° N of W from the island. In the next 1.50 h, it travels 37.5 km due north. The components of these two displacements are as follows:

Displacement	***x*-component (eastward)**	***y*-component (northward)**
123 km	–61.5 km	+107 km
37.5 km	0	+37.5 km
Resultant	–61.5 km	144 km

Therefore, the eye of the hurricane is now

$$R = \sqrt{(-61.5 \text{ km})^2 + (144 \text{ km})^2} = \boxed{157 \text{ km from the island}}$$

3.18 Choose the positive x-direction to be eastward and positive y as northward. Then, the components of the resultant displacement from Dallas to Chicago are

$$R_x = \Sigma x = (730 \text{ mi})\cos 5.00° - (560 \text{ mi})\sin 21.0° = 527 \text{ mi}$$

and

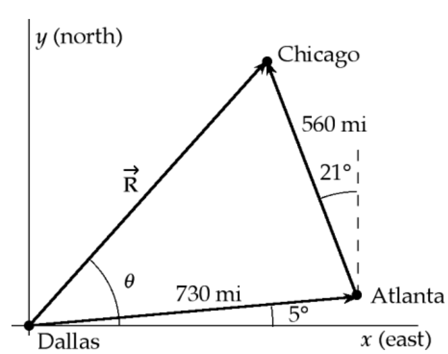
$$R_y = \Sigma y = (730 \text{ mi})\sin 5.00° + (560 \text{ mi})\cos 21.0° = 586 \text{ mi}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(527 \text{ mi})^2 + (586 \text{ mi})^2} = 788 \text{ mi}$$

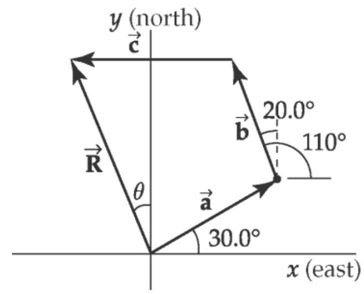
$$\theta = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right) = \tan^{-1}(1.11) = 48.1°$$

Thus, the displacement from Dallas to Chicago is

$\vec{\mathbf{R}} = \boxed{788 \text{ mi at } 48.1°\text{N of E}}$.



3.19 The components of the displacements $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, and $\vec{\mathbf{c}}$ are



$$a_x = a \cdot \cos 30.0° = +152 \text{ km}$$

$$b_x = b \cdot \cos 110° = -51.3 \text{ km}$$

$$c_x = c \cdot \cos 180° = -190 \text{ km}$$

and

$$a_y = a \cdot \sin 30.0° = +87.5 \text{ km}$$

$$b_y = b \cdot \sin 110° = +141 \text{ km}$$

$$c_y = c \cdot \sin 180° = 0$$

Thus,

$R_x = a_x + b_x + c_x = -89.7 \text{ km}$, and $R_y = a_y + b_y + c_y = +228 \text{ km}$

so

$R = \sqrt{R_x^2 + R_y^2} = 245 \text{ km}$, and $\theta = \tan^{-1}\left(\dfrac{|R_x|}{R_y}\right) = \tan^{-1}\ 1.11\ = 21.4°$

City C is $\boxed{\text{245 km at 21.4°W of N}}$ from the starting point.

3.20 (a) $F_1 = 120 \text{ N}$ $\quad F_{1x} = 120 \text{ N} \cos 60.0° = 60.0 \text{ N}$ $\quad F_{1y} = 120 \text{ N} \sin 60.0° = 104 \text{ N}$

$F_2 = 80.0 \text{ N}$ $\quad F_{2x} = -\ 80.0 \text{ N} \cos 75.0° = -20.7 \text{ N}$ $\quad F_{2y} = 80.0 \text{ N} \sin 75.0° = 77.3 \text{ N}$

$$F_R = \sqrt{\left(\Sigma F_x\right)^2 + \left(\Sigma F_y\right)^2} = \sqrt{\left(39.3 \text{ N}\right)^2 + \left(181 \text{ N}\right)^2} = 185 \text{ N}$$

and $\theta = \tan^{-1}\left(\dfrac{181 \text{ N}}{39.3 \text{ N}}\right) = \tan^{-1}\ 4.61\ = 77.8°$

The resultant force is $\vec{\mathbf{F}}_R = \boxed{\text{185 N at 77.8° from the } x\text{-axis}}$

(b) To have zero net force on the mule, the resultant above must be cancelled by a force equal in magnitude and oppositely directed. Thus, the required force is

$$\boxed{185\text{ N at }258°\text{from the }x\text{-axis}}$$

3.21 The single displacement required to sink the putt in one stroke is equal to the resultant of the three actual putts used by the novice. Taking east as the positive x-direction and north as the positive y-direction, the components of the three individual putts and their resultant are

$$A_x = 0 \qquad\qquad A_y = +4.00\text{ m}$$

$$B_x = (2.00\text{ m})\cos 45.0° = +1.41\text{ m} \qquad\qquad B_y = (2.00\text{ m})\sin 45.0° = +1.41\text{ m}$$

$$C_x = -(1.00\text{ m})\sin 30.0° = -0.500\text{ m} \qquad\qquad C_y = -(1.00\text{ m})\cos 30.0° = -0.866\text{ m}$$

$$R_x = A_x + B_x + C_x = +0.914\text{ m} \qquad\qquad R_y = A_y + B_y + C_y = +4.55\text{ m}$$

The magnitude and direction of the desired resultant is then

$$R = \sqrt{R_x^2 + R_y^2} = 4.64\text{ m} \qquad \text{and} \qquad \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = +78.6°$$

Thus, $\mathbf{R} = \boxed{4.64\text{ m at }78.6°\text{ north of east}}$.

3.22 $$v_{0x} = (101\text{ mi/h})\left(\frac{0.477\text{ m/s}}{1\text{ mi/h}}\right) = 45.1\text{ m/s and }\Delta x = (60.5\text{ ft})\left(\frac{1\text{ m}}{3.281\text{ ft}}\right) = 18.4\text{ m}$$

The time to reach home plate is

$$t = \frac{\Delta x}{v_{0x}} = \frac{18.4\text{ m}}{45.1\text{ m/s}} = 0.408\text{ s}$$

In this time interval, the vertical displacement is

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-9.80\text{ m/s}^2)(0.408\text{ s})^2 = -0.817\text{ m}$$

Thus, the ball drops vertically $0.817\ \text{m}\ (3.281\ \text{ft/1 m}) = \boxed{2.68\ \text{ft}}$.

3.23 (a) With the origin chosen at point O as shown in Figure P3.23, the coordinates of the original position of the stone are $\boxed{x_0 = 0 \text{ and } y_0 = +50.0\ \text{m}}$.

(b) The components of the initial velocity of the stone are $\boxed{v_{0x} = +18.0\ \text{m/s} \text{ and } v_{0y} = 0}$.

(c) The components of the stone's velocity during its flight are given as functions of time by

$$v_x = v_{0x} + a_x t = 18.0\ \text{m/s} + \ 0\ \ t \quad \text{or} \quad \boxed{v_x = 18.0\ \text{m/s}}$$

and

$$v_y = v_{0y} + a_y t = 0 + \ -g\ \ t \quad \text{or} \quad \boxed{v_x = -\ 9.80\ \text{m/s}^2\ \ t}$$

(d) The coordinates of the stone during its flight are

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 = 0 + \ 18.0\ \text{m/s}\ \ t + \frac{1}{2}\ \ 0\ \ t^2 \quad \text{or} \quad \boxed{x = \ 18.0\ \text{m/s}\ \ t}$$

and

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 = 50.0\ \text{m} + \ 0\ \ t + \frac{1}{2}\ \ -g\ \ t^2 \quad \text{or} \quad \boxed{y = 50.0\ \text{m} - \ 4.90\ \text{m/s}^2\ \ t^2}$$

(e) We find the time of fall from $\Delta y = v_{0y} t + \frac{1}{2} a_y t^2$ with $v_{0y} = 0$:

$$t = \sqrt{\frac{2\ \ \Delta y}{a}} = \sqrt{\frac{2\ \ -50.0\ \text{m}}{-9.80\ \text{m/s}^2}} = \boxed{3.19\ \text{s}}$$

(f) At impact, $v_x = v_{0x} = 18.0\ \text{m/s}$, and the vertical component is

$$v_y = v_{0y} + a_y t = 0 + \ -9.80\ \text{m/s}^2\ \ \ 3.19\ \text{s}\ \ = -31.3\ \text{m/s}$$

Thus,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(18.0 \text{ m/s})^2 + (-31.3 \text{ m/s})^2} = 36.1 \text{ m/s}$$

and

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-31.3}{18.0}\right) = -60.1°$$

or $\vec{\mathbf{v}} = \boxed{36.1 \text{ m/s at } 60.1° \text{below the horizontal}}$.

3.24 The constant horizontal speed of the falcon is

$$v_x = \left(200 \frac{\text{mi}}{\text{h}}\right)\left(\frac{0.447 \text{ m/s}}{1 \text{ mi/h}}\right) = 89.4 \text{ m/s}$$

The time required to travel 100 m horizontally is

$$t = \frac{\Delta x}{v_x} = \frac{100 \text{ m}}{89.4 \text{ m/s}} = 1.12 \text{ s}$$

The vertical displacement during this time is

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)(1.12 \text{ s})^2 = -6.13 \text{ m}$$

or the falcon has a vertical fall of $\boxed{6.13 \text{ m}}$.

3.25 At the maximum height $v_y = 0$, and the time to reach this height is found from

$$v_y = v_{0y} + a_y t \text{ as } t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - v_{0y}}{-g} = \frac{v_{0y}}{g}$$

The vertical displacement that has occurred during this time is

$$(\Delta y)_{\text{max}} = (v_y)_{\text{av}} t = \left(\frac{v_y + v_{0y}}{2}\right)t = \left(\frac{0 + v_{0y}}{2}\right)\left(\frac{v_{0y}}{g}\right) = \frac{v_{0y}^2}{2g}$$

Thus, if $(\Delta y)_{\max} = 12 \text{ ft}\,(1 \text{ m}/3.281 \text{ ft}) = 3.7 \text{ m}$, then

$$v_{0y} = \sqrt{2g\,(\Delta y)_{\max}} = \sqrt{2\,(9.80 \text{ m/s}^2)\,(3.7 \text{ m})} = 8.5 \text{ m/s}$$

and if the angle of projection is $\theta = 45°$, the launch speed is

$$v_0 = \frac{v_{0y}}{\sin\theta} = \frac{8.5 \text{ m/s}}{\sin 45°} = \boxed{12 \text{ m/s}}$$

3.26 (a) When a projectile is launched with speed v_0 at angle θ_0 above the horizontal, the initial velocity components are $v_{0x} = v_0 \cos\theta_0$ and $v_{0y} = v_0 \sin\theta_0$. Neglecting air resistance, the vertical velocity when the projectile returns to the level from which it was launched (in this case, the ground) will be $v_y = -v_{0y}$. From this information, the total time of flight is found from $v_y = v_{0y} + a_y t$ to be

$$t_{\text{total}} = \frac{v_{yf} - v_{0y}}{a_y} = \frac{-v_{0y} - v_{0y}}{-g} = \frac{2v_{0y}}{g} \quad \text{or} \quad t_{\text{total}} = \frac{2v_0 \sin\theta_0}{g}$$

Since the horizontal velocity of a projectile with no air resistance is constant, the horizontal distance it will travel in this time (i.e., its range) is given by

$$R = v_{0x} t_{\text{total}} = (v_0 \cos\theta_0)\left(\frac{2v_0 \sin\theta_0}{g}\right) = \frac{v_0^2}{g}(2\sin\theta_0\cos\theta_0) = \frac{v_0^2 \sin(2\theta_0)}{g}$$

Thus, if the projectile is to have a range of $R = 81.1$ when launched at an angle of $\theta_0 = 45.0°$, the required initial speed is

$$v_0 = \sqrt{\frac{Rg}{\sin(2\theta_0)}} = \sqrt{\frac{(81.1 \text{ m})(9.80 \text{ m/s}^2)}{\sin(90.0°)}} = \boxed{28.2 \text{ m/s}}$$

(b) With $v_0 = 28.2 \text{ m/s}$ and $\theta_0 = 45.0°$ the total time of flight (as found above) will be

$$t_{\text{total}} = \frac{2v_0 \sin\theta_0}{g} = \frac{2\,(28.2 \text{ m/s})\sin(45.0°)}{9.80 \text{ m/s}^2} = \boxed{4.07 \text{ s}}$$

(c) Note that at $\theta_0 = 45.0°$, $\sin(2\theta_0) = 1$, and that $\sin(2\theta_0)$ will decrease as θ_0 is increased above this optimum launch angle. Thus, if the range is to be kept constant while the launch angle is increased above 45.0°, we see from $v_0 = \sqrt{Rg/\sin 2\theta_0}$ that

$\boxed{\text{the required initial velocity will increase}}$.

