# PROBLEM SOLUTIONS

2.1 We assume that you are approximately 2 m tall and that the nerve impulse travels at uniform speed. The elapsed time is then

$$\Delta t = \frac{\Delta x}{v} = \frac{2 \text{ m}}{100 \text{ m/s}} = 2 \times 10^{-2} \text{ s} = 0.02 \text{ s}$$

**2.2** At constant speed,  $c = 3 \times 10^8$  m/s the distance light travels in 0.1 s is

$$\Delta x = c \ \Delta t = 3 \times 10^8 \text{ m/s} \ 0.1 \text{ s} = \left( \times 10^7 \text{ m} \right) \left( \frac{1 \text{ mi}}{1.609 \text{ km}} \right) \left( \frac{1 \text{ km}}{10^3 \text{ m}} \right) = \boxed{2 \times 10^4 \text{ mi}}$$

Comparing this to the diameter of the Earth,  $D_E$ , we find

$$\frac{\Delta x}{D_E} = \frac{\Delta x}{2R_E} = \frac{3.0 \times 10^7 \text{ pm}}{2 (6.38 \times 10^6 \text{ pm})} \approx \boxed{2.4} \text{ (with } R_E = \text{Earth's radius)}$$

## 2.3 Distances traveled between pairs of cities are

$$\Delta x_1 = v_1 \ \Delta t_1 = 80.0 \ \text{km/h} \ 0.500 \ \text{h} = 40.0 \ \text{km}$$

 $\Delta x_2 = v_2 \ \Delta t_2 = 100 \text{ km/h} \ 0.200 \text{ h} = 20.0 \text{ km}$ 

$$\Delta x_3 = v_3 \ \Delta t_3 = 40.0 \ \text{km/h} \ 0.750 \ \text{h} = 30.0 \ \text{km/h}$$

Thus, the total distance traveled  $\Delta x = (40.0 + 20.0 + 30.0) \text{ km} = 90.0 \text{ km}$ , and the elapsed time is  $\Delta t = 0.500 \text{ h} + 0.200 + 0.750 \text{ h} + 0.250 \text{ h} = 1.70 \text{ h}$ ).

(a) 
$$v = \frac{\Delta x}{\Delta t} = \frac{90.0 \text{ km}}{1.70 \text{ h}} = 52.9 \text{ km/h}$$

(b) 
$$\Delta x = 90.0 \text{ km}$$
 (see above)

2.4 (a) 
$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{20 \text{ ft}}{1 \text{ yf}} \left( \frac{1 \text{ m}}{3.281 \text{ ft}} \right) \left( \frac{1 \text{ yf}}{3.156 \times 10^7 \text{ s}} \right) = \boxed{2 \times 10^{-7} \text{ m/s}}$$

or in particularly windy times,

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{100 \text{ ft}}{1 \text{ yf}} \left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) \left(\frac{1 \text{ yf}}{3.156 \times 10^7 \text{ s}}\right) = \boxed{1 \times 10^{-6} \text{ m/s}}$$

(b) The time required must have been

$$\Delta t = \frac{\Delta x}{\overline{v}} = \frac{3 \times 10^3 \text{ mi}}{10 \text{ mm}/\text{yr}} \left(\frac{1609 \text{ m}}{1 \text{ mi}}\right) \left(\frac{10^3 \text{ mm}}{1 \text{ m}}\right) = \boxed{5 \times 10^8 \text{ yr}}$$

2.5

(a) Boat A requires 1.0 h to cross the lake and 1.0 h to return, total time 2.0 h. Boat B requires 2.0 h to cross the lake at which time the race is over.

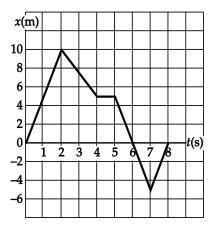
Boat A wins, being 60 km ahead of B when the race ends

- (b) Average velocity is the net displacement of the boat divided by the total elapsed time. The winning boat is back where it started, its displacement thus being zero, yielding an average velocity of zero.
- 2.6 The average velocity over any time interval is

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

(a) 
$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{10.0 \text{ m} - 0}{2.00 \text{ s} - 0} = 5.00 \text{ m/s}$$

- (b)  $\overline{v} = \frac{\Delta x}{\Delta t} = \frac{5.00 \text{ m} 0}{4.00 \text{ s} 0} = \boxed{1.25 \text{ m/s}}$
- (c)  $\overline{v} = \frac{\Delta x}{\Delta t} = \frac{5.00 \text{ m} 10.0 \text{ m}}{4.00 \text{ s} 2.00 \text{ s}} = \boxed{-2.50 \text{ m/s}}$



(d) 
$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{-5.00 \text{ m} - 5.00 \text{ m}}{7.00 \text{ s} - 4.00 \text{ s}} = \boxed{-3.33 \text{ m/s}}$$

(e) 
$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{0 - 0}{8.00 \text{ s} - 0} = \boxed{0}$$

2.7 (a) Displacement = 
$$\Delta x = 85.0 \text{ km/h} 35.0 \min \left(\frac{1 \text{ h}}{60.0 \min}\right) + 130 \text{ km} = 180 \text{ km}$$

(b) The total elapsed time is

$$\Delta t = 35.0 \min + 15.0 \min \left(\frac{1 \text{ h}}{60.0 \min}\right) + 2.00 \text{ h} = 2.84 \text{ h}$$

so,

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{180 \text{ km}}{2.84 \text{ h}} = \boxed{63.4 \text{ km/h}}$$

**2.8** The average velocity over any time interval is

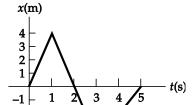
$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

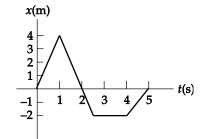
(a) 
$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{4.0 \text{ m} - 0}{1.0 \text{ s} - 0} = \boxed{+4.0 \text{ m/s}}$$

(b) 
$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{-2.0 \text{ m} - 0}{4.0 \text{ s} - 0} = \boxed{-0.50 \text{ m/s}}$$

(c) 
$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{0 - 4.0 \text{ m}}{5.0 \text{ s} - 1.0 \text{ s}} = \boxed{-1.0 \text{ m/s}}$$

(d) 
$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{0 - 0}{5.0 \text{ s} - 0} = \boxed{0}$$





time. We compute this slope by using two points on a straight segment of the curve, one point on each side of the point of interest.

(a) 
$$v|_{0.50 \text{ s}} = \frac{x|_{1.0 \text{ s}} - x|_{t=0}}{1.0 \text{ s} - 0} = \frac{4.0 \text{ m}}{1.0 \text{ s}} = 4.0 \text{ m/s}$$

(b) 
$$v|_{2.0 \text{ s}} = \frac{x|_{2.5 \text{ s}} - x|_{1.0 \text{ s}}}{2.5 \text{ s} - 1.0 \text{ s}} = \frac{-6.0 \text{ m}}{1.5 \text{ s}} = \boxed{-4.0 \text{ m/s}}$$

(c) 
$$v|_{3.0 \text{ s}} = \frac{x|_{4.0 \text{ s}} - x|_{2.5 \text{ s}}}{4.0 \text{ s} - 2.5 \text{ s}} = \frac{0}{1.5 \text{ s}} = \boxed{0}$$

(d) 
$$v|_{4.5 \text{ s}} = \frac{x|_{5.0 \text{ s}} - x|_{4.0 \text{ s}}}{5.0 \text{ s} - 4.0 \text{ s}} = \frac{+2.0 \text{ m}}{1.0 \text{ s}} = \boxed{2.0 \text{ m/s}}$$

2.10 (a) The time for a car to make the trip is  $t = \frac{\Delta x}{v}$ . Thus, the difference in the times for the

two cars to complete the same 10 mile trip is

$$\Delta t = t_1 - t_2 = \frac{\Delta x}{v_1} - \frac{\Delta x}{v_2} = \left(\frac{10 \text{ mi}}{55 \text{ mi/h}} - \frac{10 \text{ mi}}{70 \text{ mi/h}}\right) \left(\frac{60 \text{ min}}{1 \text{ h}}\right) = \boxed{2.3 \text{ min}}$$

(b) When the faster car has a 15.0 min lead, it is ahead by a distance equal to that traveled by the slower car in a time of 15.0 min. This distance is given by  $\Delta x_1 = v_1 (\Delta t) = (55 \text{ mi/h}) (15 \text{ min}).$ 

The faster car pulls ahead of the slower car at a rate of

 $v_{\text{relative}} = 70 \text{ mi/h} - 55 \text{ mi/h} = 15 \text{ mi/h}$ 

Thus, the time required for it to get distance  $\Delta x_1$  ahead is

$$\Delta t = \frac{\Delta x_1}{v_{\text{relative}}} = \frac{55 \text{ mi/h} 15 \text{ min}}{15.0 \text{ mi/h}} = 55 \text{ min}$$

Finally, the distance the faster car has traveled during this time is

$$\Delta x_2 = v_2 \quad \Delta t = 70 \text{ mi/h} \quad 55 \text{ min} \left(\frac{1 \text{ h}}{60 \text{ min}}\right) = \boxed{64 \text{ mi}}$$

## 2.11 The distance traveled by the space shuttle in one orbit is

Circumference of Orbit = 
$$2\pi r = 2\pi$$
 Earth's radius + 200 miles  
=  $2\pi$  3963 + 200 mi =  $2.61 \times 10^4$  mi

Thus, the required time is

$$t = \frac{Circumference}{average speed} = \frac{2.61 \times 10^4 \text{ mi}}{19\ 800 \text{ mi/h}} = \boxed{1.32 \text{ h}}$$

