## ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. At maximum height ( $\Delta y = h_{\text{max}}$ ), the vertical velocity of the stone will be zero. Thus,  $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$  gives

$$h_{\text{max}} = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - v_0^2 \sin^2 \theta}{2 - g} = \frac{-45 \text{ m/s}^2 \sin^2 55.0^\circ}{2 - 9.80 \text{ m/s}^2} = 69.3 \text{ m}$$

and we see that choice (c) is the correct answer.

The skier has zero initial velocity in the vertical direction ( $v_{0y}=0$ ) and undergoes a vertical displacement of  $\Delta y=-3.20$  m. The constant acceleration in the vertical direction is  $a_y=-g$ , so we use  $\Delta y=v_{0y}t+\frac{1}{2}a_yt^2$  to find the time of flight as

$$-3.20 \text{ m} = 0 + \frac{1}{2} -9.80 \text{ m/s}^2$$
  $t^2 \text{ or } t = \sqrt{\frac{2 - 3.20 \text{ m}}{-9.80 \text{ m/s}^2}} = 0.808 \text{ s}$ 

During this time, the object moves with constant horizontal velocity  $v_x = v_{0x} = 22.0$  m/s The horizontal distance traveled during the flight is

$$\Delta x = v_x t = 22.0 \text{ m/s} \quad 0.808 \text{ s} = 17.8 \text{ m}$$

which is choice (d).

Choose coordinate system with north as the positive y-direction and east as the positive x-direction. The velocity of the cruise ship relative to Earth is  $\vec{\mathbf{v}}_{CE} = 4.50$  m/s due north, with components of  $(\vec{\mathbf{v}}_{CE})_x = 0$  and  $(\vec{\mathbf{v}}_{CE})_y = 4.50$  m/s. The velocity of the patrol boat relative to Earth is  $\vec{\mathbf{v}}_{PE} = 5.20$  m/s at  $45.0^\circ$  north of west, with components of

$$\vec{\mathbf{v}}_{PE}_{x} = -|\vec{\mathbf{v}}_{PE}|\cos 45.0^{\circ} = -5.20 \text{ m/s} \quad 0.707 = -3.68 \text{ m/s}$$

and

$$\vec{\mathbf{v}}_{PE}_{v} = + |\vec{\mathbf{v}}_{PE}| \sin 45.0^{\circ} = 5.20 \text{ m/s} \quad 0.707 = 3.68 \text{ m/s}$$

Thus, the velocity of the cruise ship relative to the patrol boat is  $\vec{v}_{CP} = \vec{v}_{CE} - \vec{v}_{PE}$ , which has

components of

$$\vec{\mathbf{v}}_{\mathbf{CP}} = \vec{\mathbf{v}}_{\mathbf{CE}} - \vec{\mathbf{v}}_{\mathbf{PE}} = 0 - -3.68 \text{ m/s} = +3.68 \text{ m/s}$$

and

$$\vec{\mathbf{v}}_{\mathbf{CP}_y} = \vec{\mathbf{v}}_{\mathbf{CE}_y} - \vec{\mathbf{v}}_{\mathbf{PE}_y} = 4.50 \text{ m/s} - 3.68 \text{ m/s} = +0.823 \text{ m/s}$$

Choice (a) is then the correct answer.

4. For vectors in the x-y plane, their components their components have the signs indicated in the following table:

	Quadrant of Vector			
	First	Second	Third	Fourth
x-component	Positive	Negative	Negative	Positive
y-component	Positive	Positive	Negative	Negative

Thus, a vector having components of opposite sign must lie in either the second or fourth quadrants and choice (e) is the correct answer.

5. The path followed (and distance traveled) by the athlete is shown in the sketch, along with the vectors for the initial position, final position, and change in position.

The average speed for the elapsed time interval  $\Delta t$  is

$$v_{\rm av} = \frac{d}{\Delta t}$$

and the magnitude of the average velocity for this time interval is

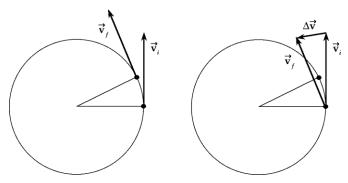
$$\left|\vec{\mathbf{v}}_{\mathrm{av}}\right| = \frac{\left|\Delta\vec{\mathbf{r}}\right|}{\Delta t}$$

The sketch clearly shows that  $d > \left| \vec{\Delta r} \right|$  in this case, meaning that  $v_{\rm av} > \left| \vec{\bf v}_{\rm av} \right|$  and that (a) is the correct choice.

 $\vec{\mathbf{r}}_i$ 

6. Consider any two very closely spaced points on a circular path and draw vectors of the same length (to

represent a constant velocity magnitude or speed) tangent to the path at each of these points as shown in the leftmost diagram below. Now carefully move the velocity vector  $\vec{\mathbf{v}}_f$  at the second point down so its tail is at the first point as shown in the rightmost diagram. Then, draw the vector difference  $\Delta \vec{\mathbf{v}} = \vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i$  and observe that if the start of this vector were located on the circular path midway between the two points, its direction would be inward toward the center of the circle.



Thus, for an object following the circular path at constant speed, its instantaneous acceleration,

 $\vec{\mathbf{a}} = \lim_{\Delta t \to 0} \Delta \vec{\mathbf{v}} / \Delta t$ , at the point midway between your initial and end points is directed toward the center of the circle, and the only correct choice for this question is (d).