Observe that for $\theta_0 < 90°$, the function $\sin\theta_0$ increases as θ_0 is increased. Thus, increasing the launch angle above 45.0° while keeping the range constant means that both v_0 and $\sin\theta_0$ will increase. Considering the expression for t_{total} given above, we see that $\boxed{\text{the total time of flight will increase}}$.

3.27 When $\Delta y = (\Delta y)_{\max}$, $v_y = 0$.

Thus, $v_y = v_{0y} + a_y t$ yields $0 = v_0 \sin 3.00° - gt$, or

$$t = \frac{v_0 \sin 3.00°}{g}$$

The vertical displacement is $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$. At the maximum height, this becomes

$$\Delta y_{\max} = v_0 \sin 3.00°\left(\frac{v_0 \sin 3.00°}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin 3.00°}{g}\right)^2 = \frac{v_0^2 \sin^2 3.00°}{2g}$$

If $(\Delta y)_{\max} = 0.330\text{ m}$, the initial speed is

$$v_0 = \frac{\sqrt{2g\,\Delta y_{\max}}}{\sin 3.00°} = \frac{\sqrt{2\left(9.80\text{ m/s}^2\right)\left(0.330\text{ m}\right)}}{\sin 3.00°} = \boxed{48.6\text{ m/s}}$$

Note that it was unnecessary to use the horizontal distance of 12.6 m in this solution.

3.28 (a) With the origin at ground level directly below the window, the original coordinates of the ball are $(x, y) = \boxed{(0, y_0)}$.

(b) $v_{0x} = v_0 \cos\theta_0 = \left(8.00\text{ m/s}\right)\cos\left(-20.0°\right) = \boxed{+7.52\text{ m/s}}$

$$v_{0y} = v_0 \sin\theta_0 = (8.00 \text{ m/s})\sin(-20.0°) = \boxed{-2.74 \text{ m/s}}$$

(c) $$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = 0 + (7.52 \text{ m/s})t + \frac{1}{2}(0)t^2 \quad \text{or} \quad \boxed{x = (7.52 \text{ m/s})t}$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = y_0 + (-2.74 \text{ m/s})t + \frac{1}{2}(-9.80 \text{ m/s}^2)t^2$$

or $$\boxed{y = y_0 - (2.74 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2}$$

(d) Since the ball hits the ground at $t = 3.00$ s, the x-coordinate at the landing site is

$$x_{\text{landing}} = x\big|_{t=3.00\text{ s}} = (7.52 \text{ m/s})(3.00 \text{ s}) = \boxed{22.6 \text{ m}}$$

(e) Since $y = 0$ when the ball reaches the ground at $t = 3.00$ s, the result of (c) above gives

$$y_0 = \left[y + \left(2.74 \frac{\text{m}}{\text{s}}\right)t + \left(4.90 \frac{\text{m}}{\text{s}^2}\right)t^2\right]\Bigg|_{t=3.00\text{ s}} = 0 + \left(2.74 \frac{\text{m}}{\text{s}}\right)(3.00 \text{ s}) + \left(4.90 \frac{\text{m}}{\text{s}^2}\right)(3.00 \text{ s})^2$$

Or $y_0 = \boxed{52.3 \text{ m}}$.

(f) When the ball has a vertical displacement of $\Delta y = -10.0$, it will be moving downward with a velocity given by $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$ as

$$v_y = -\sqrt{v_{0y}^2 + 2a_y(\Delta y)} = -\sqrt{(-2.74 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-10.0 \text{ m})} = -14.3 \text{ m/s}$$

The elapsed time at this point is then

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-14.3 \text{ m/s} - (-2.74 \text{ m/s})}{-9.80 \text{ m/s}^2} = \boxed{1.18 \text{ s}}$$

3.29 We choose our origin at the initial position of the projectile. After 3.00 s, it is at ground level, so the vertical displacement is $\Delta y = -H$.

To find H, we use $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, which becomes

$$-H = \left[(15\ \text{m/s})\sin 25^\circ\right](3.0\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(3.0\ \text{s})^2, \text{ or } H = \boxed{25\ \text{m}}$$

3.30 The initial velocity components of the projectile are

$$v_{0x} = (300\ \text{m/s})\cos 55.0^\circ = 172\ \text{m/s} \quad \text{and} \quad v_{0y} = (300\ \text{m/s})\sin 55.0^\circ = 246\ \text{m/s}$$

while the constant acceleration components are

$$a_x = 0 \quad \text{and} \quad a_y = -g = -9.80\ \text{m/s}^2$$

The coordinates of where the shell strikes the mountain at $t = 42.0$ s are

$$x = v_{0x}t + \frac{1}{2}a_x t^2 = (172\ \text{m/s})(42.0\ \text{s}) + 0 = 7.22 \times 10^3\ \text{m} = \boxed{7.22\ \text{km}}$$

and

$$y = v_{0y}t + \frac{1}{2}a_y t^2$$
$$= (246\ \text{m/s})(42.0\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(42.0\ \text{s})^2 = 1.69 \times 10^3\ \text{m} = \boxed{1.69\ \text{km}}$$

3.31 The speed of the car when it reaches the edge of the cliff is

$$v = \sqrt{v_0^2 + 2a(\Delta x)} = \sqrt{0 + 2(4.00\ \text{m/s}^2)(50.0\ \text{m})} = 20.0\ \text{m/s}$$

Now, consider the projectile phase of the car's motion. The vertical velocity of the car as it reaches the water is

$$v_y = -\sqrt{v_{0y}^2 + 2a_y(\Delta y)} = -\sqrt{\left[-(20.0\ \text{m/s})\sin 24.0^\circ\right]^2 + 2(-9.80\ \text{m/s}^2)(-30.0\ \text{m})}$$

or

$$v_y = -25.6\ \text{m/s}$$

(b) The time of flight is

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-25.6\ \text{m/s} - \left[-(20.0\ \text{m/s})\sin 24.0^\circ\right]}{-9.80\ \text{m/s}^2} = \boxed{1.78\ \text{s}}$$

(a) The horizontal displacement of the car during this time is

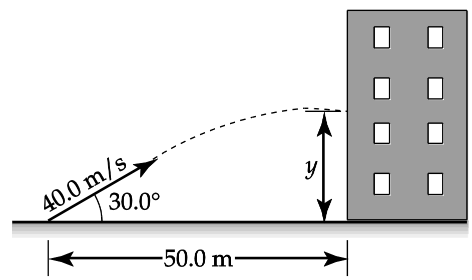
$$\Delta x = v_{0x}t = \left[(20.0 \text{ m/s})\cos 24.0° \right](1.78 \text{ s}) = \boxed{32.5 \text{ m}}$$

3.32 The components of the initial velocity are

$$v_{0x} = (40.0 \text{ m/s})\cos 30.0° = 34.6 \text{ m/s}$$

and

$$v_{0y} = (40.0 \text{ m/s})\sin 30.0° = 20.0 \text{ m/s}$$



The time for the water to reach the building is

$$t = \frac{\Delta x}{v_{0x}} = \frac{50.0 \text{ m}}{34.6 \text{ m}} = 1.44 \text{ s}$$

The height of the water at this time is

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 = (20.0 \text{ m/s})(1.44 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(1.44 \text{ s})^2 = \boxed{18.7 \text{ m}}$$

3.33 (a) At the highest point of the trajectory, the projectile is moving horizontally with velocity components of and

$v_y = 0$ and

$$v_x = v_{0x} = v_0 \cos\theta = (60.0 \text{ m/s})\cos 30.0° = \boxed{52.0 \text{ m/s}}$$

(b) The horizontal displacement is $\Delta x = v_{0x}t = (52.0 \text{ m/s})(4.00 \text{ s}) = 208 \text{ m}$ and, from $\Delta y = (v_0 \sin\theta)t + \frac{1}{2}a_y t^2$, the vertical displacement is

$$\Delta y = (60.0 \text{ m/s})(\sin 30.0°)(4.00 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(4.00 \text{ s})^2 = 41.6 \text{ m}$$

The straight line distance is

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(208 \text{ m})^2 + (41.6 \text{ m})^2} = \boxed{212 \text{ m}}$$

3.34 The horizontal kick gives zero initial vertical velocity to the rock. Then, from $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, the time of flight is

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-40.0\text{ m})}{-9.80\text{ m/s}^2}} = \sqrt{8.16}\text{ s}$$

The extra time $\Delta t = 3.00\text{ s} - \sqrt{8.16}\text{ s} = 0.143\text{ s}$ is the time required for the sound to travel in a straight line back to the player. The distance the sound travels is

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = v_{\text{sound}}(\Delta t) = (343\text{ m/s})(0.143\text{ s}) = 49.0\text{ m}$$

where Δx represents the horizontal displacement of the rock when it hits the water. Thus,

$$\Delta x = \sqrt{d^2 - (\Delta y)^2} = \sqrt{(49.0\text{ m})^2 - (-40.0\text{ m})^2} = 28.3\text{ m}$$

The initial velocity given the ball must have been

$$v_0 = v_{0x} = \frac{\Delta x}{t} = \frac{28.3\text{ m}}{\sqrt{8.16}\text{ s}} = \boxed{9.91\text{ m/s}}$$

3.35 (a) The jet moves at 3.00×10^2 mi/h due east relative to the air. Choosing a coordinate system with the positive x-direction eastward and the positive y-direction northward, the components of this velocity are

$$\boxed{(\vec{\mathbf{v}}_{JA})_x = 3.00 \times 10^2\text{ mi/h} \quad \text{and} \quad (\vec{\mathbf{v}}_{JA})_y = 0}$$

(b) The velocity of the air relative to Earth is 1.00×10^2 mi/h at 30.0° north of east. Using the coordinate system adopted in (a) above, the components of this velocity are

$$(\vec{\mathbf{v}}_{AE})_x = |\vec{\mathbf{v}}_{AE}|\cos\theta = (1.00 \times 10^2\text{ mi/h})\cos 30.0° = \boxed{86.6\text{ mi/h}}$$

and

$$(\vec{\mathbf{v}}_{AE})_y = |\vec{\mathbf{v}}_{AE}|\sin\theta = (1.00 \times 10^2\text{ mi/h})\sin 30.0° = \boxed{50.0\text{ mi/h}}$$

(c) Carefully observe the pattern of the subscripts in Equation 3.16 of the textbook. There, two objects (cars A and B) both move relative to a third object (Earth, E). The velocity of object A relative to object B is given in terms of the velocities of these objects relative to E as $\vec{\mathbf{v}}_{AB} = \vec{\mathbf{v}}_{AE} - \vec{\mathbf{v}}_{BE}$. In the present case, we have two objects, a jet (J) and the air (A), both moving relative to a third object, Earth (E). Using the same pattern of subscripts as that in Equation 3.16, the velocity of the jet relative to the air is given by

$$\boxed{\vec{\mathbf{v}}_{JA} = \vec{\mathbf{v}}_{JE} - \vec{\mathbf{v}}_{AE}}$$