**2.12** (a) 
$$\overline{v}_1 = \frac{\Delta x_1}{\Delta t_1} = \frac{+L}{t_1} = \frac{+L/t_1}{-L/t_1}$$

(b) 
$$\overline{v}_2 = \frac{\Delta x_2}{\Delta t_2} = \frac{-L}{t_2} = \frac{-L/t_2}{-L/t_2}$$

(c) 
$$\overline{v}_{\text{total}} = \frac{\Delta x_{\text{total}}}{\Delta t_{\text{total}}} = \frac{\Delta x_1 + \Delta x_2}{t_1 + t_2} = \frac{+L - L}{t_1 + t_2} = \frac{0}{t_1 + t_2} = \boxed{0}$$

(d) ave. speed trip = 
$$\frac{\text{total distance traveled}}{\Delta t_{\text{total}}} = \frac{\left| \Delta x_1 \right| + \left| \Delta x_2 \right|}{t_1 + t_2} = \frac{\left| +L \right| + \left| -L \right|}{t_1 + t_2} = \frac{2L}{t_1 + t_2}$$

**2.13** The total time for the trip is  $t = t_1 + 22.0$  min =  $t_1 + 0.367$  h, where  $t_1$  is the time spent

traveling at  $v_1 = 89.5$  km/h. Thus, the distance traveled is  $\Delta x = v_1 t_1 = \overline{v} t$ , which gives  $\Delta x = 89.5$  km/h  $t_1 = 77.8$  km/h  $t_1 + 0.367$  h = 77.8 km/h  $t_1 + 28.5$  km

or

89.5 km/h – 77.8 km/h 
$$t_1 = 28.5$$
 km

From which,  $t_1 = 2.44$  for a total time of.  $t = t_1 + 0.367$  h = 2.80h

Therefore,  $\Delta x = \bar{v}t = 77.8 \text{ km/h} 2.80 \text{ h} = 218 \text{ km}$ .

2.14 (a) At the end of the race, the tortoise has been moving for time *t* and the hare for a time t - 2.0 min = t - 120 s.The speed of the tortoise is  $v_t = 0.100 \text{ m/s}$ , and the speed of the hare is  $v_h = 20 v_t = 2.0 \text{ m/s}$ . The tortoise travels distance  $x_t$ , which is 0.20 m larger than the distance  $x_h$  traveled by the hare. Hence,  $x_t = x_h + 0.20 \text{ m}$ 

which becomes

$$v_t t = v_h t - 120 s + 0.20 m$$

or

$$0.100 \text{ m/s} t = 2.0 \text{ m/s} t - 120 \text{ s} + 0.20 \text{ m}$$

This gives the time of the race as  $t = 1.3 \times 10^2$  s

(b) 
$$x_t = v_t t = 0.100 \text{ m/s} \quad 1.3 \times 10^2 \text{ s} = 13 \text{ m}$$

2.15 The maximum allowed time to complete the trip is

$$t_{\text{total}} = \frac{\text{total distance}}{\text{required average speed}} = \frac{1\,600 \text{ m}}{250 \text{ km/h}} \left(\frac{1 \text{ km/h}}{0.278 \text{ m/s}}\right) = 23.0 \text{ s}$$

The time spent in the first half of the trip is

$$t_1 = \frac{half \ distance}{\overline{v}_1} = \frac{800 \ \text{m}}{230 \ \text{km/h}} \left(\frac{1 \ \text{km/h}}{0.278 \ \text{m/s}}\right) = 12.5 \ \text{s}$$

Thus, the maximum time that can be spent on the second half of the trip is

$$t_2 = t_{total} - t_1 = 2.30 s - 12.5 s = 10.5 s$$

and the required average speed on the second half is

$$\overline{v}_2 = \frac{half \, distance}{t_2} = \frac{800 \, \text{m}}{10.5 \, \text{s}} = 76.2 \, \text{m/s} \left(\frac{1 \, \text{km/h}}{0.278 \, \text{m/s}}\right) = 274 \, \text{km/h}$$

2.16 (a) In order for the trailing athlete to be able to catch the leader, his speed  $(v_1)$  must be greater than that of the leading athlete  $(v_2)$ , and the distance between the leading athlete and the finish line must be great enough to give the trailing athlete sufficient time to make up the deficient distance, *d*.

(b) During a time t the leading athlete will travel a distance  $d_2 = v_2 t$  and the trailing athlete will travel a distance

 $d_1 = v_1 t$ . Only when  $d_1 = d_2 + d$  (where *d* is the initial distance the trailing athlete was behind the leader) will the trailing athlete have caught the leader. Requiring that this condition be satisfied gives the elapsed time required for the second athlete to overtake the first:

$$d_1 = d_2 + d$$
 or  $v_1 t = v_2 t + d$ 

giving

$$v_1 t - v_2 t = d$$
 or  $t = \boxed{\frac{d}{v_1 - v_2}}$ 

(c) In order for the trailing athlete to be able to at least tie for first place, the initial distance D between the leader and the finish line must be greater than or equal to the distance the leader can travel in the time t calculated above (i.e., the time required to overtake the leader). That is, we must require that

$$D \ge d_2 = v_2 t = v_2 \left[ \frac{d}{v_1 - v_2} \right]$$
 or  $D \ge \frac{v_2 d}{v_1 - v_2}$ 

2.17 The instantaneous velocity at any time is the slope of the x vs. t graph at that time. We compute this slope by using two points on a straight segment of the curve, one point on each side of the point of interest.

(a) 
$$v_{t=1.00 \text{ s}} = \frac{10.0 \text{ m} - 0}{2.00 \text{ s} - 0} = 5.00 \text{ m/s}$$

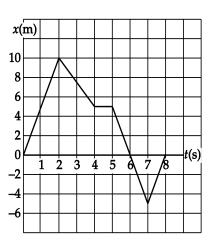
(b) 
$$v_{t=3.00 \text{ s}} = \frac{5.00 - 10.0 \text{ m}}{4.00 - 2.00 \text{ s}} = \boxed{-2.50 \text{ m/s}}$$

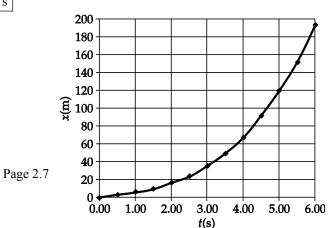
(c) 
$$v_{t=4.50 \text{ s}} = \frac{5.00 - 5.00 \text{ m}}{5.00 - 4.00 \text{ s}} = \boxed{0}$$

(d) 
$$v_{t=7.50 \text{ s}} = \frac{0 - -5.00 \text{ m}}{8.00 - 7.00 \text{ s}} = 5.00 \text{ m/s}$$

**2.18** (a) A few typical values are

t(s) = x(m)





1.00	5.75
2.0	16.0
3.00	35.3
4.00	68.0
5.00	119
6.00	192

(b) We will use a 0.400 s interval centered at t = 4.00 s. We find at t = 3.80 s, x = 60.2 m and at t = 4.20 s, x = 76.6. Therefore,

$$v = \frac{\Delta x}{\Delta t} = \frac{16.4 \text{ m}}{0.400 \text{ s}} = 41.0 \text{ m/s}$$

Using a time interval of 0.200 s, we find the corresponding values to be: at t = 3.90 s, x = 64.0 m and at t = 4.10 s, x = 72.2 m. Thus,

$$v = \frac{\Delta x}{\Delta t} = \frac{8.20 \text{ m}}{0.200 \text{ s}} = 41.0 \text{ m/s}$$

For a time interval of 0.100 s, the values are: at t = 3.95 s, x = 66.0 m, and at t = 4.05 s, x = 70.1 m Therefore,

$$v = \frac{\Delta x}{\Delta t} = \frac{4.10 \text{ m}}{0.100 \text{ s}} = 41.0 \text{ m/s}$$

(c) At t = 4.00 s, x = 68.0 m. Thus, for the first 4.00 s,

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{68.0 \text{ m} - 0}{4.00 \text{ s} - 0} = \boxed{17.0 \text{ m/s}}$$

This value is much less than the instantaneous velocity at t = 4.00 s.

2.19 Choose a coordinate axis with the origin at the flagpole and east as the positive direction. Then, using  $x = x_0 + v_0 t + \frac{1}{2} at^2$  with a = 0 for each runner, the x-coordinate of each runner at time t is

 $x_A = -4.0 \text{ mi} + 6.0 \text{ mi/h} t$  and  $x_B = 3.0 \text{ mi} + -5.0 \text{ mi/h} t$ 

When the runners meet,  $x_A = x_B$ , giving -4.0 mi +(6.0 mi/h) t = 3.0 mi + (-5.0 mi/h)t, Or(6.0 mi/h + 5.0 mi/h)t = 3.0 mi + 4.0 mi. This gives the elapsed time when they meet as

$$t = \frac{7.0 \text{ mi}}{11.0 \text{ mi/h}} = 0.64 \text{ h}$$

At this time,  $x_A = x_B = -0.18$  mi. Thus, they meet 0.18 mi west of the flagpole.