Whether on Earth or the Moon, a golf ball is in free fall with a constant downward acceleration of magnitude determined by local gravity from the time it leaves the tee until it strikes the ground or other object. Thus, the vertical component of velocity is constantly changing while the horizontal component of velocity is constant. Note that the speed (or magnitude of the velocity)

$$v = \left| \vec{\mathbf{v}} \right| = \sqrt{v_x^2 + v_y^2}$$

will change in time since  $v_y$  changes in time. Thus, the only correct choices for this question are (b) and (d).

8. At maximum altitude, the projectile's vertical component of velocity is zero. The time for the projectile to reach its maximum height is found from  $v_y = v_{0y} + a_y t$  as

$$t_{\text{max}} = \frac{v_y \Big|_{\Delta y = h_{\text{max}}} - v_{0y}}{a_y} = \frac{0 - v_0 \sin \theta_0}{-g} = \frac{v_0 \sin \theta_0}{g}$$

Since the acceleration of gravity on the Moon is one-sixth that of Earth, we see that (with  $v_0$  and  $\theta_0$  kept constant)

$$t_{\text{max}}\big|_{\text{Moon}} = \frac{v_0 \sin \theta_0}{g_{\text{Moon}}} = \frac{v_0 \sin \theta_0}{g_{\text{Earth}}/6} = 6\bigg(\frac{v_0 \sin \theta_0}{g_{\text{Earth}}}\bigg) = 6 t_{\text{max}}\big|_{\text{Earth}}$$

and the correct answer for this question is (e).

The boat moves with a constant horizontal velocity (or its velocity relative to Earth has components of  $(\vec{\mathbf{v}}_{\mathrm{BE}})_x = v_0 = \mathrm{constant}$ , and  $(\vec{\mathbf{v}}_{\mathrm{BE}})_y = 0$ ), where the y-axis is vertical and the x-axis is parallel to the keel of the boat. Once the wrench is released, it is a projectile whose velocity relative to Earth has components of

$$\vec{\mathbf{v}}_{\text{WE}_x} = v_{0x} + a_x t = v_0 + 0 = v_0$$
 and  $\vec{\mathbf{v}}_{\text{WE}_y} = v_{0y} + a_y t = 0 - gt = -gt$ 

The velocity of the wrench relative to the boat  $(\vec{v}_{WB} = \vec{v}_{WE} - \vec{v}_{BE})$  has components of

$$\vec{\mathbf{v}}_{\mathrm{WB}}_{x} = \vec{\mathbf{v}}_{\mathrm{WE}}_{x} - \vec{\mathbf{v}}_{\mathrm{BE}}_{x} = v_{0} - v_{0} = 0$$
 and  $\vec{\mathbf{v}}_{\mathrm{WB}}_{y} = \vec{\mathbf{v}}_{\mathrm{WE}}_{y} - \vec{\mathbf{v}}_{\mathrm{BE}}_{y} = -gt - 0 = -gt$ 

Thus, the wrench has zero horizontal velocity relative to the boat and will land on the deck at a point directly below where it was released (i.e., at the base of the mast). The correct choice is (b).

- While in the air, the baseball is a projectile whose velocity always has a constant horizontal component  $(v_x = v_{0x})$  and a vertical component that changes at a constant rate  $(\Delta v_y/\Delta t = a_y = -g)$ . At the highest point on the path, the vertical velocity of the ball is momentarily zero. Thus, at this point, the resultant velocity of the ball is horizontal and its acceleration continues to be directed downward  $(a_x = 0, a_y = -g)$ . The only correct choice given for this question is (c).
- Note that for each ball,  $v_{0y}=0$ , Thus, the vertical velocity of each ball when it reaches the ground  $(\Delta y=-h) \text{ is given by } v_y^2=v_{0y}^2+2a_y(\Delta y) \text{ as}$

$$v_v = -\sqrt{0 + 2 - g - h} = -\sqrt{2gh}$$

and the time required for each ball to reach the ground is given by  $v_y = v_{0y} + a_y t$  as

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-\sqrt{2gh} - 0}{-g} = \sqrt{\frac{2h}{g}}$$

The speeds (i.e., magnitudes of total velocities) of the balls at ground level are

Red Ball: 
$$v_R = \sqrt{v_x^2 + v_y^2} = \sqrt{v_{0x}^2 + -\sqrt{2gh}^2} = \sqrt{v_0^2 + 2gh}$$

Blue Ball: 
$$v_B = \sqrt{v_x^2 + v_y^2} = \sqrt{v_{0x}^2 + -\sqrt{2gh}^2} = \sqrt{0 + 2gh} = \sqrt{2gh}$$

Therefore, we see that the two balls reach the ground at the same time but with different speeds  $(v_R > v_B)$ , so only choice (b) is correct.

- When the apple first comes off the tree, it is moving forward with the same horizontal velocity as the truck. Since, while in free fall, the apple has zero horizontal acceleration, it will maintain this constant horizontal velocity as it falls. Also, while in free fall, the apple has constant downward acceleration  $(a_y = -g)$ , so its downward speed increases uniformly in time.
  - (i) As the truck moves left to right past an observer stationary on the ground, this observer will see both the constant velocity horizontal motion and the uniformly accelerated downward motion of the apple. The curve that best describes the path of the apple as seen by this observer is (a).
  - (ii) An observer on the truck moves with the same horizontal motion as does the apple. This observer does not detect any horizontal motion of the apple relative to him. However, this observer does detect the uniformly accelerated vertical motion of the apple. The curve best describing the path of the apple as seen by the observer on the truck is (b).
- Of the choices listed, the quantities that have magnitude or size, but no direction, associated with them (i.e., scalar quantities) are (b) temperature, (c) volume, and (e) height. The other quantities, (a) velocity of a sports car and (d) displacement of a tennis player who moves from the court's backline to the net, have both magnitude and direction associated with them, and are both vector quantities.