(d) From the expression for $\vec{\mathbf{v}}_{JA}$ found in (c) above, the velocity of the jet relative to the ground is $\vec{\mathbf{v}}_{JE} = \vec{\mathbf{v}}_{JA} + \vec{\mathbf{v}}_{AE}$. Its components are then

$$\left(\vec{\mathbf{v}}_{JE}\right)_x = \left(\vec{\mathbf{v}}_{JA}\right)_x + \left(\vec{\mathbf{v}}_{AE}\right)_x = 3.00 \times 10^2 \text{ mi/h} + 86.6 \text{ mi/h} = 3.87 \times 10^2 \text{ mi/h}$$

and

$$\left(\vec{\mathbf{v}}_{JE}\right)_y = \left(\vec{\mathbf{v}}_{JA}\right)_y + \left(\vec{\mathbf{v}}_{AE}\right)_y = 0 + 50.0 \text{ mi/h} = 50.0 \text{ mi/h}$$

This gives the magnitude and direction of the jet's motion relative to Earth as

$$\left|\vec{\mathbf{v}}_{JE}\right| = \sqrt{\left|\vec{\mathbf{v}}_{JE}\right|_x^2 + \left|\vec{\mathbf{v}}_{JE}\right|_y^2} = \sqrt{\left(3.87 \times 10^2 \text{ mi/h}\right)^2 + \left(50.0 \text{ mi/h}\right)^2} = \boxed{3.90 \times 10^2 \text{ mi/h}}$$

and

$$\theta = \tan^{-1}\left(\frac{\left(\vec{\mathbf{v}}_{JE}\right)_y}{\left(\vec{\mathbf{v}}_{JE}\right)_x}\right) = \tan^{-1}\left(\frac{50.0 \text{ mi/h}}{3.87 \times 10^2 \text{ mi/h}}\right) = 7.37°$$

Therefore, $\boxed{\vec{\mathbf{v}}_{JE} = 3.90 \times 10^2 \text{ mi/h at } 7.37° \text{ north of east}}$.

3.36 We use the following notation:

$\vec{\mathbf{v}}_{BS}$ = velocity of boat relative to the shore

$\vec{\mathbf{v}}_{BW}$ = velocity of boat relative to the water,

and $\vec{\mathbf{v}}_{WS}$ = velocity of water relative to the shore.

If we take downstream as the positive direction, then $\vec{\mathbf{v}}_{WS} = +1.5 \text{ m/s}$ for both parts of the trip.

Also, $\vec{\mathbf{v}}_{BW} = +10 \text{ m/s}$ while going downstream and $\vec{\mathbf{v}}_{BW} = -10 \text{ m/s}$ for the upstream part of the trip.

The velocity of the boat relative to the boat relative to the water is $\vec{\mathbf{v}}_{BW} = \vec{\mathbf{v}}_{BS} - \vec{\mathbf{v}}_{WS}$, so the velocity of the boat relative to the shore is $\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}$.

While going downstream, $\vec{\mathbf{v}}_{BS} = 10\ \text{m/s} + 1.5\ \text{m/s}$ and the time to go 300 m downstream is

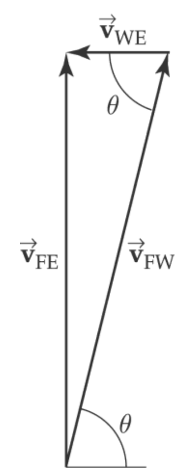
$$t_{down} = \frac{d}{|\vec{\mathbf{v}}_{BS}|} = \frac{300\ \text{m}}{10 + 1.5\quad \text{m/s}} = 26\ \text{s}$$

When going upstream, $\vec{\mathbf{v}}_{BS} = -10\ \text{m/s} + 1.5\ \text{m/s} = -8.5\ \text{m/s}$ and the time required to move 300 m upstream is

$$t_{up} = \frac{d}{|\vec{\mathbf{v}}_{BS}|} = \frac{300\ \text{m}}{8.5\ \text{m/s}} = 35\ \text{s}$$

The time for the round trip is $t = t_{down} + t_{up} = \ 26 + 35\quad \text{s} = \boxed{61\ \text{s}}$.

3.37 Prior to the leap, the salmon swims upstream through water flowing at speed $|\vec{\mathbf{v}}_{WE}| = 1.50\ \text{m/s}$ relative to Earth. The fish swims at $|\vec{\mathbf{v}}_{FW}| = 6.26\ \text{m/s}$ relative to the water in such a direction to make its velocity relative to Earth, $\vec{\mathbf{v}}_{FE}$, vertical. Since $\vec{\mathbf{v}}_{FE} = \vec{\mathbf{v}}_{FW} + \vec{\mathbf{v}}_{WE}$, as shown in the diagram at the right, we find that



$$\theta = \cos^{-1}\left(\frac{|\vec{\mathbf{v}}_{WE}|}{|\vec{\mathbf{v}}_{FW}|}\right) = \cos^{-1}\left(\frac{1.50\ \text{m/s}}{6.26\ \text{m/s}}\right) = 76.1°$$

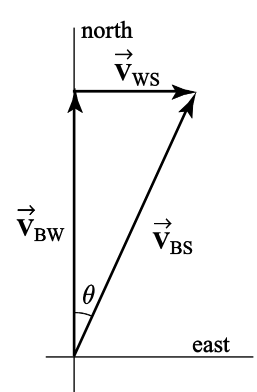
and the vertical velocity of the fish as it leaves the water is

$$v_{0y} = |\vec{\mathbf{v}}_{FE}| = |\vec{\mathbf{v}}_{FW}|\sin\theta = \ 6.26\ \text{m/s}\ \sin 76.1° = 6.08\ \text{m/s}$$

The height of the salmon above the water at the top of its leap (that is, when $v_y = 0$) is given by

$$\Delta y = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - \ 6.08\ \text{m/s}\ ^2}{2\ -9.80\ \text{m/s}^2} = \boxed{1.88\ \text{m}}$$

3.38 (a) The velocity of the boat relative to the water is



$$\vec{\mathbf{v}}_{BW} = \vec{\mathbf{v}}_{BS} - \vec{\mathbf{v}}_{WS}$$

where $\vec{\mathbf{v}}_{BS}$ is the velocity of the boat relative to shore, and is $\vec{\mathbf{v}}_{WS}$ is the velocity of the water relative to shore. Thus, we may write $\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}$. Note, in the vector diagram at the right, that these vectors form a right triangle. The Pythagorean theorem then gives the magnitude of the resultant $|\vec{\mathbf{v}}_{BS}|$ as

$$|\vec{\mathbf{v}}_{BS}| = \sqrt{|\vec{\mathbf{v}}_{BW}|^2 + |\vec{\mathbf{v}}_{WS}|^2} = \sqrt{10.0\ \text{m/s}^{\ 2} + 1.50\ \text{m/s}^{\ 2}} = 10.1\ \text{m/s}$$

also,

$$\theta = \tan^{-1}\left(\frac{|\vec{\mathbf{v}}_{WS}|}{|\vec{\mathbf{v}}_{BW}|}\right) = \tan^{-1}\left(\frac{1.50\ \text{m/s}}{10.0\ \text{m/s}}\right) = 8.53°$$

so $\vec{\mathbf{v}}_{BS} = \boxed{10.1\ \text{m/s at } 8.53° \text{ east of north}}$.

(b) The boat has a constant northward velocity of $\vec{\mathbf{v}}_{BW} = 10.0\ \text{m/s}$, and must travel a distance $d = 300$ m northward to reach the opposite shore. The time required will be

$$t = \frac{d}{|\vec{\mathbf{v}}_{BW}|} = \frac{300\ \text{m}}{10.0\ \text{m/s}} = 30.0\ \text{s}$$

During this time, the boat drifts downstream at a constant rate of $|\vec{\mathbf{v}}_{WS}| = 1.50\ \text{m/s}$.

The distance it drifts downstream during the crossing is

$$drift = |\vec{\mathbf{v}}_{WS}| \cdot t = 1.50\ \text{m/s}\quad 30.0\ \text{s} = \boxed{45.0\ \text{m}}$$

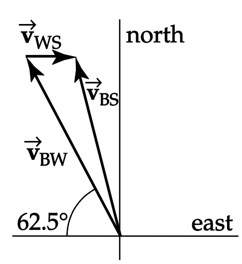
3.39 $\vec{\mathbf{v}}_{BW}$ = velocity of boat relative to the water

$\vec{\mathbf{v}}_{WS}$ = velocity of water relative to the shore

$\vec{\mathbf{v}}_{BS}$ = velocity of boat relative to the shore

$\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}$ (as shown in the diagram)

The northward (that is, cross-stream) component of $\vec{\mathbf{v}}_{BS}$ is



$$\left(\vec{\mathbf{v}}_{\mathrm{BS}}\right)_{\mathrm{north}} = \left(v_{\mathrm{BW}}\right)\sin 62.5° + 0 = \left(3.30\ \mathrm{mi/h}\right)\sin 62.5° + 0 = 2.93\ \mathrm{mi/h}$$

The time required to cross the stream is then

$$t = \frac{0.505\ \mathrm{mi}}{2.93\ \mathrm{mi/h}} = 0.173\ \mathrm{h}$$

The eastward (that is, downstream) component of $\vec{\mathbf{v}}_{\mathrm{BS}}$ is

$$\left(\vec{\mathbf{v}}_{\mathrm{BS}}\right)_{\mathrm{east}} = -\left(v_{\mathrm{BW}}\right)\cos 62.5° + v_{\mathrm{WS}}$$

$$= -\left(3.30\ \mathrm{mi/h}\right)\cos 62.5° + 1.25\ \mathrm{mi/h} = -0.274\ \mathrm{mi/h}$$

Since the last result is negative, it is seen that the boat moves upstream as it crosses the river. The distance it moves upstream is

$$d = \left|\left(\vec{\mathbf{v}}_{\mathrm{BS}}\right)_{\mathrm{east}}\right| t = \left(0.274\ \mathrm{mi/h}\right)\left(0.173\ \mathrm{h}\right) = \left(4.72\times 10^{-2}\ \mathrm{mi}\right)\left(\frac{5\,280\ \mathrm{ft}}{1\ \mathrm{mi}}\right) = \boxed{249\ \mathrm{ft}}$$

3.40 If the salmon (a projectile) is to have $v_y = 0$ when $\Delta y = +1.50\ \mathrm{m}$, the required initial velocity in the vertical direction is given by $v_y^2 = v_{0y}^2 + 2a_y\Delta y$ as

$$v_{0y} = +\sqrt{v_y^2 - 2a_y\Delta y} = \sqrt{0 - 2\left(-9.80\ \mathrm{m/s^2}\right)\left(+1.50\ \mathrm{m}\right)} = 5.42\ \mathrm{m/s}$$

The elapsed time for the upward flight will be

$$\Delta t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - 5.42\ \mathrm{m/s}}{-9.80\ \mathrm{m/s^2}} = 0.553\ \mathrm{s}$$

If the horizontal displacement at this time is to be $\Delta x = +1.00\ \mathrm{m}$, the required constant horizontal component of the salmon's velocity must be

$$v_{0x} = \frac{\Delta x}{\Delta t} = \frac{1.00\ \mathrm{m}}{0.553\ \mathrm{s}} = 1.81\ \mathrm{m/s}$$

The speed with which the salmon must leave the water is then

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{\left(1.81\ \mathrm{m/s}\right)^2 + \left(5.42\ \mathrm{m/s}\right)^2} = 5.72\ \mathrm{m/s}$$

$\boxed{\text{Yes}}$, since $v_0 < 6.26 \text{ m/s}$ the salmon is capable of making this jump.