2.20 (a) Using  $v = v_0 + at$  with an initial velocity of  $v_0 = 13.0$  m/s and a constant acceleration of a = -4.00 m/s<sup>2</sup>, the velocity after an elapsed time of t = 1.00 s is

$$v = v_0 + at = 13.0 \text{ m/s} + -4.00 \text{ m/s}^2 \quad 1.00 \text{ s} = 9.00 \text{ m/s}$$

- (b) At an elapsed time of t = 2.00 s, v = 13.0 m/s + -4.00 m/s<sup>2</sup> 2.00 s = 5.00 m/s
- (c) When t = 2.50 s, v = 13.0 m/s + -4.00 m/s<sup>2</sup> 2.50 s = 3.00 m/s
- (d) At t = 4.00 s, v = 13.0 m/s + -4.00 m/s<sup>2</sup> 4.00 s = -3.00 m/s.
- (e) The graph of velocity versus time for this canister is a straight line passing through 13.0 m/s at t = 0 and sloping downward, decreasing by 4.00 m/s for each second thereafter.
- (f) If the canister's velocity at time t = 0 and the value of its (constant) acceleration are known, one can predict the velocity of the canister at any later time.
- 2.21 The average speed during a time interval is

$$\overline{v} = \frac{distance traveled}{\Delta t}$$

During any quarter mile segment, the distance traveled is

$$\Delta x = \frac{1 \text{ mi}}{4} \left( \frac{5 280 \text{ ft}}{1 \text{ mi}} \right) = 1 320 \text{ ft}$$

(a) During the first quarter mile segment, Secretariat's average speed was

$$\overline{v}_1 = \frac{1\ 320\ \text{ft}}{25.2\ \text{s}} = 52.4\ \text{ft/s}$$

During the second quarter mile segment,

$$\overline{v}_2 = \frac{1\ 320\ \text{ft}}{24.0\ \text{s}} = 55.0\ \text{ft/s}$$

For the third quarter mile of the race,

$$\overline{v}_3 = \frac{1\ 320\ \text{ft}}{23.8\ \text{s}} = 55.5\ \text{ft/s}$$

For the fourth final quarter mile,

$$\overline{v}_4 = \frac{1\ 320\ \text{ft}}{23.2\ \text{s}} = 56.9\ \text{ft/s}$$

and during the final quarter mile,

$$\overline{v}_5 = \frac{1\ 320\ \text{ft}}{23.0\ \text{s}} = 57.4\ \text{ft/s}$$

(b) Assuming that  $v_{\text{final}} = \overline{v}_5$  and recognizing that  $v_0 = 0$ , the average acceleration for the entire race was

$$\overline{a} = \frac{v_{\text{final}} - v_0}{\text{total elapsed time}} = \frac{57.4 \text{ ft/s} - 0}{25.2 + 24.0 + 23.8 + 23.2 + 23.0 \text{ s}} = \boxed{0.481 \text{ ft/s}^2}$$

**2.22** From  $a\mathcal{U} = \Delta \mathcal{U}/\Delta t$ , the required time is seen to be

$$\Delta t = \frac{\Delta v}{a} = \left(\frac{60.0 \text{ mi/h} - 0}{7g}\right) \left(\frac{1g}{9.80 \text{ m/s}^2}\right) \left(\frac{0.447 \text{ m/s}}{1 \text{ mi/h}}\right) = \boxed{0.391 \text{ s}}$$

2.23 From 
$$a = \Delta v / \Delta t$$
, we have  $\Delta t = \frac{\Delta v}{a} = \frac{-60 - 55 \text{ mi/h}}{0.60 \text{ m/s}^2} \left( \frac{0.447 \text{ m/s}}{1 \text{ mi/h}} \right) = 3.7 \text{ s}.$ 

2.24 (a) From 
$$t = 0$$
 to  $t = 5.0$  s,  
Page 2.10
$$r(m/s)$$

$$\frac{v(m/s)}{4 - 2} - \frac{1}{2} - \frac{1}{2}$$

$$\overline{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{-8.0 \text{ m/s} - -8.0 \text{ m/s}}{5.0 \text{ s} - 0} = \boxed{0}$$

From to t = 15 s,

$$\overline{a} = \frac{8.0 \text{ m/s} - -8.0 \text{ m/s}}{15 \text{ s} - 5.0 \text{ s}} = 1.6 \text{ m/s}^2$$

and from t = 0 to t = 20 s,

$$\overline{a} = \frac{8.0 \text{ m/s} - -8.0 \text{ m/s}}{20 \text{ s} - 0} = \boxed{0.80 \text{ m/s}^2}$$

(b) At any instant, the instantaneous acceleration equals the slope of the line tangent to the v vs. t graph at that point in time. At t = 2.0 s, the slope of the tangent line to the curve is 0. At t = 10 s, the slope of the tangent line is  $1.6 \text{ m/s}^2$ , and at t = 18 s, the slope of the tangent line is 0.

2.25 (a) 
$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{175 \text{ mi/h} - 0}{2.5 \text{ s}} = \boxed{70.0 \text{ mi/h} \cdot \text{s}}$$

or

$$\overline{a} = \left(70.0 \ \frac{\text{mi}}{\text{M} \cdot \text{s}}\right) \left(\frac{1609 \text{ m}}{1 \text{ mi}}\right) \left(\frac{1 \text{ M}}{3 600 \text{ s}}\right) = \boxed{31.3 \text{ m/s}^2}$$

Alternatively,

$$\overline{a} = \left(31.3 \ \frac{\mathrm{m}}{\mathrm{s}^2}\right) \left(\frac{1 \ \mathrm{g}}{9.80 \ \mathrm{m/s^2}}\right) = \boxed{3.19 \ \mathrm{g}}$$

(b) If the acceleration is constant,  $\Delta x = v_0 t + \frac{1}{2} a t^2$ :

$$\Delta x = 0 + \frac{1}{2} \left( 31.3 \ \frac{\text{m}}{\text{s}^2} \right) \ 2.50 \ \text{s}^2 = \boxed{97.8 \ \text{m}}$$

or

$$\Delta x = 97.8 \text{ m} \left(\frac{3.281 \text{ ft}}{1 \text{ m}}\right) = \boxed{321 \text{ ft}}$$

2.26 We choose eastward as the positive direction so the initial velocity of the car is given by  $v_0 = +25.0 \text{ m/s}$ .

(a) In this case, the acceleration is  $a = +0.750 \text{ m/s}^2$  and the final velocity will be

$$v = v_0 + at = +25.0 \text{ m/s} + +0.750 \text{ m/s}^2 = 8.50 \text{ s} = +31.4 \text{ m/s}$$

or

$$v = 31.4$$
 m/s eastward

(b) When the acceleration is directed westward,  $a = -0.750 \text{ m/s}^2$ , the final velocity is

$$v = v_0 + at = +25.0 \text{ m/s} + -0.750 \text{ m/s}^2 \quad 8.50 \text{ s} = +18.6 \text{ m/s}, \text{ or } v = 18.6 \text{ m/s} \text{ eastward}$$
.

2.27 Choose the direction of the car's motion (eastward) as the positive direction. Then, the initial velocity of the car is  $v_0 = +40.0 \text{ m/s}$  and the final velocity (after an elapsed time of  $\Delta t = 3.50 \text{ s}$ ) is v = +25.0 m/s.

(a) The car's acceleration is

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{\Delta t} = \frac{25.0 \text{ m/s} - 40.0 \text{ m/s}}{3.50 \text{ s}} = -4.29 \text{ m/s}^2$$
 or  $a = 4.29 \text{ m/s}^2$  westward

(b) The distance traveled during the 3.50 s time interval is

$$\Delta x = v_{av} \quad \Delta t = \left(\frac{v + v_0}{2}\right) \Delta t = \left(\frac{25.0 \text{ m/s} + 40.0 \text{ m/s}}{2}\right) 3.50 \text{ s} = \boxed{114 \text{ m}}$$

**2.28** From  $v^2 = v_0^2 + 2a \Delta x$ , we have  $10.97 \times 10^3$  m/s<sup>2</sup> = 0 + 2a 220 m so that

$$a = \frac{v^2 - v_0^2}{2 \Delta x} = \frac{10.97 \times 10^3 \text{ m/s}^2 - 0}{2 220 \text{ m}} = \boxed{2.74 \times 10^5 \text{ m/s}^2}$$
$$= 2.74 \times 10^5 \text{ m/s}^2 \left(\frac{1 \text{ g}}{9.80 \text{ m/s}^2}\right) = \boxed{2.79 \times 10^4 \text{ times g !}}$$

2.29 (a) 
$$\Delta x = v_{av} \Delta t = v + v_0/2 \Delta t$$
 becomes 40.0 m =  $\left(\frac{2.80 \text{ m/s} + v_0}{2}\right)$  8.50 s,

which yields 
$$v_0 = \frac{2}{8.50 \text{ s}} 40.0 \text{ m} - 2.80 \text{ m/s} = 6.61 \text{ m/s}$$

(b) 
$$a = \frac{v - v_0}{\Delta t} = \frac{2.80 \text{ m/s} - 6.61 \text{ m/s}}{8.50 \text{ s}} = \boxed{-0.448 \text{ m/s}^2}$$

2.30 (a) 
$$v_i \rightarrow v_f \rightarrow x (m)$$
  
0 2.00 × 10<sup>2</sup> m

(b) The known quantities are initial velocity, final velocity, and displacement. The kinematics equation that relates these quantities to acceleration is  $v_f^2 = v_i^2 + 2a \Delta x$ .

(c) 
$$a = \boxed{\frac{v_f^2 - v_i^2}{2 \Delta x}}$$

(d) 
$$a = \frac{v_f^2 - v_i^2}{2 \Delta x} = \frac{30.0 \text{ m/s}^2 - 20.0 \text{ m/s}^2}{2 2.00 \times 10^2 \text{ m}} = 1.25 \text{ m/s}^2$$

(e) Using 
$$a = \Delta v / \Delta t$$
, we find that  $\Delta t = \frac{\Delta v}{a} = \frac{v_f - v_i}{a} = \frac{30.0 \text{ m/s} - 20.0 \text{ m/s}}{1.25 \text{ m/s}^2} = 8.00 \text{ s}$ 

**2.31** (a) With v = 120 km/h,  $v^2 = v_0^2 + 2a \Delta x$  yields

$$a = \frac{v^2 - v_0^2}{2 \Delta x} = \frac{\left[ 120 \text{ km/h}^2 - 0 \right]}{2 240 \text{ m}} \left( \frac{0.278 \text{ m/s}}{1 \text{ km/h}} \right)^2 = \boxed{2.32 \text{ m/s}^2}$$

(b) The required time is 
$$\Delta t = \frac{v - v_0}{a} = \frac{120 \text{ km/h} - 0}{2.32 \text{ m/s}^2} \left(\frac{0.278 \text{ m/s}}{1 \text{ km/h}}\right) = 14.4 \text{ s}.$$

2.32

(a) The time for the truck to reach 20 m/s, starting from rest, is found from:  $v = v_0 + at$ :

$$t_{\text{speed}} = \frac{v - v_0}{a} = \frac{20 \text{ m/s} - 0}{2.0 \text{ m/s}^2} = 10 \text{ s}$$

The total time for the trip is  $t_{\text{total}} = t_{\text{speed}} + t_{\text{constant}} + t_{\text{braking}} = 10 \text{ s} + 20 \text{ s} + 5.0 \text{ s} = 35 \text{ s}$ .