3.41 (a) Both the student (S) and the water (W) move relative to Earth (E). The velocity of the student relative to the water is given by $\vec{\mathbf{v}}_{SW} = \vec{\mathbf{v}}_{SE} - \vec{\mathbf{v}}_{WE}$, where $\vec{\mathbf{v}}_{SE}$ and $\vec{\mathbf{v}}_{WE}$ are the velocities of the student relative to Earth and the water relative to Earth, respectively. If we choose downstream as the positive direction, then $\vec{\mathbf{v}}_{WE} = +0.500 \text{ m/s}$, $\vec{\mathbf{v}}_{SW} = -1.20 \text{ m/s}$ when the student is going up stream, and $\vec{\mathbf{v}}_{SW} = +1.20 \text{ m/s}$ when the student moves downstream.

The velocity of the student relative to Earth for each leg of the trip is

$$\left(\vec{\mathbf{v}}_{SE}\right)_{\text{upstream}} = \vec{\mathbf{v}}_{WE} + \left(\vec{\mathbf{v}}_{SW}\right)_{\text{upstream}} = 0.500 \text{ m/s} + \left(-1.20 \text{ m/s}\right) = -0.700 \text{ m/s}$$

and

$$\left(\vec{\mathbf{v}}_{SE}\right)_{\text{downstream}} = \vec{\mathbf{v}}_{WE} + \left(\vec{\mathbf{v}}_{SW}\right)_{\text{downstream}} = 0.500 \text{ m/s} + \left(+1.20 \text{ m/s}\right) = +1.70 \text{ m/s}$$

The distance (measured relative to Earth) for each leg of the trip is $d = 1.00 \text{ km} = 1.00 \times 10^3 \text{m}$. The times required for each of the two legs are

$$t_{\text{upstream}} = \frac{d}{\left|\vec{\mathbf{v}}_{SE}\right|_{\text{upstream}}} = \frac{1.00 \times 10^3 \text{ m}}{0.700 \text{ m/s}} = 1.43 \times 10^3 \text{ s}$$

and

$$t_{\text{downstream}} = \frac{d}{\left|\vec{\mathbf{v}}_{SE}\right|_{\text{downstream}}} = \frac{1.00 \times 10^3 \text{ m}}{1.70 \text{ m/s}} = 5.88 \times 10^{s} \text{ s}$$

so the time for the total trip is

$$t_{\text{total}} = t_{\text{upstream}} + t_{\text{downstream}} = 1.43 \times 10^3 \text{ s} + 5.88 \times 10^{s} \text{ s} = \boxed{2.02 \times 10^3 \text{ s}}$$

(b) If the water is still $\left(\left|\vec{\mathbf{v}}_{WE}\right| = 0\right)$, the speed of the student relative to Earth would have been the same for each leg of the trip, $\left|\vec{\mathbf{v}}_{SE}\right| = \left|\vec{\mathbf{v}}_{SE}\right|_{\text{upstream}} = \left|\vec{\mathbf{v}}_{SE}\right|_{\text{downstream}} = 1.20 \text{ m/s}$. In this case, the time for each leg, and the total time would have been

$$t_{\text{leg}} = \frac{d}{\left|\vec{\mathbf{v}}_{SE}\right|} = \frac{1.00 \times 10^3 \text{ m}}{1.20 \text{ m/s}} = 8.33 \times 10^3 \text{ s} \quad \text{and} \quad t_{\text{total}} = 2t_{\text{leg}} = \boxed{1.67 \times 10^3 \text{ s}}$$

(c) $\boxed{\text{The time savings going downstream with the current is always less than the extra time required to go the same distance against the current.}}$

3.42 (a) The speed of the student relative to shore is $v_{\text{up}} = v - v_s$ while swimming upstream and $v_{\text{down}} = v + v_s$ while swimming downstream. The time required to travel distance *d* upstream is then

$$t_{\text{up}} = \frac{d}{v_{\text{up}}} = \boxed{\frac{d}{v - v_s}}$$

(b) The time required to swim the same distance *d* downstream is

$$t_{\text{down}} = \frac{d}{v_{\text{down}}} = \boxed{\frac{d}{v + v_s}}$$

(c) The total time for the trip is therefore

$$t_a = t_{\text{up}} + t_{\text{down}} = \frac{d}{v - v_s} + \frac{d}{v + v_s} = \frac{d(v + v_s) + d(v - v_s)}{(v - v_s)(v + v_s)} = \frac{2dv}{v^2 - v_s^2} = \boxed{\frac{2d/v}{1 - v_s^2/v^2}}$$

(d) In still water, $v_s = 0$ and the time for the complete trip is seen to be

$$t_b = t_a\Big|_{v_s = 0} = \frac{2d/v}{1 - 0/v^2} = \boxed{\frac{2d}{v}}$$

(e) Note that $t_b = \dfrac{2d}{v} = \dfrac{2dv}{v^2}$ and that $t_a = \dfrac{2dv}{v^2 - v_s^2}$.

Thus, when there is a current $(v_s > 0)$, $\boxed{\text{it is always true that } t_a > t_b}$.

3.43 (a) The bomb starts its fall with $v_{0y} = 0$ and $v_{0x} = v_{\text{plane}} = 275\ \text{m/s}$. Choosing the origin at the location of the plane when the bomb is released and upward as positive, the *y* coordinate of the bomb at ground level is $y = -h = -3.00 \times 10^3\ \text{m}$. The time required for the bomb to fall is given by $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ as

$$-3.00 \times 10^3\ \text{m} = 0 + 0 + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)t_{\text{fall}}^2 \quad \text{or} \quad t_{\text{fall}} = \sqrt{\frac{2\left(3.00 \times 10^3\ \text{m}\right)}{9.80\ \text{m/s}^2}} = 24.7\ \text{s}$$

With $a_x = 0$, the horizontal distance the bomb travels during this time is

$$d = v_{0x}t_{\text{fall}} = (275 \text{ m/s})(24.7 \text{ s}) = 6.80 \times 10^3 \text{ m} = \boxed{6.80 \text{ km}}$$

(b) While the bomb is falling, the plane travels in the same horizontal direction with the same constant horizontal speed, $v_x = v_{0x} = v_{\text{plane}}$, as the bomb. Thus, the plane remains directly above the bomb as the bomb falls to the ground. When impact occurs, the plane is $\boxed{\text{directly over the impact point, at an altitude of 3.00 km.}}$

(c) The angle, measured in the forward direction from the vertical, at which the bombsight must have been set is

$$\theta = \tan^{-1}\left(\frac{d}{h}\right) = \tan^{-1}\left(\frac{6.80 \text{ km}}{3.00 \text{ km}}\right) = \boxed{66.2°}$$

3.44 (a) The time required for the woman, traveling at constant speed v_1 relative to the ground, to travel distance L relative to the ground is $t_{\text{woman}} = \boxed{L/v_1}$.

(b) With both the walkway (W) and the man (M) moving relative to Earth (E), we know that the velocity of the man relative to the moving walkway is $\vec{\mathbf{v}}_{\text{MW}} = \vec{\mathbf{v}}_{\text{ME}} - \vec{\mathbf{v}}_{\text{WE}}$. His velocity relative to Earth is then $\vec{\mathbf{v}}_{\text{ME}} = \vec{\mathbf{v}}_{\text{MW}} + \vec{\mathbf{v}}_{\text{WE}}$. Since all of these velocities are in the same direction, his speed relative to Earth is $|\vec{\mathbf{v}}_{\text{ME}}| = |\vec{\mathbf{v}}_{\text{MW}}| + |\vec{\mathbf{v}}_{\text{WE}}| = v_2 + v_1$. The time required for the man to travel distance L relative to the ground is then

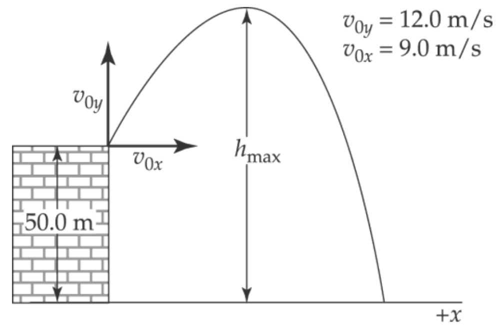
$$t_{\text{man}} = \frac{L}{|\vec{\mathbf{v}}_{\text{ME}}|} = \boxed{\frac{L}{v_1 + v_2}}$$

3.45 Choose the positive direction to be the direction of each car's motion relative to Earth. The velocity of the faster car relative to the slower car is given by $\vec{\mathbf{v}}_{\text{FS}} = \vec{\mathbf{v}}_{\text{FE}} - \vec{\mathbf{v}}_{\text{SE}}$, where $\vec{\mathbf{v}}_{\text{SE}} = 40.0 \text{ km/h}$ is the velocity of the faster car relative to Earth, and $\vec{\mathbf{v}}_{\text{FE}} = +60.0 \text{ km/h}$ is the velocity of the slower car relative to Earth.

Thus, $\vec{\mathbf{v}}_{\text{FS}} = +60.0 \text{ km/h} - 40.0 \text{ km/h} = +20.0 \text{ km/h}$ and the time required for the faster car to move 100 m (0.100 km) closer to the slower car is

$$t = \frac{d}{v_{\text{FS}}} = \frac{0.100 \text{ km}}{20.0 \text{ km/h}} = 5.00 \times 10^{-3} \text{ h}\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = \boxed{18.0 \text{ s}}$$

3.46 The vertical displacement from the launch point (top of the building) to the top of the arc may be found from $v_y^2 = v_{0y}^2 + 2a_y\Delta y$ with $v_y = 0$ at the top of the arc. This yields



$$\Delta y = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (12.0 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = +7.35 \text{ m}$$

and $\Delta y = y_{max} - y_0$ gives

$$y_{max} = y_0 + \Delta y = y_0 + 7.35 \text{ m}$$

(a) If the origin is chosen at the top of the building, then $y_0 = 0$ and $y_{max} = 7.35$ m.

Thus, the maximum height above the ground is

$$h_{max} = 50.0 \text{ m} + y_{max} = 50.0 \text{ m} + 7.35 \text{ m} = \boxed{57.4 \text{ m}}$$

The elapsed time from the point of release to the top of the arc is found from $v_y = v_{0y} + a_y t$ as

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - 12.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = \boxed{1.22 \text{ s}}$$

(b) If the origin is chosen at the base of the building (ground level), then $y_0 = +50.0$ m and $h_{max} = y_{max}$, giving

$$h_{max} = y_0 + \Delta y = 50.0 \text{ m} + 7.35 \text{ m} = \boxed{57.4 \text{ m}}$$

The calculation for the time required to reach maximum height is exactly the same as that given above. Thus, $t = \boxed{1.22 \text{ s}}$.

3.47 (a) The known parameters for this jump are: $\theta_0 = -10.0°$, $\Delta x = 108$ m, $\Delta y = -55.0$ m, $a_x = 0$ and $a_y = -g = -9.80$ m/s.

Since $a_x = 0$, the horizontal displacement is $\Delta x = v_{0x}t = (v_0 \cos\theta_0)t$ where t is the total time of the flight. Thus, $t = \Delta x / (v_0 \cos\theta_0)$.