(b) The distance traveled during the first 10 s is

$$\Delta x_{\text{speed}} = \overline{v}_{\text{speed}} t_{\text{speed}} = \left(\frac{v + v_0}{2}\right) t_{\text{speed}} = \left(\frac{20 \text{ m/s} + 0}{2}\right) 10 \text{ s} = 100 \text{ m}$$

The distance traveled during the next 20 s (with a = 0) is

$$\Delta x_{\text{constant}} = v \cdot t_{\text{constant}} = 20 \text{ m/s} \quad 20 \text{ s} = 400 \text{ m}$$

The distance traveled in the last 5.0 s is

$$\Delta x_{\text{braking}} = \overline{v}_{\text{braking}} t_{\text{braking}} = \left(\frac{v_f + v}{2}\right) t_{\text{braking}} = \left(\frac{0 + 20 \text{ m/s}}{2}\right) 5.0 \text{ s} = 50 \text{ m}$$

The total displacement is then

$$\Delta x_{\text{total}} = \Delta x_{\text{speed}} + \Delta x_{\text{constant}} + \Delta x_{\text{braking}} = 100 \text{ m} + 400 \text{ m} + 50 \text{ m} = 550 \text{ m}$$

and the average velocity for the entire trip is

$$\overline{v}_{\text{trip}} = \frac{\Delta x_{\text{total}}}{t_{\text{total}}} = \frac{550 \text{ m}}{35 \text{ s}} = 16 \text{ m/s}$$

**2.33** (a) 
$$a = \frac{v - v_0}{\Delta t} = \frac{24.0 \text{ m/s}^2 - 0}{2.95 \text{ s}} = \boxed{8.14 \text{ m/s}^2}$$

(b) From 
$$a = \Delta v / \Delta t$$
, the required time is  $\Delta t = \frac{v_f - v_i}{a} = \frac{20.0 \text{ m/s} - 10.0 \text{ m/s}}{8.14 \text{ m/s}^2} = 123 \text{ s}$ .

- (c) Yes. For uniform acceleration, the change in velocity  $\Delta v$  generated in time  $\Delta t$  is given by  $\Delta v = (\Delta t)$ . From this, it is seen that doubling the length of the time interval  $\Delta t$  will always double the change in velocity  $\Delta v$ . A more precise way of stating this is: "When acceleration is constant, velocity is a linear function of time."
- 2.34 (a) The time required to stop the plane is

$$t = \frac{v - v_0}{a} = \frac{0 - 100 \text{ m/s}}{-5.00 \text{ m/s}^2} = 20.0 \text{ s}$$

(b) The minimum distance needed to stop is

$$\Delta x = \overline{v} \cdot t = \left(\frac{v + v_0}{2}\right) \cdot t = \left(\frac{0 + 100 \text{ m/s}}{2}\right) 20.0 \text{ s} = 1000 \text{ m} = 1.00 \text{ km}$$

Thus, the plane requires a minimum runway length of 1.00 km.

## It cannot land safely on a 0.800 km runway.

2.35 We choose x = 0 and t = 0 at location of Sue's car when she first spots the van and applies the brakes. Then, the initial conditions for Sue's car  $x_{0S} = 0$  are and  $v_{0S} = 30.0$  m/s. Her constant acceleration for  $t \ge 0$  is  $a_S = -2.00$  m/s<sup>2</sup>. The initial conditions for the van are  $x_{0V} = 155$  m,  $v_{0V} = 5.00$  m/s and its constant acceleration is  $a_V = 0$ . We then use  $\Delta x = x - x_0 = v_0 t + \frac{1}{2} a t^2$  to write an equation for the x coordinate of each vehicle for  $t \ge 0$ . This gives

Sue's Car:

$$x_S - 0 = 30.0 \text{ m/s} t + \frac{1}{2} -2.00 \text{ m/s}^2 t^2$$
 or  $x_S = 30.0 \text{ m/s} t - 1.00 \text{ m/s}^2 t^2$ 

Van:

$$x_V - 155 \text{ m} = 5.00 \text{ m/s} t + \frac{1}{2} 0 t^2$$
 or  $x_V = 155 \text{ m} + 5.00 \text{ m/s} t$ 

In order for a collision to occur, the two vehicles must be at the same location i.e.,  $x_S = x_V$ . Thus, we test for a collision by equating the two equations for the *x*-coordinates and see if the resulting equation has any real solutions.

$$x_S = x_V \implies 30.0 \text{ m/s} t - 1.00 \text{ m/s}^2 t^2 = 155 \text{ m} + 5.00 \text{ m/s} t$$
  
or  $1.00 \text{ m/s}^2 t^2 - 25.00 \text{ m/s} + 155 \text{ m} = 0$ 

Using the quadratic formula yields

$$t = \frac{-25.00 \text{ m/s} \pm \sqrt{-25.00 \text{ m/s}^2 - 4 \ 1.00 \text{ m/s}^2 \ 155 \text{ m}}}{2 \ 1.00 \text{ m/s}^2} = 13.6 \text{ s or } 11.4 \text{ s}$$

The solutions are real, not imaginary, so  $\boxed{a \text{ collision will occur}}$ . The smaller of the two solutions is the collision time. (The larger solution tells when the van would pull ahead of the car again if the vehicles could pass harmlessly through each other.) The *x*-coordinate where the collision occurs is given by

$$x_{\text{collision}} = x_{S}\Big|_{t=11.4 \text{ s}} = x_{V}\Big|_{t=11.4 \text{ s}} = 155 \text{ m} + 5.00 \text{ m/s} \quad 11.4 \text{ s} = 212 \text{ m}$$

2.36 The velocity at the end of the first interval is

$$v = v_0 + at = 0 + (2.77 \text{ m/s}) 15.0 \text{ s} = 41.6 \text{ m/s}$$

This is also the constant velocity during the second interval and the initial velocity for the third interval.

(a) From 
$$\Delta x = v_0 t + \frac{1}{2} a t^2$$
, the total displacement is

$$\Delta x_{\text{total}} = \Delta x_{1} + \Delta x_{2} + \Delta x_{3}$$
$$= \left[ 0 + \frac{1}{2} \ 2.77 \ \text{m/s}^{2} \ 15.0 \ \text{s}^{2} \right] + \left[ 41.6 \ \text{m/s} \ 123 \ \text{s} + 0 \right]$$
$$+ \left[ 41.6 \ \text{m/s} \ 4.39 \ \text{s} + \frac{1}{2} \ -9.47 \ \text{m/s}^{2} \ 4.39 \ \text{s}^{2} \right]$$

or

$$\Delta x_{\text{total}} = 312 \text{ m} + 5.11 \times 10^3 \text{ m} + 91.2 \text{ m} = 5.51 \times 10^3 \text{ m} = 5.51 \text{ km}$$

(b) 
$$\overline{v}_1 = \frac{\Delta x_1}{t_1} = \frac{312 \text{ m}}{15.0 \text{ s}} = 20.8 \text{ m/s}$$

$$\overline{v}_2 = \frac{\Delta x_2}{t_2} = \frac{5.11 \times 10^3 \text{ m}}{123 \text{ s}} = 41.6 \text{ m/s}$$

$$\overline{v}_3 = \frac{\Delta x_3}{t_3} = \frac{91.2 \text{ m}}{4.39 \text{ s}} = 20.8 \text{ m/s}$$

and the average velocity for the total trip is

$$\overline{v}_{\text{total}} = \frac{\Delta x_{\text{total}}}{t_{\text{total}}} = \frac{5.51 \times 10^3 \text{ m}}{15.0 + 123 + 4.39 \text{ s}} = 38.7 \text{ m/s}$$

2.37 Using the uniformly accelerated motion equation  $\Delta x = v_0 t + \frac{1}{2} a t^2$  for the full 40 s interval yields  $\Delta x = 20 \text{ m/s} \quad 40 \text{ s} + \frac{1}{2} -1.0 \text{ m/s}^2 \quad 40 \text{ s}^2 = 0$ , which is obviously wrong. The source of the error is found by computing the time required for the train to come to rest. This time is

$$t = \frac{v - v_0}{a} = \frac{0 - 20 \text{ m/s}}{-1.0 \text{ m/s}^2} = 20 \text{ s}$$

Thus, the train is slowing down for the first 20 s and is at rest for the last 20 s of the 40 s interval.