The vertical displacement during the flight is given by

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 = (v_0 \sin\theta_0)t - \frac{gt^2}{2}$$

or

$$\Delta y = (\cancel{v_0}\sin\theta_0)\left(\frac{\Delta x}{\cancel{v_0}\cos\theta_0}\right) - \frac{g}{2}\left(\frac{\Delta x}{v_0\cos\theta_0}\right)^2 = (\Delta x)\tan\theta_0 - \left[\frac{g(\Delta x)^2}{2\cos^2\theta_0}\right]\frac{1}{v_0^2}$$

Thus,

$$\left[\Delta y - (\Delta x)\tan\theta_0\right] = -\left[\frac{g(\Delta x)^2}{2\cos^2\theta_0}\right]\frac{1}{v_0^2}$$

or

$$v_0 = \sqrt{\frac{-g(\Delta x)^2}{2\left[\Delta y - (\Delta x)\tan\theta_0\right]\cos^2\theta_0}} = \sqrt{\frac{-(9.80\ \text{m/s}^2)(108\ \text{m})^2}{2\left[-55.0\ \text{m} - (108\ \text{m})\tan(-10.0°)\right]\cos^2(-10.0°)}}$$

yielding

$$v_0 = \sqrt{\frac{-1.143\times 10^5\ \text{m}^3/\text{s}^2}{-69.75\ \text{m}}} = \boxed{40.5\ \text{m/s}}$$

(b) Rather than falling like a rock, $\boxed{\text{the skier glides through the air}}$ much like a bird, prolonging the jump.

3.48 The cup leaves the counter with initial velocity components of $(v_{0x} = v_i,\ v_{0y} = 0)$ and has acceleration components of $(a_x = 0,\ a_y = -g)$ while in flight.

(a) Applying $\Delta y = v_{0y}t + a_y t^2/2$ from when the cup leaves the counter until it reaches the floor gives

$$-h = 0 + \frac{-g}{2}t^2$$

so the time of the fall is

$$\boxed{t = \sqrt{\frac{2h}{g}}}$$

(b) If the cup travels a horizontal distance d while falling to the floor, $\Delta x = v_{0x}t$ gives

$$v_i = v_{0x} = \frac{\Delta x}{t} = \frac{d}{\sqrt{2h/g}} \qquad \text{or} \qquad \boxed{v_i = d\sqrt{\frac{g}{2h}}}$$

(c) The components of the cup's velocity just before hitting the floor are

$$v_x = v_{0x} = v_i = d\sqrt{\frac{g}{2h}} \qquad \text{and} \qquad v_y = v_{0y} + a_y t = 0 - g\sqrt{\frac{2h}{g}} = -\sqrt{2gh}$$

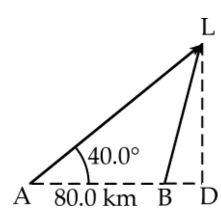
Thus, the total speed at this point is

$$v = \sqrt{v_x^2 + v_y^2} = \boxed{\sqrt{\frac{d^2 g}{2h} + 2gh}}$$

(d) The direction of the cup's motion just before it hits the floor is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-\sqrt{2gh}}{d\sqrt{g/2h}}\right) = \tan^{-1}\left(\frac{-1}{d}\sqrt{2\cancel{g}h\left(\frac{2h}{\cancel{g}}\right)}\right) = \boxed{\tan^{-1}\left(\frac{-2h}{d}\right)}$$

3.49 $\overline{AL} = v_1 t = \quad 90.0\ \text{km/h} \quad 2.50\ \text{h} \quad = 225\ \text{km}$



$\overline{BD} = \overline{AD} - \overline{AB} = \overline{AL}\cos 40.0° - 80.0\ \text{km} = 92.4\ \text{km}$

From the triangle BCD,

$$\overline{BL} = \sqrt{\overline{BD}^{\;2} + \overline{DL}^{\;2}}$$

$$= \sqrt{92.4\ \text{km}^{\;2} + \overline{AL}\sin 40.0°^{\;2}} = 172\ \text{km}$$

Since car 2 travels this distance in 2.50 h, its constant speed is

$$v_2 = \frac{172\ \text{km}}{2.50\ \text{h}} = \boxed{68.6\ \text{km/h}}$$

3.50 After leaving the ledge, the water has a constant horizontal component of velocity.

$$v_x = v_{0x} = 1.50\ \text{m/s}^2$$

Thus, when the speed of the water is $v = 3.00\ \text{m/s}$, the vertical component of its velocity will be

$$v_y = -\sqrt{v^2 - v_x^2} = -\sqrt{(3.00\ \text{m/s})^2 - (1.50\ \text{m/s})^2} = -2.60\ \text{m/s}$$

The vertical displacement of the water at this point is

$$\Delta y = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{(-2.60\ \text{m/s})^2 - 0}{2(-9.80\ \text{m/s}^2)} = -0.344\ \text{m}$$

or the water is $\boxed{0.344\ \text{m}}$ below the ledge.

If its speed leaving the water is 6.26 m/s, the maximum vertical leap of the salmon is

$$\Delta y_{\text{leap}} = \frac{0 - v_{0y}^2}{2a_y} = \frac{0 - (6.26\ \text{m/s})^2}{2(-9.80\ \text{m/s})} = 2.00\ \text{m}$$

Therefore, the maximum height waterfall the salmon can clear is

$$h_{\text{max}} = \Delta y_{\text{leap}} + 0.344\ \text{m} = \boxed{2.34\ \text{m}}$$

3.51 The distance, s, moved in the first 3.00 seconds is given by

$$s = v_0 t + \frac{1}{2}at^2 = (100\ \text{m/s})(3.00\ \text{s}) + \frac{1}{2}(30.0\ \text{m/s}^2)(3.00\ \text{s})^2 = 435\ \text{m}$$

Choosing the origin at the point where the rocket was launched, the coordinates of the rocket at the end of powered flight are:

$$x_1 = s(\cos 53.0°) = 262\ \text{m} \qquad \text{and} \qquad y_1 = s(\sin 53.0°) = 347\ \text{m}$$

The speed of the rocket at the end of powered flight is

$$v_1 = v_0 + at = 100\ \text{m/s} + \left(30.0\ \text{m/s}^2\right)\left(3.00\ \text{s}\right) = 190\ \text{m/s}$$

so the initial velocity components for the free-fall phase of the flight are

$$v_{0x} = v_1 \cos 53.0° = 114\ \text{m/s} \qquad \text{and} \qquad v_{0y} = v_1 \sin 53.0° = 152\ \text{m/s}$$

(a) When the rocket is at maximum altitude, $v_y = 0$. The rise time during the free-fall phase can be found from $v_y = v_{0y} + a_y t$ as

$$t_{\text{rise}} = \frac{0 - v_{0y}}{a_y} = \frac{0 - 152\ \text{m}}{-9.80\ \text{m/s}^2} = 15.5\ \text{s}$$

The vertical displacement occurring during this time is

$$\Delta y = \left(\frac{v_y + v_{0y}}{2}\right) t_{\text{rise}} = \left(\frac{0 + 152\ \text{m/s}}{2}\right)\left(15.5\ \text{s}\right) = 1.17 \times 10^3\ \text{m}$$

The maximum altitude reached is then

$$H = y_1 + \Delta y = 347\ \text{m} + 1.17 \times 10^3\ \text{m} = \boxed{1.52 \times 10^3\ \text{m}}$$

(b) After reaching the top of the arc, the rocket falls 1.52×10^3 to the ground, starting with zero vertical velocity. $(v_{0y} = 0)$ The time for this fall is found from $\Delta y = v_{0y}\, t + \frac{1}{2} a_y t^2$ as

$$t_{\text{fall}} = \sqrt{\frac{2\left(\Delta y\right)}{a_y}} = \sqrt{\frac{2\left(-1.52 \times 10^3\ \text{m}\right)}{-9.80\ \text{m/s}^2}} = 17.6\ \text{s}$$

The total time of flight is

$$t = t_{\text{powered}} + t_{\text{rise}} + t_{\text{fall}} = \left(3.00 + 15.5 + 17.6\right)\ \text{s} = \boxed{36.1\ \text{s}}$$

(c) The free-fall phase of the flight lasts for

$$t_2 = t_{\text{rise}} + t_{\text{fall}} = \left(15.5 + 17.6\right)\ \text{s} = 33.1\ \text{s}$$

The horizontal displacement occurring during this time is

$$\Delta x = v_{0x} t_2 = \left(114 \text{ m/s}\right)\left(33.1 \text{ s}\right) = 3.78 \times 10^3 \text{ m}$$

and the full horizontal range is

$$R = x_1 + \Delta x = 262 \text{ m} + 3.78 \times 10^3 \text{ m} = \boxed{4.05 \times 10^3 \text{ m}}$$

3.52 Taking downstream as the positive direction, the velocity of the water relative to shore is $\vec{\mathbf{v}}_{WS} = +v_{WS}$, where v_{WS} is the speed of the flowing water. Also, if v_{CW} is the common *speed* of the two canoes relative to the water, their velocities relative to the water are

$$\left(\vec{\mathbf{v}}_{CW}\right)_{\text{downstream}} = +v_{CW} \qquad \text{and} \qquad \left(\vec{\mathbf{v}}_{CW}\right)_{\text{upstream}} = -v_{CW}$$

The velocity of a canoe relative to the water can also be expressed as $\vec{\mathbf{v}}_{CW} = \vec{\mathbf{v}}_{CS} - \vec{\mathbf{v}}_{WS}$.

Applying this to the canoe moving downstream gives

$$+v_{CW} = +2.9 \text{ m/s} - v_{WS} \qquad [1]$$

and for the canoe going upstream

$$-v_{CW} = -1.2 \text{ m/s} - v_{WS} \qquad [2]$$

(a) Adding equations [1] and [2] gives

$$2v_{WS} = 1.7 \text{ m/s}, \text{ so } v_{WS} = \boxed{0.85 \text{ m/s}}$$

(b) Subtracting [2] from [1] yields

$$2v_{CW} = 4.1 \text{ m/s}, \text{ or } v_{CW} = \boxed{2.1 \text{ m/s}}$$

3.53 The time of flight is found from $\Delta y = v_{0y}\, t + \frac{1}{2} a_y t^2$ with $\Delta y = 0$, as

$$t = \frac{2v_{0y}}{g}$$

This gives the range as

$$R = v_{0x}t = \frac{2v_{0x}v_{0y}}{g}$$

On Earth this becomes

$$R_{\text{Earth}} = \frac{2v_{0x}v_{0y}}{g_{\text{Earth}}}$$

and on the moon,

$$R_{\text{Moon}} = \frac{2v_{0x}v_{0y}}{g_{\text{Moon}}}$$

Dividing R_{Moon} by R_{Earth}, we find $R_{\text{Moon}} = (g_{\text{Earth}}/g_{\text{Moon}})R_{\text{Earth}}$. With $g_{\text{Moon}} = (1/6)\,g_{\text{Earth}}$, this gives

$R_{\text{Moon}} = 6R_{\text{Earth}} = 6\ 3.0\text{ m} = \boxed{18\text{ m}}$.