The acceleration is not constant during the full 40 s. It is, however, constant during the first 20 s as the train slows to rest. Application of  $\Delta x = v_0 t + \frac{1}{2} a t^2$  to this interval gives the stopping distance as

$$\Delta x = 20 \text{ m/s} \quad 20 \text{ s} + \frac{1}{2} - 1.0 \text{ m/s}^2 \quad 20 \text{ s}^2 = 200 \text{ m}$$

**2.38** 
$$v_0 = 0$$
 and  $v_f = \left(40.0 \ \frac{\text{mi}}{\text{h}}\right) \left(\frac{0.447 \ \text{m/s}}{1 \ \text{mi/h}}\right) = 17.9 \ \text{m/s}$ 

(a) To find the distance traveled, we use

$$\Delta x = \overline{v} \cdot t = \left(\frac{v_f + v_0}{2}\right) \cdot t = \left(\frac{17.9 \text{ m/s} + 0}{2}\right) 12.0 \text{ s} = \boxed{107 \text{ m}}$$

(b) The constant acceleration is

$$a = \frac{v_f - v_0}{t} = \frac{17.9 \text{ m/s} - 0}{12.0 \text{ s}} = \boxed{1.49 \text{ m/s}^2}$$

**2.39** At the end of the acceleration period, the velocity is

$$v = v_0 + at_{accel} = 0 + 1.5 \text{ m/s}^2 5.0 \text{ s} = 7.5 \text{ m/s}$$

This is also the initial velocity for the braking period.

(a) After braking, 
$$v_f = v + at_{\text{brake}} = 7.5 \text{ m/s} + -2.0 \text{ m/s}^2 \quad 3.0 \text{ s} = 1.5 \text{ m/s}$$

(b) The total distance traveled is

$$\Delta x_{\text{total}} = \Delta x_{\text{accel}} + \Delta x_{\text{brake}} = \overline{v} \cdot t_{\text{accel}} + \overline{v} \cdot t_{\text{brake}} = \left(\frac{v + v_0}{2}\right) t_{\text{accel}} + \left(\frac{v_f + v}{2}\right) t_{\text{brake}}$$

$$\Delta x_{\text{total}} = \left(\frac{7.5 \text{ m/s} + 0}{2}\right) 5.0 \text{ s} + \left(\frac{1.5 \text{ m/s} + 7.5 \text{ m/s}}{2}\right) 3.0 \text{ s} = \boxed{32 \text{ m}}$$

- 2.40 For the acceleration period, the parameters for the car are: initial velocity =  $v_{ia} = 0$ , acceleration =  $a_a = a_1$ , elapsed time =  $(\Delta t)_a = t_1$  and final velociety =  $v_{fa}$ . For the braking period, the parameters are: initial velocity =  $v_{ib}$  = final vel. of accel. period =  $v_{fa}$ , acceleration =  $a_b = a_2$ , and elapsed time =  $(\Delta t)_b = t_2$ .
  - (a) To determine the velocity of the car just before the brakes are engaged, we apply  $v_f = v_i + a \Delta t$  to the acceleration period and find

$$v_{ib} = v_{fa} = v_{ia} + a_a \Delta t_a = 0 + a_1 t_1 \text{ or } v_{ib} = a_1 t_1$$

(b) We may use  $\Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2$  to determine the distance traveled during the acceleration period (i.e., before the driver begins to brake). This gives

$$\Delta x_{a} = v_{ia} \Delta t_{a} + \frac{1}{2}a_{a} \Delta t_{a}^{2} = 0 + \frac{1}{2}a_{1}t_{1}^{2} \quad \text{or} \quad \Delta x_{a} = \boxed{\frac{1}{2}a_{1}t_{1}^{2}}$$

(c) The displacement occurring during the braking period is

$$\Delta x_{b} = v_{ib} \Delta t_{b} + \frac{1}{2}a_{b} \Delta t_{b}^{2} = a_{1}t_{1} t_{2} + \frac{1}{2}a_{2}t_{2}^{2}$$

Thus, the total displacement of the car during the two intervals combined is

$$\Delta x_{\text{total}} = \Delta x_{a} + \Delta x_{b} = \boxed{\frac{1}{2}a_{1}t_{1}^{2} + a_{1}t_{1}t_{2} + \frac{1}{2}a_{2}t_{2}^{2}}$$

2.41 The time the Thunderbird spends slowing down is

$$\Delta t_1 = \frac{\Delta x_1}{\frac{1}{V_1}} = \frac{2}{\frac{1}{V_1}} \frac{\Delta x_1}{\frac{1}{V_1} + \frac{1}{V_0}} = \frac{2}{\frac{1}{0}} \frac{250}{\frac{1}{0}} \frac{1}{100} \frac{1}$$

The time required to regain speed after the pit stop is

$$\Delta t_2 = \frac{\Delta x_2}{\overline{v}_2} = \frac{2 \ \Delta x_2}{v + v_0} = \frac{2 \ 350 \text{ m}}{71.5 \text{ m/s} + 0} = 9.79 \text{ s}$$

Thus, the total elapsed time before the Thunderbird is back up to speed is

 $\Delta t = \Delta t_1 + 5.00 \text{ s} + \Delta t_2 = 6.99 \text{ s} + 5.00 \text{ s} + 9.79 \text{ s} = 21.8 \text{ s}$ 

During this time, the Mercedes has traveled (at constant speed) a distance

 $\Delta x_M = v_0 \ \Delta t = 71.5 \text{ m/s} \ 21.8 \text{ s} = 1558 \text{ m}$ 

and the Thunderbird has fallen behind a distance

$$d = \Delta x_M - \Delta x_1 - \Delta x_2 = 1558 \text{ m} - 250 \text{ m} - 350 \text{ m} = 958 \text{ m}$$

**2.42** The car is distance *d* from the dog and has initial velocity  $v_0$  when the brakes are applied, giving it a constant acceleration *a*.

Apply  $\overline{v} = \Delta x / \Delta t = v + v_0 / 2$  to the entire trip (for which  $\Delta x = d + 4.0$  m,  $\Delta t = 10$  s, and v = 0) to obtain

$$\frac{d+4.0 \text{ m}}{10 \text{ s}} = \frac{0+v_0}{2} \text{ or } v_0 = \frac{d+4.0 \text{ m}}{5.0 \text{ s}}$$
[1]

Then, applying  $v^2 = v_0^2 + 2a \Delta x$  to the entire trip yields  $0 = v_0^2 + 2a d + 4.0 \text{ m}$ .

Substitute for  $v_0$  from Equation [1] to find that

$$0 = \frac{d + 4.0 \text{ m}^2}{25 \text{ s}^2} + 2a \ d + 4.0 \text{ m} \text{ and } a = -\frac{d + 4.0 \text{ m}}{50 \text{ s}^2}$$
[2]

Finally, apply  $\Delta x = v_0 t + \frac{1}{2} a t^2$  to the first 8.0 s of the trip (for which  $\Delta x = d$ ).

This gives

$$d = v_0 \ 8.0 \ s \ + \frac{1}{2} \ a \ 64 \ s^2$$
[3]

Substitute Equations [1] and [2] into Equation [3] to obtain

$$d = \left(\frac{d+4.0 \text{ m}}{5.0 \text{ s}}\right) 8.0 \text{ s} + \frac{1}{2} \left(-\frac{d+4.0 \text{ m}}{50 \text{ s}^2}\right) 64 \text{ s}^2 = 0.96d + 3.84 \text{ m}$$

which yields d = 3.84 m/0.04 = 96 m.

2.43 (a) Take t = 0 at the time when the player starts to chase his opponent. At this time, the opponent is distance  $d = 12 \text{ m/s} \quad 3.0 \text{ s} = 36 \text{ m}$  in front of the player. At time t > 0, the displacements of the players from their initial positions are

$$\Delta x_{\text{player}} = v_0 \,_{\text{player}} t + \frac{1}{2} \, a_{\text{player}} t^2 = 0 + \frac{1}{2} \, 4.0 \, \text{m/s}^2 \, t^2$$
[1]

and

$$\Delta x_{\text{opponent}} = v_0 \quad \text{opponent} \quad t + \frac{1}{2} a_{\text{opponent}} t^2 = 12 \text{ m/s} \quad t + 0 \quad [2]$$

When the players are side-by-side,

$$\Delta x_{\text{player}} = \Delta x_{\text{opponent}} + 36 \text{ m}$$
[3]

Substituting Equations [1] and [2] into Equation [3] gives

$$\frac{1}{2} 4.0 \text{ m/s}^2 t^2 = 12 \text{ m/s} t + 36 \text{ m} \text{ or } t^2 + -6.0 \text{ s} t + -18 \text{ s}^2 = 0$$

Applying the quadratic formula to this result gives

$$t = \frac{-6.0 \text{ s} \pm \sqrt{-6.0 \text{ s}^2 - 4 \text{ 1} -18 \text{ s}^2}}{2 \text{ 1}}$$

which has solutions of t = -2.2 s and t = +8.2 s Since the time must be greater than zero, we must choose t = 8.2 s as the proper answer.