Similarly,

$$R_{\text{Mars}} = \left(\frac{g_{\text{Earth}}}{g_{\text{Mars}}}\right)R_{\text{Earth}} = \frac{3.0\text{ m}}{0.38} = \boxed{7.9\text{ m}}.$$

3.54 The time to reach the opposite side is

$$t = \frac{\Delta x}{v_{0x}} = \frac{10\text{ m}}{v_0 \cos 15°}$$

When the motorcycle returns to the original level, the vertical displacement is $\Delta y = 0$. Using this in the relation $\Delta y = v_{0y}\,t + \frac{1}{2}a_y t^2$ gives a second relation between the takeoff speed and the time of flight as

$$0 = v_0 \sin 15°\, t + \frac{1}{2}\ -g\ t^2 \text{ or } v_0 = \left(\frac{g}{2\sin 15°}\right)t$$

Substituting the time found earlier into this result yields

$$v_0 = \left(\frac{g}{2\sin 15°}\right)\left(\frac{10\text{ m}}{v_0\cos 15°}\right) \quad \text{or} \quad v_0 = \sqrt{\frac{9.80\text{ m/s}^2\quad 10\text{ m}}{2\ \sin 15°\ \cos 15°}} = \boxed{14\text{ m/s}}$$

3.55 (a) The time for the ball to reach the fence is

$$t = \frac{\Delta x}{v_{0x}} = \frac{130 \text{ m}}{v_0 \cos 35°} = \frac{159 \text{ m}}{v_0}$$

At this time, the ball must be $\Delta y = 21\text{m} - 1.0 \text{ m} = 20 \text{ m}$ above its launch position, so $\Delta y = v_{0y}\, t + \frac{1}{2} a_y t^2$ gives

$$20 \text{ m} = \left(\cancel{v_0} \sin 35°\right)\left(\frac{159 \text{ m}}{\cancel{v_0}}\right) - \left(4.90 \text{ m/s}^2\right)\left(\frac{159 \text{ m}}{v_0}\right)^2$$

or

$$\left(159 \text{ m}\right) \sin 35° - 20 \text{ m} = \frac{\left(4.90 \text{ m/s}^2\right)\left(159 \text{ m}\right)^2}{v_0^2}$$

From which, $v_0 = \sqrt{\dfrac{\left(4.90 \text{ m/s}^2\right)\left(159 \text{ m}\right)^2}{\left(159 \text{ m}\right) \sin 35° - 20 \text{ m}}} = \boxed{42 \text{ m/s}}$

(b) From above, $t = \dfrac{159 \text{ m}}{v_0} = \dfrac{159 \text{ m}}{42 \text{ m/s}} = \boxed{3.8 \text{ s}}$.

(c) When the ball reaches the wall (at $t = 3.8 \text{ s}$),

$$v_x = v_{0x} = \left(42 \text{ m/s}\right) \cos 35° = \boxed{34 \text{ m/s}}$$

$$v_y = v_{0y} + a_y t = \left(42 \text{ m/s}\right) \sin 35° - \left(9.80 \text{ m/s}^2\right)\left(3.8 \text{ s}\right) = \boxed{-13 \text{ m/s}}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(34 \text{ m/s}\right)^2 + \left(-13 \text{ m/s}\right)^2} = \boxed{37 \text{ m/s}}$$

3.56 We shall first find the initial velocity of the ball thrown vertically upward, recognizing that it takes descend from its maximum height as was required to reach this height. Thus, the elapsed time when it reaches maximum height is t = 1.50 s. Also, at this time, $v_y = 0$, and, $v_y = v_{0y} + a_y t$ gives

$$0 = v_{0y} - \left(9.80 \text{ m/s}^2\right)\left(1.50 \text{ s}\right) \qquad \text{or} \qquad v_{0y} = 14.7 \text{ m/s}$$

In order for the second ball to reach the same vertical height as the first, the second must have the same initial vertical velocity as the first. Thus, we find v_0 as

$$v_0 = \frac{v_{0y}}{\sin 30.0°} = \frac{14.7 \text{ m/s}}{0.500} = \boxed{29.4 \text{ m/s}}$$

3.57 The time of flight of the ball is given by $\Delta y = v_{0y}\, t + \frac{1}{2} a_y t^2$, with $\Delta y = 0$, as

$$0 = \left[\left(20 \text{ m/s}\right)\sin 30°\right] t + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right) t^2$$

or $t = 2.0$ s.

The horizontal distance the football moves in this time is

$$\Delta x = v_{0x} t = \left[\left(20 \text{ m/s}\right)\cos 30°\right]\left(2.0 \text{ s}\right) = 35 \text{ m}$$

Therefore, the receiver must run a distance of $\Delta x = 35 \text{ m} - 20 \text{ m} = 15 \text{ m}$ away from the quarterback, $\boxed{\text{in the direction the ball was thrown}}$ to catch the ball. He has a time of 2.0 s to do this, so the required speed is

$$v = \frac{\Delta x}{t} = \frac{15 \text{ m}}{2.0 \text{ s}} = \boxed{7.5 \text{ m/s}}$$

3.58 The horizontal component of the initial velocity is $v_{0x} = v_0 \cos 40° = 0.766\, v_0$, and the time required for the ball to move 10.0 m horizontally is

$$t = \frac{\Delta x}{v_{0x}} = \frac{10.0 \text{ m}}{0.766\, v_0} = \frac{13.1 \text{ m}}{v_0}$$

At this time, the vertical displacement of the ball must be

$$\Delta y = y - y_0 = 3.05 \text{ m} - 2.00 \text{ m} = 1.05 \text{ m}$$

Thus, $\Delta y = v_{0y}\, t + \frac{1}{2} a_y t^2$ becomes

$$1.05\ \text{m} = \left(\cancel{v_0}\sin 40.0°\right)\frac{13.1\ \text{m}}{\cancel{v_0}} + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)\frac{\left(13.1\ \text{m}\right)^2}{v_0^2}$$

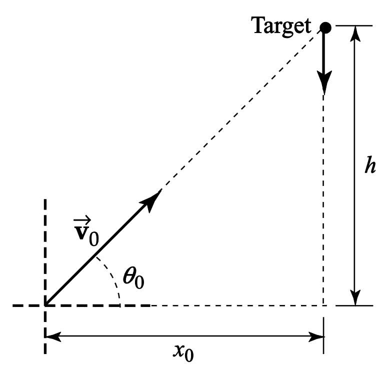
or

$$1.05\ \text{m} = 8.39\ \text{m} - \frac{835\ \text{m}^3/\text{s}^2}{v_0^2}$$

which yields

$$v_0 = \sqrt{\frac{835\ \text{m}^3/\text{s}^2}{8.39\ \text{m} - 1.05\ \text{m}}} = \boxed{10.7\ \text{m/s}}$$

3.59 Choose an origin where the projectile leaves the gun and let the y-coordinates of the projectile and the target at time t be labeled y_p and y_T, respectively.



Then, $(\Delta y)_p = y_p - 0 = (v_0 \sin\theta_0)\, t - \dfrac{g}{2}t^2$, and

$$(\Delta y)_T = y_T - h = 0 - \frac{g}{2}t^2 \text{ or } y_T = h - \frac{g}{2}t^2$$

The time when the projectile will have the same x-coordinate as the target is

$$t = \frac{\Delta x}{v_{0x}} = \frac{x_0}{v_0 \cos\theta_0}$$

For a collision to occur, it is necessary that $y_p = y_T$ at this time, or

$$\left(\cancel{v_0}\sin\theta_0\right)\left(\frac{x_0}{\cancel{v_0}\cos\theta_0}\right) - \cancel{\frac{g}{2}t^2} = h - \cancel{\frac{g}{2}t^2}$$

which reduces to

$$\tan\theta_0 = \frac{h}{x_0}$$

This requirement is satisfied provided that the gun is aimed at the initial location of the target. Thus, $\boxed{\text{a collision is guaranteed if the shooter aims the gun in this manner.}}$

3.60 (a) The components of the vectors are as follows:

Vector	***x*-component (cm)**	***y*-component (cm)**
$\vec{\mathbf{d}}_{1m}$	0	104
$\vec{\mathbf{d}}_{2m}$	46.0	19.5
$\vec{\mathbf{d}}_{1f}$	0	84.0
$\vec{\mathbf{d}}_{2f}$	38.0	20.2

The sums $\vec{\mathbf{d}}_m = \vec{\mathbf{d}}_{1m} + \vec{\mathbf{d}}_{2m}$ and $\vec{\mathbf{d}}_f = \vec{\mathbf{d}}_{1f} + \vec{\mathbf{d}}_{2f}$ are computed as

$$d_m = \sqrt{(0 + 46.0)^2 + (104 + 19.5)^2} = 132 \text{ cm} \text{ and } \theta = \tan^{-1}\left(\frac{104 + 19.5}{0 + 46.0}\right) = 69.6°$$

$$d_f = \sqrt{(0 + 38.0)^2 + (84.0 + 20.2)^2} = 111 \text{ cm and } \theta = \tan^{-1}\left(\frac{84.0 + 20.2}{0 + 38.0}\right) = 70.0°$$

or $\boxed{\vec{\mathbf{d}}_m = 132 \text{ cm at } 69.6° \quad\text{and}\quad \vec{\mathbf{d}}_f = 111 \text{ cm at } 70.0°}$.

(b) To normalize, multiply each component in the above calculation by the appropriate scale factor. The scale factor required for the components of $\vec{\mathbf{d}}_{1m}$ and $\vec{\mathbf{d}}_{2m}$ is

$$s_m = \frac{200 \text{ cm}}{180 \text{ cm}} = 1.11$$

and the scale factor needed for components of $\vec{\mathbf{d}}_{1f}$ and $\vec{\mathbf{d}}_{2f}$ is

$$s_f = \frac{200 \text{ cm}}{168 \text{ cm}} = 1.19$$

After using these scale factors and recomputing the vector sums, the results are

$$\boxed{\vec{\mathbf{d}}'_m = 146 \text{ cm at } 69.6° \quad \text{and} \quad \vec{\mathbf{d}}'_f = 132 \text{ cm at } 70.0°}$$

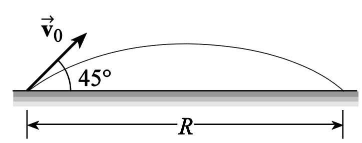
The difference in the normalized vector sums is $\Delta\vec{\mathbf{d}}' = \vec{\mathbf{d}}'_m - \vec{\mathbf{d}}'_f$.

Vector	*x*-component (cm)	*y*-component (cm)
$\vec{\mathbf{d}}'_m$	50.9	137
$-\vec{\mathbf{d}}'_f$	–45.1	–124
$\Delta\vec{\mathbf{d}}'$	$\Sigma x = 5.74$	$\Sigma y = 12.8$

Therefore, $\Delta d' = \sqrt{\left(\Sigma x\right)^2 + \left(\Sigma y\right)^2} = \sqrt{\left(5.74\right)^2 + \left(12.8\right)^2} \text{ cm} = 14.0 \text{ cm}$, and

$$\theta = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right) = \tan^{-1}\left(\frac{12.8}{5.74}\right) = 65.8° \text{ or } \boxed{\Delta\vec{\mathbf{d}}' = 14.0 \text{ cm at } 65.8°}$$

3.60 To achieve maximum range, the projectile should be launched at 45° above the horizontal. In this case, the initial velocity components are



$$v_{0x} = v_{0y} = \frac{v_0}{\sqrt{2}}$$

The time of flight may be found from $v_y = v_{0y} - gt$ by recognizing that when the projectile returns to the original level, $v_y = -v_{0y}$.

Thus, the time of flight is

$$t = \frac{-v_{0y} - v_{0y}}{-g} = \frac{2v_{0y}}{g} = \frac{2}{g}\left(\frac{v_0}{\sqrt{2}}\right) = \frac{v_0\sqrt{2}}{g}$$

The maximum horizontal range is then

$$R = v_{0x}t = \left(\frac{v_0}{\sqrt{2}}\right)\left(\frac{v_0\sqrt{2}}{g}\right) = \frac{v_0^2}{g} \qquad [1]$$

Now, consider throwing the projectile straight upward at speed v_0. At maximum height, $v_y = 0$, and the time required to reach this height is found from $v_y = v_{0y} - gt$ as $0 = v_0 - gt$, which yields $t = v_0/g$.

Therefore, the maximum height the projectile will reach is

$$\Delta y_{\text{max}} = v_{y\ \text{av}}\, t = \left(\frac{0 + v_0}{2}\right)\left(\frac{v_0}{g}\right) = \frac{v_0^2}{2g}$$

Comparing this result with the maximum range found in equation [1] above reveals that $(\Delta y)_{\text{max}} = \boxed{R/2}$ provided the projectile is given the same initial speed in the two tosses.If the girl takes a step when she makes the horizontal throw, she can likely give a higher initial speed for that throw than for the vertical throw.