(b) 
$$\Delta x_{\text{player}} = v_0 v_{\text{player}} t + \frac{1}{2} a_{\text{player}} t^2 = 0 + \frac{1}{2} 4.0 \text{ m/s}^2 8.2 \text{ s}^2 = 1.3 \times 10^2 \text{ m}$$

2.44 The initial velocity of the train is  $v_0 = 82.4$  km/h and the final velocity is v = 16.4 km/h. The time required for the 400 m train to pass the crossing is found from

$$\Delta x = \overline{v} \cdot t = \left[ v + v_0 / 2 \right] t$$
 as

$$t = \frac{2 \Delta x}{v + v_0} = \frac{2 0.400 \text{ km}}{82.4 + 16.4 \text{ km/h}} = 8.10 \times 10^{-3} \text{ h} \left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = 29.1 \text{ s}$$

2.45 (a) From  $v^2 = v_0^2 + 2a \Delta y$  with v = 0, we have

$$\Delta y_{\text{max}} = \frac{v^2 - v_0^2}{2 a} = \frac{0 - 25.0 \text{ m/s}^2}{2 - 9.80 \text{ m/s}^2} = \boxed{31.9 \text{ m}}$$

(b) The time to reach the highest point is

$$t_{\rm up} = \frac{v - v_0}{a} = \frac{0 - 25.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = 2.55 \text{ s}$$

(c) The time required for the ball to fall 31.9 m, starting from rest, is found from

$$\Delta y = 0 \ t + \frac{1}{2} a t^2 \text{ as } t = \sqrt{\frac{2 \ \Delta y}{a}} = \sqrt{\frac{2 \ -31.9 \text{ m}}{-9.80 \text{ m/s}^2}} = \boxed{2.55 \text{ s}}$$

- (d) The velocity of the ball when it returns to the original level (2.55 s after it starts to fall from rest) is  $v = v_0 + at = 0 + -9.80 \text{ m/s}^2 + 2.55 \text{ s} = -25.0 \text{ m/s}$
- 2.46 (a) For the upward flight of the arrow,  $v_0 = +100 \text{ m/s}$ ,  $a = -g = -9.80 \text{ m/s}^2$ , and the final velocity is v = 0. Thus,  $v^2 = v_0^2 + 2a \Delta y$  yields

$$\Delta y_{\text{max}} = \frac{v^2 - v_0^2}{2a} = \frac{0 - 100 \text{ m/s}^2}{2 - 9.80 \text{ m/s}^2} = 510 \text{ m}$$

(b) The time for the upward flight is

$$t_{\rm up} = \frac{\Delta y_{\rm max}}{\overline{v}_{\rm up}} = \frac{2 \Delta y_{\rm max}}{v_{\rm o} + v} = \frac{2 510 \text{ m}}{100 \text{ m/s} + 0} = 10.2 \text{ s}$$

For the downward flight,  $\Delta y = -\Delta y_{\text{max}} = -510 \text{ m}$ ,  $v_0 = 0$ , and  $a = -9.8 \text{ m/s}^2$  Thus,

$$\Delta y = v_0 t + \frac{1}{2} a t^2$$
 gives  $t_{\text{down}} = \sqrt{\frac{2 \Delta y}{a}} = \sqrt{\frac{2 - 510 \text{ m}}{-9.80 \text{ m/s}^2}} = 10.2 \text{ s}$ 

and the total time of the flight is  $t_{\text{total}} = t_{\text{down}} + t_{\text{down}} = 10.2 \text{ s} + 10.2 \text{ s} = 20.4 \text{ s}$ .

2.47 The velocity of the object when it was 30.0 m above the ground can be determined by applying  $\Delta y = v_0 t + \frac{1}{2} a t^2$  to the last 1.50 s of the fall. This gives

$$-30.0 \text{ m} = v_0 \ 1.50 \text{ s} \ + \frac{1}{2} \left( -9.80 \ \frac{\text{m}}{\text{s}^2} \right) \ 1.50 \text{ s}^2 \qquad \text{or} \qquad v_0 = -12.7 \text{ m/s}$$

The displacement the object must have undergone, starting from rest, to achieve this velocity at a point 30.0 m above the ground is given by  $v^2 = v_0^2 + 2a \Delta y$  as

$$\Delta y_{1} = \frac{v^{2} - v_{0}^{2}}{2a} = \frac{-12.7 \text{ m/s}^{2} - 0}{2 - 9.80 \text{ m/s}^{2}} = -8.23 \text{ m}$$

The total distance the object drops during the fall is

$$\Delta y_{\text{total}} = \Delta y_1 + -30.0 \text{ m} = 38.2 \text{ m}$$

2.48

(a) Consider the rock's entire upward flight, for which 
$$v_0 = +7.40 \text{ m/s}$$
,  $v_f = 0$ ,  $a = -g = -9.80 \text{ m/s}^2$ ,  
 $y_i = 1.55 \text{ m}$  (taking y = 0 at ground level), and  $y_f = h_{\text{max}} =$  maximum altitude reached by rock. Then  
applying  $v_f^2 = v_i^2 + 2a \Delta y$  to this upward flight gives

$$0 = 7.40 \text{ m/s}^2 + 2 -9.80 \text{ m/s}^2 h_{\text{max}} - 1.55 \text{ m}$$

and solving for the maximum altitude of the rock gives

$$h_{\text{max}} = 1.55 \text{ m} + \frac{7.40 \text{ m/s}^2}{2 9.80 \text{ m/s}^2} = 4.34 \text{ m}$$

Since  $h_{\text{max}} > 3.65$  m (height of the wall). the rock does reach the top of the wall.

(b) To find the velocity of the rock when it reaches the top of the wall, we use  $v_f^2 = v_i^2 + 2a \Delta y$  and solve for  $v_f$  when  $y_f = 3.65$  m (starting with  $v_i = +7.40$  m/s at  $y_i = 1.55$  m). This yields

$$v_f = \sqrt{v_i^2 + 2a y_f - y_i} = \sqrt{7.40 \text{ m/s}^2 + 2 -9.80 \text{ m/s}^2} \quad 3.65 \text{ m} - 1.55 \text{ m} = 3.69 \text{ m/s}$$

(c) A rock thrown *downward* at a speed of 7.40 m/s  $v_i = -7.40$  m/s from the top of the wall undergoes a displacement of  $(\Delta y) = y_f - y_i = 1.55$  m -3.65 m = -2.10 m before reaching the level of the attacker. Its velocity when it reaches the attacker is

$$v_f = -\sqrt{v_i^2 + 2a} \Delta y = -\sqrt{-7.40} \text{ m/s}^2 + 2 -9.80 \text{ m/s}^2 -2.10 \text{ m} = -9.79 \text{ m/s}^2$$

so the change in speed of this rock as it goes between the 2 points located at the top of the wall and the attacker is given by

$$\Delta speed_{\text{down}} = \left\| v_f \right| - \left| v_i \right\| = \left\| -9.79 \text{ m/s} \right| - \left| -7.40 \text{ m/s} \right\| = \boxed{2.39 \text{ m/s}}$$

(d) Observe that the change in speed of the ball thrown upward as it went from the attacker to the top of the wall was

$$\Delta \text{ speed }_{up} = \left\| v_f \right\| - \left| v_i \right\| = |3.69 \text{ m/s} - 7.40 \text{ m/s}| = 3.71 \text{ m/s}$$

Thus, the two rocks *do not* undergo the same magnitude change in speeds. As the two rocks travel between the level of the attacker and the level of the top of the wall, the rock thrown upward undergoes a greater change in speed than does the rock thrown downward. The reason for this is that the rock thrown upward has a smaller average speed between these two levels:

$$\overline{v}_{up} = \frac{\left|v_{i}\right|_{up} + \left|v_{f}\right|_{up}}{2} = \frac{7.40 \text{ m/s} + 3.69 \text{ m/s}}{2} = 5.55 \text{ m/s}$$

and

$$\overline{v}_{down} = \frac{\left|\overline{v}_{i}\right|_{down} + \left|v_{f}\right|_{down}}{2} = \frac{7.40 \text{ m/s} + 9.79 \text{ m/s}}{2} = 8.60 \text{ m/s}$$

Thus, the rock thrown upward spends more time travelling between the two levels. with gravity changing its speed by 9.80 m/s for each second that passes.

2.49 The velocity of the child's head just before impact (after falling a distance of 0.40 m, starting from rest) is given by  $v^2 = v_0^2 + 2a \Delta y$  as

$$v_I = -\sqrt{v_0^2 + 2a \Delta y} = -\sqrt{0 + 2 - 9.8 \text{ m/s}^2} - 0.40 \text{ m} = -2.8 \text{ m/s}^2$$

If, upon impact, the child's head undergoes an additional displacement  $\Delta y = -h$  before coming to rest, the acceleration during the impact can be found from  $v^2 = v_0^2 + 2a \Delta y$  to be  $a = (0 - v_I^2)/2(-h) = v_I^2/2h$ . The duration of the impact is found from  $v = v_0 + at$  as  $t = \Delta v/a = -v_I/(v_I^2/2h)$ , or  $t = -2h/v_I$ .

Applying these results to the two cases yields:

Hardwood Floor ( $h = 2.0 \times 10^{-3}$  m):

$$a = \frac{v_I^2}{2h} = \frac{-2.8 \text{ m/s}^2}{2 \ 2.0 \times 10^{-3} \text{ m}} = 2.0 \times 10^3 \text{ m/s}^2$$

and 
$$t = \frac{-2h}{v_I} = \frac{-2 \ 2.0 \times 10^{-3} \text{ m}}{-2.8 \text{ m/s}} = 1.4 \times 10^{-3} \text{ s} = \boxed{7.1 \text{ ms}}$$

Carpeted Floor ( $h = 1.0 \times 10^{-2}$  m):

$$a = \frac{v_I^2}{2h} = \frac{-2.8 \text{ m/s}^2}{2 1.0 \times 10^{-2} \text{ m}} = \frac{3.9 \times 10^2 \text{ m/s}^2}{3.9 \times 10^2 \text{ m/s}^2}$$

and 
$$t = \frac{-2h}{v_I} = \frac{-2 \ 1.0 \times 10^{-2} \ \text{m}}{-2.8 \ \text{m/s}} = 7.1 \times 10^{-3} \ \text{s} = 7.1 \ \text{ms}$$

**2.50** (a) After 2.00 s, the velocity of the mailbag is

$$v_{\text{bag}} = v_0 + at = -1.50 \text{ m/s} + -9.80 \text{ m/s}^2 \quad 2.00 \text{ s} = -21.1 \text{ m/s}^2$$

The negative sign tells that the bag is moving downward and the magnitude of the velocity gives the speed as 21.1 m/s.

(b) The displacement of the mailbag after 2.00 s is

$$\Delta y_{\text{bag}} = \left(\frac{v + v_0}{2}\right) t = \left[\frac{-21.1 \text{ m/s} + -1.50 \text{ m/s}}{2}\right] 2.00 \text{ s} = -22.6 \text{ m}$$

During this time, the helicopter, moving downward with constant velocity, undergoes a displacement of

$$\Delta y_{\text{copter}} = v_0 t + \frac{1}{2} a t^2 = -1.5 \text{ m/s} \quad 2.00 \text{ s} + 0 = -3.00 \text{ m}$$

The distance separating the package and the helicopter at this time is then

$$d = \left| \Delta y_p - \Delta y_h \right| = \left| -22.6 \text{ m} - -3.00 \text{ m} \right| = \left| -19.6 \text{ m} \right| = \boxed{19.6 \text{ m}}$$

(c) Here,  $v_{0 \text{ bag}} = v_{0 \text{ copter}} = +1.50 \text{ m/s}$  and  $a_{\text{bag}} = -9.80 \text{ m/s}^2$  while  $a_{\text{copter}} = 0$ . After 2.00 s, the velocity of the mailbag is

$$v_{\text{bag}} = 1.50 \ \frac{\text{m}}{\text{s}} + \left(-9.80 \ \frac{\text{m}}{\text{s}^2}\right) \ 2.00 \ \text{s} = -18.1 \ \frac{\text{m}}{\text{s}}$$

and its speed is

$$\left| v_{\text{bag}} \right| = \left| 18.1 \ \frac{\text{m}}{\text{s}} \right|$$

In this case, the displacement of the helicopter during the 2.00 s interval is

$$\Delta y_{\text{copter}} = +1.50 \text{ m/s} \quad 2.00 \text{ s} + 0 = +3.00 \text{ m}$$

Meanwhile, the mailbag has a displacement of

$$\Delta y_{\text{bag}} = \left(\frac{v_{\text{bag}} + v_0}{2}\right) t = \left[\frac{-18.1 \text{ m/s} + 1.50 \text{ m/s}}{2}\right] 2.00 \text{ s} = -16.6 \text{ m}$$

The distance separating the package and the helicopter at this time is then

$$d = \left| \Delta y_p - \Delta y_h \right| = \left| -16.6 \text{ m} - +3.00 \text{ m} \right| = \left| -19.6 \text{ m} \right| = \boxed{19.6 \text{ m}}$$

**2.51** (a) From the instant the ball leaves the player's hand until it is caught, the ball is a freely falling body with an acceleration of

$$a = -g = -9.80 \text{ m/s}^2 = 9.80 \text{ m/s}^2 \text{ downward}$$

- (b) At its maximum height, the ball comes to rest momentarily and then begins to fall back downward. Thus,  $v_{\text{max}} = \boxed{0}$ .
- (c) Consider the relation  $\Delta y = v_0 t + \frac{1}{2} a t^2$  with a = -g. When the ball is at the thrower's hand, the displacement is  $\Delta y = 0$ , giving  $0 = v_0 t \frac{1}{2} g t^2$

This equation has two solutions, t = 0 which corresponds to when the ball was thrown, and  $t = 2v_0/g$  corresponding to when the ball is caught. Therefore, if the ball is caught at t = 2.00 s, the initial velocity must have been

$$v_0 = \frac{gt}{2} = \frac{9.80 \text{ m/s}^2 + 2.00 \text{ s}}{2} = \frac{9.80 \text{ m/s}}{2}$$

(d) From  $v^2 = v_0^2 + 2a \Delta y$ , with v = 0 at the maximum height,

$$\Delta y_{\text{max}} = \frac{v^2 - v_0^2}{2 a} = \frac{0 - 9.80 \text{ m/s}^2}{2 - 9.80 \text{ m/s}^2} = \boxed{4.90 \text{ m}}$$

2.52 (a) Let t = 0 be the instant the package leaves the helicopter, so the package and the helicopter have a common initial velocity of  $v_i = -v_0$ . (choosing upward as positive).

At times t > 0, the velocity of the package (in free-fall with constant acceleration  $a_p = -g$ ) is given by

$$v = v_i + at$$
 as  $v_p = -v_0 - gt = -v_0 + gt$  and speed =  $\left| v_p \right| = v_0 + gt$ 

(b) After an elapsed time t, the downward displacement of the package from its point of release will be

$$\Delta y_{p} = v_{t}t + \frac{1}{2}a_{p}t^{2} = -v_{0}t - \frac{1}{2}gt^{2} = -\left(v_{0}t + \frac{1}{2}gt^{2}\right)$$

and the downward displacement of the helicopter (moving with constant velocity, or acceleration  $a_h = 0$ ) from the release point at this time is

$$\Delta y_{h} = v_{i}t + \frac{1}{2}a_{h}t^{2} = -v_{0}t + 0 = -v_{0}t$$

The distance separating the package and the helicopter at this time is then

$$d = \left| \Delta y_p - \Delta y_h \right| = \left| -\left( v_0 t + \frac{1}{2} g t^2 \right) - -v_0 t \right| = \boxed{\frac{1}{2} g t^2}$$

(c) If the helicopter and package are moving upward at the instant of release, then the common initial velocity is  $v_i = +v_0$ . The accelerations of the helicopter (moving with constant velocity) and the package (a freely falling object) remain unchanged from the previous case ( $a_p = -g$  and  $a_h = 0$ ).

In this case, the package speed at time t > 0 is  $1 |v_p| = |v_i + a_p t| = |v_0 - gt| = |gt - v_0|$ 

At this time, the displacements from the release point of the package and the helicopter are given by

$$\Delta y_{p} = v_{i}t + \frac{1}{2}a_{p}t^{2} = v_{0}t - \frac{1}{2}gt^{2} \qquad \text{and} \qquad \Delta y_{h} = v_{i}t + \frac{1}{2}a_{h}t^{2} = v_{0}t + 0 = +v_{0}t$$

The distance separating the package and helicopter at time t is now given by

$$d = \left| \Delta y_p - \Delta y_h \right| = \left| v_0 t - \frac{1}{2} g t^2 - v_0 t \right| = \left| \frac{1}{2} g t^2 \right|$$
(the same as earlier!)

2.53

(a) After its engines stop, the rocket is a freely falling body. It continues upward, slowing under the influence of gravity until it comes to rest momentarily at its maximum altitude. Then it falls back to Earth, gaining speed as it falls.

(b) When it reaches a height of 150 m, the speed of the rocket is

$$v = \sqrt{v_0^2 + 2a} \Delta y = \sqrt{50.0 \text{ m/s}^2 + 2 2.00 \text{ m/s}^2} 150 \text{ m} = 55.7 \text{ m/s}^2$$

After the engines stop, the rocket continues moving upward with an initial velocity of  $v_0 = 55.7$  m/s and acceleration a = -g = -9.80 m/s<sup>2</sup>. When the rocket reaches maximum height, v = 0. The displacement of the rocket above the point where the engines stopped (that is, above the 150-m level) is

$$\Delta y = \frac{v^2 - v_0^2}{2a} = \frac{0 - 55.7 \text{ m/s}^2}{2 - 9.80 \text{ m/s}^2} = 158 \text{ m}$$

The maximum height above ground that the rocket reaches is then given by

$$h_{\rm max} = 150 \text{ m} + 158 \text{ m} = 308 \text{ m}$$
.

(c) The total time of the upward motion of the rocket is the sum of two intervals. The first is the time for the rocket to go from  $v_0 = 50.0$  m/s at the ground to a velocity of v = 55.7 m/s at an altitude of 150 m. This time is given by

$$t_1 = \frac{\Delta y_1}{\overline{v}_1} = \frac{\Delta y_1}{v + v_0/2} = \frac{2\ 150\ \text{m}}{55.7 + 50.0\ \text{m/s}} = 2.84\ \text{s}$$

The second interval is the time to rise 158 m starting with  $v_0 = 55.7$  m/s and ending with v = 0. This time is

$$t_2 = \frac{\Delta y_2}{\overline{v}_2} = \frac{\Delta y_2}{v + v_0/2} = \frac{2.158 \text{ m}}{0 + 55.7 \text{ m/s}} = 5.67 \text{ s}$$

The total time of the upward flight is then  $t_{up} = t_1 + t_2 = 2.84 + 5.67$  s = 8.51 s

(d) The time for the rocket to fall 308 m back to the ground, with  $v_0 = 0$  and acceleration  $a = -g = -9.80 \text{ m/s}^2$ , is found from  $\Delta y = v_0 t + \frac{1}{2} a t^2$  as

$$t_{\rm down} = \sqrt{\frac{2 \ \Delta y}{a}} = \sqrt{\frac{2 \ -308 \ \rm{m}}{-9.80 \ \rm{m/s^2}}} = 7.93 \ \rm{s}$$

so the total time of the flight is  $t_{\text{flight}} = t_{\text{up}} + t_{\text{down}} = 8.51 + 7.93$  s = 16.4 s.

2.54 (a) The camera falls 50 m with a free-fall acceleration, starting with  $v_0 = -10$  m/s. Its velocity when it reaches the ground is

$$v = \sqrt{v_0^2 + 2 a \Delta y} = \sqrt{-10 \text{ m/s}^2 + 2 - 9.80 \text{ m/s}^2} - 50 \text{ m} = -33 \text{ m/s}^2$$

The time to reach the ground is given by

$$t = \frac{v - v_0}{a} = \frac{-33 \text{ m/s} - -10 \text{ m/s}}{-9.80 \text{ m/s}^2} = 2.3 \text{ s}$$

(b) This velocity was found to be  $v = \boxed{-33 \text{ m/s}}$  in part (a) above.