3.62 (a) $x = v_{0x}t$, so the time may be written as $t = x/v_{0x}$.

Thus, $y = v_{0y}t - \frac{1}{2}gt^2$ becomes

$$y = v_{0y}\left(\frac{x}{v_{0x}}\right) - \frac{1}{2}g\left(\frac{x}{v_{0x}}\right)^2$$

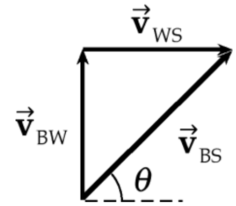
or

$$\boxed{y = \left(-\frac{g}{2v_{0x}^2}\right)x^2 + \left(\frac{v_{0y}}{v_{0x}}\right)x + 0}$$

(b) Note that this result is of the general form $y = ax^2 + bx + c$ with

$$\boxed{a = \left(-\frac{g}{2v_{0x}^2}\right), \quad b = \left(\frac{v_{0y}}{v_{0x}}\right), \quad \text{and} \quad c = 0}$$

3.63 In order to cross the river in minimum time, the velocity of the boat relative to the water ($\vec{\mathbf{v}}_{\text{BW}}$) must be perpendicular to the banks (and hence perpendicular to the velocity $\vec{\mathbf{v}}_{\text{WS}}$ of the water relative to shore).



The velocity of the boat relative to the water is $\vec{\mathbf{v}}_{\text{BW}} = \vec{\mathbf{v}}_{\text{BS}} - \vec{\mathbf{v}}_{\text{WS}}$, where $\vec{\mathbf{v}}_{\text{BS}}$ is the velocity of the boat relative to shore. Note that this vector equation can be rewritten as $\vec{\mathbf{v}}_{\text{BS}} = \vec{\mathbf{v}}_{\text{BW}} + \vec{\mathbf{v}}_{\text{WS}}$. Since $\vec{\mathbf{v}}_{\text{BW}}$ and $\vec{\mathbf{v}}_{\text{WS}}$ are to be perpendicular in this case, the vector diagram for this equation is a right triangle with $\vec{\mathbf{v}}_{\text{BS}}$ as the hypotenuse.

Hence, velocity of the boat relative to shore must have magnitude

$$v_{\text{BS}} = \sqrt{v_{\text{BW}}^2 + v_{\text{WS}}^2} = \sqrt{12\ \text{km/h}^{\,2} + 5.0\ \text{km/h}^{\,2}} = 13\ \text{km/h}$$

and be directed at

$$\theta = \tan^{-1}\left(\frac{v_{\text{BW}}}{v_{\text{WS}}}\right) = \tan^{-1}\left(\frac{12\ \text{km/h}}{5.0\ \text{km/h}}\right) = 67°$$

to the direction of the current in the river (which is the same as the line of the riverbank).

The minimum time to cross the river is

$$t = \frac{\text{width of river}}{v_{\text{BW}}} = \frac{1.5\text{ km}}{12\ \text{km/h}}\left(\frac{60\text{ min}}{1\text{ h}}\right) = \boxed{7.5\text{ min}}$$

During this time, the boat drifts downstream a distance of

$$d = v_{\text{WS}}\, t = \ 5.0\ \text{km/h}\ \ 7.5\text{ min}\ \left(\frac{1\text{ h}}{60\text{ min}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right) = 6.3\times 10^2\text{ m}$$

3.64 For the ball thrown at 45.0°, the time of flight is found from

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 \text{ as } 0 = \left(\frac{v_0}{\sqrt{2}}\right)t_1 - \frac{g}{2}t_1^2$$

which has the single nonzero solution of

$$t_1 = \frac{v_0\sqrt{2}}{g}$$

The horizontal range of this ball is

$$R_1 = v_{0x}\,t_1 = \left(\frac{v_0}{\sqrt{2}}\right)\left(\frac{v_0\sqrt{2}}{g}\right) = \frac{v_0^2}{g}$$

Now consider the first arc in the motion of the second ball, started at angle θ with initial speed v_0.

Applied to this arc, $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ becomes

$$0 = v_0 \sin\theta\; t_{21} - \frac{g}{2}t_{21}^2$$

with nonzero solution

$$t_{21} = \frac{2\,v_0 \sin\theta}{g}$$

Similarly, the time of flight for the second arc (started at angle θ with initial speed $v_0/2$) of this ball's motion is found to be

$$t_{22} = \frac{2\;v_0/2\;\sin\theta}{g} = \frac{v_0\sin\theta}{g}$$

The horizontal displacement of the second ball during the first arc of its motion is

$$R_{21} = v_{0x}\,t_{21} = v_0\cos\theta\left(\frac{2\,v_0\sin\theta}{g}\right) = \frac{v_0^2\;2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin\;2\theta}{g}$$

Similarly, the horizontal displacement during the second arc of this motion is

$$R_{22} = \frac{(v_0/2)^2 \sin(2\theta)}{g} = \frac{1}{4}\frac{v_0^2 \sin(2\theta)}{g}$$

The total horizontal distance traveled in the two arcs is then

$$R_2 = R_{21} + R_{22} = \frac{5}{4}\frac{v_0^2 \sin(2\theta)}{g}$$

(a) Requiring that the two balls cover the same horizontal distance (that is, requiring that $R_2 = R_1$) gives

$$\frac{5}{4}\frac{v_0^2 \sin(2\theta)}{g} = \frac{v_0^2}{g}$$

This reduces to $\sin(2\theta) = \frac{4}{5}$, which yields $2\theta = 53.1°$, so $\theta = \boxed{26.6°}$ is the required projection angle for the second ball.

(b) The total time of flight for the second ball is

$$t_2 = t_{21} + t_{22} = \frac{2v_0 \sin\theta}{g} + \frac{v_0 \sin\theta}{g} = \frac{3v_0 \sin\theta}{g}$$

Therefore, the ratio of the times of flight for the two balls is

$$\frac{t_2}{t_1} = \frac{3v_0 \sin\theta / g}{v_0 \sqrt{2} / g} = \frac{3}{\sqrt{2}}\sin\theta$$

With $\theta = 26.6°$ as found in (a), this becomes

$$\frac{t_2}{t_1} = \frac{3}{\sqrt{2}}\sin(26.6°) = \boxed{0.950}$$

3.65 The initial velocity components for the daredevil are $v_{0x} = v_0 \cos 45°$ and $v_{0y} = v_0 \sin 45°$, or

$$v_{0x} = v_{0y} = \frac{v_0}{\sqrt{2}} = \frac{25.0\ \text{m/s}}{\sqrt{2}}$$

The time required to travel 50.0 m horizontally is

$$t = \frac{\Delta x}{v_{0x}} = \frac{50.0\ \text{m}\ \sqrt{2}}{25.0\ \text{m/s}} = 2\sqrt{2}\ \text{s}$$

The vertical displacement of the daredevil at this time, and the proper height above the level of the cannon to place the net, is

$$\Delta y = v_{0y}\, t + \frac{1}{2} a_y t^2 = \left(\frac{25.0\ \text{m/s}}{\sqrt{2}}\right)\ 2\sqrt{2}\ \text{s}\ - \frac{1}{2}\ 9.80\ \text{m/s}^2\ \ 2\sqrt{2}\ \text{s}^{\ 2} = \boxed{10.8\ \text{m}}$$

3.66 The vertical component of the salmon's velocity as it leaves the water is

$$v_{0y} = +v_0 \sin\theta = +\ 6.26\ \text{m/s}\ \sin 45.0° = +4.43\ \text{m/s}$$

When the salmon returns to water level at the end of the leap, the vertical component of velocity will be $v_y = -v_{0y} = -4.43\ \text{m/s}$.

The time the salmon is out of the water is given by

$$t_1 = \frac{v_y - v_{0y}}{a_y} = \frac{-4.43\ \text{m/s} - 4.43\ \text{m/s}}{-9.80\ \text{m/s}^2} = 0.903\ \text{s}$$

The horizontal distance traveled during the leap is

$$L = v_{0x} t_1 = \ v_0 \cos\theta\ t_1 = \ 6.26\ \text{m/s}\ \cos 45.0°\ 0.903\ \text{s}\ = 4.00\ \text{m}$$

To travel this same distance underwater, at speed $v = 3.58\ \text{m/s}$, requires a time of

$$t_2 = \frac{L}{v} = \frac{4.00\ \text{m}}{3.58\ \text{m/s}} = 1.12\ \text{s}$$

The average horizontal speed for the full porpoising maneuver is then

$$v_{av} = \frac{\Delta x_{total}}{\Delta t_{total}} = \frac{2L}{t_1 + t_2} = \frac{2\ \ 4.00\ \text{m}}{0.903\ \text{s} + 1.12\ \text{s}} = \boxed{3.96\ \text{m/s}}$$

3.67 (a) and (b)

Since the shot leaves the gun horizontally, $v_{0x} = v_0$ and the time required to reach the target is

$$t = \frac{\Delta x}{v_{0x}} = \frac{x}{v_0}$$

The vertical displacement occurring in this time is

$$\Delta y = -y = v_{0y}t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}g\left(\frac{x}{v_0}\right)^2$$

which gives the drop as

$$\boxed{y = \frac{1}{2}g\left(\frac{x}{v_0}\right)^2 = Ax^2 \text{ with } A = \frac{g}{2v_0^2}\text{, where } v_0 \text{ is the muzzle velocity}}$$

(c) If $x = 3.00$ m, and $y = 0.210$ m, then

$$A = \frac{y}{x^2} = \frac{0.210\text{ m}}{3.00\text{ m}^{\;2}} = 2.33 \times 10^{-2}\text{ m}^{-1}$$

and

$$v_0 = \sqrt{\frac{g}{2A}} = \sqrt{\frac{9.80\text{ m/s}^2}{2\;\;2.33 \times 10^{-2}\text{ m}^{-1}}} = \boxed{14.5\text{ m/s}}$$

3.68 The velocity of the wind relative to the boat, $\vec{\mathbf{v}}_{WB}$, is given by $\vec{\mathbf{v}}_{WB} = \vec{\mathbf{v}}_{WE} - \vec{\mathbf{v}}_{BE}$, where $\vec{\mathbf{v}}_{WE}$ and $\vec{\mathbf{v}}_{BE}$ are the velocities of the wind and the boat relative to Earth, respectively. Choosing the positive x-direction as east and positive y as north, these relative velocities have components of

$$\vec{\mathbf{v}}_{WE\;\;x} = +17\text{ knots} \qquad \vec{\mathbf{v}}_{WE\;\;y} = 0\text{ knots}$$

$$\vec{\mathbf{v}}_{BE\;\;x} = 0 \qquad \vec{\mathbf{v}}_{BE\;\;y} = +20\text{ knots}$$

so

$$\vec{\mathbf{v}}_{WB\;\;x} = \vec{\mathbf{v}}_{WE\;\;x} - \vec{\mathbf{v}}_{BE\;\;x} = +17\text{ knots} \qquad \vec{\mathbf{v}}_{WB\;\;y} = \vec{\mathbf{v}}_{WE\;\;y} - \vec{\mathbf{v}}_{BE\;\;y} = -20\text{ knots}$$

Thus,

$$|\vec{\mathbf{v}}_{WB}| = \sqrt{(\vec{\mathbf{v}}_{WB})_x^2 + (\vec{\mathbf{v}}_{WB})_y^2} = \sqrt{(17 \text{ knots})^2 + (-20 \text{ knots})^2} = 26 \text{ knots}$$

and

$$\theta = \tan^{-1}\left(\frac{(\vec{\mathbf{v}}_{WB})_y}{(\vec{\mathbf{v}}_{WB})_x}\right) = \tan^{-1}\left(\frac{-20 \text{ knots}}{17 \text{ knots}}\right) = -50°$$

or

$$\vec{\mathbf{v}}_{WB} = \boxed{26 \text{ knots at } 50° \text{ south of east}}$$

The component of this velocity parallel to the motion of the boat (that is, parallel to a north-south line) is $(\vec{\mathbf{v}}_{WB})_y = -20$ knots, or $\boxed{20 \text{ knots south}}$.

3.69 (a) Take the origin at the point where the ball is launched. Then $x_0 = y_0 = 0$, and the coordinates of the ball at time t later are

$$x = v_{0x}t = (v_0 \cos\theta_0)t \qquad \text{and} \qquad y = v_{0y}t + \frac{1}{2}a_y t^2 = (v_0 \sin\theta_0)t - \left(\frac{g}{2}\right)t^2$$

When the ball lands at $x = R = 240$ m, the y-coordinate is $y = 0$ and the elapsed time is found from

$$0 = (v_0 \sin\theta_0)t - \left(\frac{g}{2}\right)t^2$$

for which the nonzero solution is

$$t = \frac{2v_0 \sin\theta_0}{g}$$

Substituting this time into the equation for the x-coordinate gives

$$x = 240 \text{ m} = (v_0 \cos\theta_0)\left(\frac{2v_0 \sin\theta_0}{g}\right) = \left(\frac{v_0^2}{g}\right)(2\sin\theta_0 \cos\theta_0) = \left(\frac{v_0^2}{g}\right)\sin(2\theta_0)$$

Thus, if $v_0 = 50.0$ m/s, we must have

$$\sin(2\theta_0) = \frac{(240 \text{ m})g}{v_0^2} = \frac{(240 \text{ m})(9.80 \text{ m/s}^2)}{(50.0 \text{ m/s})^2} = +0.941$$

with solutions of $2\theta_0 = 70.2°$ and $2\theta_0 = 180° - 70.2° = 109.8°$.

So, the two possible launch angles are $\boxed{\theta_0 = 35.1° \text{ and } \theta_0 = 54.9°}$.

(b) At maximum height, $v_y = 0$ and the elapsed time is given by

$$t_{\text{peak}} = \frac{v_{y\,\text{peak}} - v_{0y}}{a_y} = \frac{0 - v_0 \sin\theta_0}{-g} \quad \text{or} \quad t_{\text{peak}} = \frac{v_0 \sin\theta_0}{g}$$

The y coordinate of the ball at this time will be

$$y_{\max} = (v_0 \sin\theta_0)\, t_{\text{peak}} - \left(\frac{g}{2}\right) t_{\text{peak}}^2 = (v_0 \sin\theta_0)\left(\frac{v_0 \sin\theta_0}{g}\right) - \left(\frac{g}{2}\right)\frac{v_0^2 \sin^2\theta_0}{g^2} = \frac{v_0^2 \sin^2\theta_0}{2g}$$

The maximum heights corresponding to the two possible launch angles are

$$(y_{\max})_1 = \frac{(50.0 \text{ m/s})^2 \sin^2(35.1°)}{2(9.80 \text{ m/s}^2)} = \boxed{42.2 \text{ m}}$$

and

$$(y_{\max})_2 = \frac{(50.0 \text{ m/s})^2 \sin^2(54.9°)}{2(9.80 \text{ m/s}^2)} = \boxed{85.4 \text{ m}}$$

3.70 (a) Consider the falling water to consist of droplets, each following a projectile trajectory. If the origin is chosen at the level of the pool and directly beneath the end of the channel, the parameters for these projectiles are

$x_0 = 0$ $\qquad y_0 = h = 2.35 \text{ m}$

$v_{0x} = 0.75 \text{ m/s}$ $\qquad v_{0y} = 0$

$a_x = 0$ $\qquad a_y = -g$

The elapsed time when the droplet reaches the pool is found from $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ as

$$0 - h = 0 - \frac{g}{2}t_p^2 \quad \text{or} \quad t_p = \sqrt{\frac{2h}{g}}$$

The distance from the wall where the water lands is then

$$R = x_{\max} = v_{0x}t_p = v_{0x}\sqrt{\frac{2h}{g}} = (0.750\ \mathrm{m/s})\sqrt{\frac{2(2.35\ \mathrm{m})}{9.80\ \mathrm{m/s^2}}} = \boxed{0.519\ \mathrm{m}}$$

This space is too narrow for a pedestrian walkway.

(b) It is desired to build a model in which linear dimensions, such as the height h_{model} and horizontal range of the water R_{model}, are one-twelfth the corresponding dimensions in the actual waterfall. If v_{model} is to be the speed of the water flow in the model, then we would have

$$R_{\text{model}} = v_{\text{model}}(t_p)_{\text{model}} = v_{\text{model}}\sqrt{\frac{2h_{\text{model}}}{g}}$$

or

$$v_{\text{model}} = R_{\text{model}}\sqrt{\frac{g}{2h_{\text{model}}}} = \frac{R_{\text{actual}}}{12}\sqrt{\frac{g}{2(h_{\text{actual}}/12)}} = \frac{1}{\sqrt{12}}\left(R_{\text{actual}}\sqrt{\frac{g}{2h_{\text{actual}}}}\right) = \frac{v_{\text{actual}}}{\sqrt{12}}$$

and the needed speed of flow in the model is

$$v_{\text{model}} = \frac{v_{\text{actual}}}{\sqrt{12}} = \frac{0.750\ \mathrm{m/s}}{\sqrt{12}} = \boxed{0.217\ \mathrm{m/s}}$$

3.71 (a) Applying $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ to the vertical motion of the first snowball gives

$$0 = \left[(25.0\ \mathrm{m/s})\sin 70.0°\right]t_1 + \frac{1}{2}(-9.80\ \mathrm{m/s^2})t_1^2$$

which has the nonzero solution of

$$t_1 = \frac{2(25.0\ \mathrm{m/s})\sin 70.0°}{9.80\ \mathrm{m/s^2}} = 4.79\ \mathrm{s}$$

as the time of flight for this snowball.

The horizontal displacement this snowball achieves is

$$\Delta x = v_{0x}t_1 = \left[(25.0 \text{ m/s}) \cos 70.0°\right](4.79 \text{ s}) = 41.0 \text{ m}$$

Now consider the second snowball, also given an initial speed of $v_0 = 25.0 \text{ m/s}$, thrown at angle θ, and is in the air for time t_2. Applying $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ to its vertical motion yields

$$0 = \left[(25.0 \text{ m/s}) \sin\theta\right] t_2 + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right) t_2^2$$

which has a nonzero solution of

$$t_2 = \frac{2(25.0 \text{ m/s}) \sin\theta}{9.80 \text{ m/s}^2} = (5.10 \text{ s}) \sin\theta$$

We require the horizontal range of this snowball be the same as that of the first ball, namely

$$\Delta x = v_{0x}t_2 = \left[(25.0 \text{ m/s}) \cos\theta\right]\left[(5.10 \text{ s}) \sin\theta\right] = 41.0 \text{ m}$$

This yields the equation

$$\sin\theta\cos\theta = \frac{41.0 \text{ m}}{(25.0 \text{ m/s})(5.10 \text{ s})} = 0.321$$

From the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$, this result becomes

$$\sin 2\theta = 2(0.321) = 0.642$$

so

$$2\theta = 40.0°$$

and the required angle of projection for the second snowball is

$$\theta = \boxed{20.0° \text{ above the horizontal}}$$

(b) From above, the time of flight for the first snowball is $t_1 = 4.79 \text{ s}$ and that for the second snowball is

$$t_2 = (5.10 \text{ s}) \sin\theta = (5.10 \text{ s}) \sin 20.0° = 1.74 \text{ s}$$

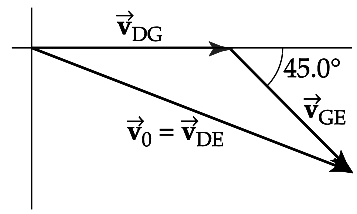
Thus, if they are to arrive simultaneously, the time delay between the first and second snowballs should be

$$\Delta t = t_1 - t_2 = 4.79\text{ s} - 1.74\text{ s} = \boxed{3.05\text{ s}}$$

3.72 First, we determine the velocity with which the dart leaves the gun by using the data collected when the dart was fired horizontally ($v_{0y} = 0$) from a stationary gun. In this case, $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ gives the time of flight as

$$t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2\left(-1.00\text{ m/s}\right)}{-9.80\text{ m/s}^2}} = 0.452\text{ s}$$

Thus, the initial speed of the dart relative to the gun is



$$v_{DG} = v_{0x} = \frac{\Delta x}{t} = \frac{5.00\text{ m}}{0.452\text{ s}} = 11.1\text{ m/s}$$

At the instant when the dart is fired horizontally from a moving gun, the velocity of the dart relative to the gun may be expressed as $\vec{\mathbf{v}}_{DG} = \vec{\mathbf{v}}_{DE} - \vec{\mathbf{v}}_{GE}$ where $\vec{\mathbf{v}}_{DE}$ and $\vec{\mathbf{v}}_{GE}$ are the velocities of the dart and gun relative to Earth respectively. The initial velocity of the dart relative to Earth is therefore

$$\vec{\mathbf{v}}_0 = \vec{\mathbf{v}}_{DE} = \vec{\mathbf{v}}_{DG} + \vec{\mathbf{v}}_{GE}$$

From the vector diagram, observe that

$$v_{0y} = -v_{GE}\sin 45.0° = -\left(2.00\text{ m/s}\right)\sin 45.0° = -1.41\text{ m/s}$$

and

$$v_{0x} = v_{DG} + v_{GE}\cos 45.0° = 11.1\text{ m/s} + \left(2.00\text{ m/s}\right)\cos 45.0° = 12.5\text{ m/s}$$

The vertical velocity of the dart after dropping 1.00 m to the ground is

$$v_y = -\sqrt{v_{0y}^2 + 2a_y\Delta y} = -\sqrt{\left(-1.41\text{ m/s}\right)^2 + 2\left(-9.80\text{ m/s}^2\right)\left(-1.00\text{ m}\right)} = -4.65\text{ m/s}$$

and the time of flight is

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-4.65\text{ m/s} - \left(-1.41\text{ m/s}\right)}{-9.80\text{ m/s}^2} = 0.330\text{ s}$$

The displacement during the flight is $\Delta x = v_{0x}t = (12.5\ \text{m/s})(0.330\ \text{s}) = \boxed{4.12\ \text{m}}$.

3.73 (a) First, use $\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$ to find the time for the coyote to travel 70 m, starting from rest with constant acceleration $a_x = 15\ \text{m/s}^2$:

$$t_1 = \sqrt{\frac{2\Delta x}{a_x}} = \sqrt{\frac{2(70\ \text{m})}{15\ \text{m/s}^2}} = 3.1\ \text{s}$$

The minimum constant speed the roadrunner must have to reach the edge in this time is

$$v = \frac{\Delta x}{t_1} = \frac{70\ \text{m}}{3.1\ \text{s}} = \boxed{23\ \text{m/s}}$$

(b) The initial velocity of the coyote as it goes over the edge of the cliff is horizontal and equal to

$$v_0 = v_{0x} = 0 + a_x t_1 = (15\ \text{m/s}^2)(3.1\ \text{s}) = 46\ \text{m/s}$$

From $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, the time for the coyote to drop 100 m with $v_{0y} = 0$ is

$$t_2 = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-100\ \text{m})}{-9.80\ \text{m/s}^2}} = 4.52\ \text{s}$$

The horizontal displacement of the coyote during his fall is

$$\Delta x = v_{0x}t_2 + \frac{1}{2}a_x t_2^2 = (46\ \text{m/s})(4.52\ \text{s}) + \frac{1}{2}(15\ \text{m/s}^2)(4.52\ \text{s})^2 = \boxed{3.6 \times 10^2\ \text{m}}$$