**2.55** During the 0.600 s required for the rig to pass completely onto the bridge, the front bumper of the tractor moves a distance equal to the length of the rig at constant velocity of v = 100 km/h. Therefore, the length of the rig is

$$L_{\rm rig} = vt = \left[ 100 \ \frac{\rm km}{\rm h} \left( \frac{0.278 \ {\rm m/s}}{1 \ {\rm km/h}} \right) \right] 0.600 \ {\rm s} = 16.7 \ {\rm m}$$

While some part of the rig is on the bridge, the front bumper moves a distance

$$\Delta x = L_{bridge} + L_{rig} = 400 \text{ m} + 16.7 \text{ m}$$

With a constant velocity of, the time for this to occur is

$$t = \frac{L_{\text{bridge}} + L_{\text{rig}}}{v} = \frac{400 \text{ m} + 16.7 \text{ m}}{100 \text{ km/h}} \left(\frac{1 \text{ km/h}}{0.278 \text{ m/s}}\right) = 15.0 \text{ s}$$

2.56 (a) From  $\Delta x = v_0 t + \frac{1}{2} a t^2$ , we have 100 m = 30.0 m/s  $t + \frac{1}{2} - 3.50$  m/s<sup>2</sup>  $t^2$ . This reduces to  $3.50 t^2 + (-60.0 \text{ s}) t + (200 \text{ s}^2) = 0$ , and the quadratic formula gives

$$t = \frac{-60.0 \text{ s} \pm \sqrt{-60.0 \text{ s}^2 - 4 3.50 200 \text{ s}^2}}{2 3.50}$$

The desired time is the smaller solution of t = 4.53 s. The larger solution of t = 12.6 s is the time when the boat would pass the buoy moving backwards, assuming it maintained a constant acceleration.

(b) The velocity of the boat when it first reaches the buoy is

$$v = v_0 + at = 30.0 \text{ m/s} + -3.50 \text{ m/s}^2 \quad 4.53 \text{ s} = 14.1 \text{ m/s}$$

**2.57** (a) The acceleration of the bullet is

$$a = \frac{\mathbf{v}^2 - \mathbf{v}_{\delta}^2}{2 \Delta x} = \frac{300 \text{ m/s}^2 - 400 \text{ m/s}^2}{2 0.100 \text{ m}} = \boxed{-3.50 \times 10^5 \text{ m/s}^2}$$

(b) The time of contact with the board is

$$t = \frac{v - v_0}{a} = \frac{300 - 400 \text{ m/s}}{-3.50 \times 10^5 \text{ m/s}^2} = \boxed{2.86 \times 10^{-4} \text{ s}}$$

- **2.58** We assume that the bullet begins to slow just as the front end touches the first surface of the board, and that it reaches its exit velocity just as the front end emerges from the opposite face of the board.
  - (a) The acceleration is

$$a = \frac{v_{\text{exit}}^2 - v_0^2}{2 \Delta x} = \frac{280 \text{ m/s}^2 - 420 \text{ m/s}^2}{2 0.100 \text{ m}} = \boxed{-4.90 \times 10^5 \text{ m/s}^2}$$

(b) The average velocity as the front of the bullet passes through the board is

$$\overline{v} = \frac{v_{\text{exit}} + v_0}{2} = \frac{280 \text{ m/s} + 420 \text{ m/s}}{2} = 350 \text{ m/s}$$

and the total time of contact with the board is the time for the front of the bullet to pass through plus the additional time for the trailing end to emerge (at speed  $v_{exit}$ ),

$$t = \frac{\Delta x_{\text{board}}}{v} + \frac{L_{\text{bullet}}}{v_{\text{exit}}} = \frac{0.100 \text{ m}}{350 \text{ m/s}} + \frac{0.0200 \text{ m}}{280 \text{ m/s}} = \boxed{3.57 \times 10^{-4} \text{ s}}$$

(c) From  $v^2 = v_0^2 + 2a \Delta x$ , with v = 0, gives the required thickness is

$$\Delta x = \frac{v^2 - v_0^2}{2 a} = \frac{0 - 420 \text{ m/s}^2}{2 - 4.90 \times 10^5 \text{ m/s}^2} = 0.180 \text{ m} = \boxed{18.0 \text{ cm}}$$

2.59 (a) The keys have acceleration  $a = -g = -9.80 \text{ m/s}^2$  from the release point until they are caught 1.50 s later. Thus,  $\Delta y = v_0 t + \frac{1}{2} a t^2$  gives

$$v_0 = \frac{\Delta y - at^2/2}{t} = \frac{+4.00 \text{ m} - -9.80 \text{ m/s}^2}{1.50 \text{ s}^2/2} = +10.0 \text{ m/s}^2$$

or

$$v_0 = 10.0 \text{ m/s upward}$$

(b) The velocity of the keys just before the catch was

$$v = v_0 + at = 10.0 \text{ m/s} + -9.80 \text{ m/s}^2 \quad 1.50 \text{ s} = -4.68 \text{ m/s}$$

or

$$v = 4.68 \text{ m/s downward}$$

2.60 (a) The keys, moving freely under the influence of gravity (a = -g), undergo a vertical displacement of  $\Delta y = +h$  in time *t*. We use  $\Delta y = v_i t + \frac{1}{2}at^2$  to find the initial velocity as

$$h = v_i t + \frac{1}{2} -g t^2$$

giving

$$v_i = \frac{h + gt^2/2}{t} = \frac{h}{t} + \frac{gt}{2}$$

(b) The velocity of the keys just before they were caught (at time t) is given by  $v = v_i + at$  as

$$v = \left(\frac{h}{t} + \frac{gt}{2}\right) + -g \quad t = \frac{h}{t} + \frac{gt}{2} - gt = \boxed{\frac{h}{t} - \frac{gt}{2}}$$

2.61 (a) From  $v^2 = v_0^2 + 2a \Delta y$ , the insect's velocity after straightening its legs is

$$v = \sqrt{v_0^2 + 2a \Delta y} = \sqrt{0 + 2 4000 \text{ m/s}^2 2.0 \times 10^{-3} \text{ m}} = 4.0 \text{ m/s}$$

and the time to reach this velocity is

$$t = \frac{v - v_0}{a} = \frac{4.0 \text{ m/s} - 0}{4\ 000 \text{ m/s}^2} = 1.0 \times 10^{-3} \text{ s} = \boxed{1.0 \text{ ms}}$$

(b) The upward displacement of the insect between when its feet leave the ground and it comes to rest momentarily at maximum altitude is

$$\Delta y = \frac{v^2 - v_0^2}{2a} = \frac{0 - v_0^2}{2 - g} = \frac{-4.0 \text{ m/s}^2}{2 - 9.8 \text{ m/s}^2} = \boxed{0.82 \text{ m}}$$

## 2.62 The distance required to stop the car after the brakes are applied is

$$\Delta x_{\text{stop}} = \frac{v^2 - v_0^2}{2 a} = \frac{0 - \left[35.0 \frac{\text{mi}}{\text{h}} \left(\frac{1.47 \text{ ft/s}}{1 \text{ mi/h}}\right)\right]^2}{2 - 9.00 \text{ ft/s}^2} = 147 \text{ ft}$$

Thus, if the deer is not to be hit, the maximum distance the car can travel before the brakes are applied is given by

$$\Delta x_{\text{before}} = 200 \text{ ft} - \Delta x_{\text{stop}} = 200 \text{ ft} - 147 \text{ ft} = 53.0 \text{ ft}$$

Before the brakes are applied, the constant speed of the car is 35.0 mi/h. Thus, the time required for it to travel 53.0 ft, and hence the maximum allowed reaction time, is

$$t_{r \max} = \frac{\Delta x_{\text{before}}}{v_0} = \frac{53.0 \text{ ft}}{\left[35.0 \frac{\text{mi}}{\text{h}} \left(\frac{1.47 \text{ ft/s}}{1 \text{ mi/h}}\right)\right]} = \left[1.03 \text{ s}\right]$$

2.63 The falling ball moves a distance of 15 m - h before they meet, where *h* is the height above the ground where they meet. Apply  $\Delta y = v_0 t + \frac{1}{2} a t^2$ , with a = -g, to obtain

$$-15 \text{ m} - h = 0 - \frac{1}{2}gt^2$$
 or  $h = 15 \text{ m} - \frac{1}{2}gt^2$  [1]

Applying  $\Delta y = v_0 t + \frac{1}{2} a t^2$  to the rising ball gives

$$h = 25 \text{ m/s} t - \frac{1}{2}gt^2$$
[2]

Combining equations [1] and [2] gives

25 m/s 
$$t - \frac{1}{2}gt^2 = 15 \text{ m} - \frac{1}{2}gt^2$$

or

$$t = \frac{15 \text{ m}}{25 \text{ m/s}} = \boxed{0.60 \text{ s}}$$

**2.64** The constant speed the student has maintained for the first 10 minutes, and hence her initial speed for the final 500 yard dash, is

$$v_0 = \frac{\Delta x_{10}}{\Delta t} = \frac{1.0 \text{ mi} - 500 \text{ yards}}{10 \text{ min}} = \frac{5280 \text{ ft} - 1500 \text{ ft}}{600 \text{ s}} \left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 1.9 \text{ m/s}$$

With an initial speed of  $v_0 = 1.9$  m/s and constant acceleration of a = 0.15 m/s<sup>2</sup>, the maximum distance the student can travel in the remaining 2.0 min (120 s) of her allotted time is

$$\Delta x_{2.0} = v_0 t + \frac{1}{2} a_{\text{max}} t^2 = \left(1.9 \ \frac{\text{m}}{\text{s}}\right) \ 120 \ \text{s} + \frac{1}{2} \left(0.15 \ \frac{\text{m}}{\text{s}^2}\right) \ 120 \ \text{s}^2 = 1.3 \times 10^3 \ \text{m}$$

or

$$\Delta x_{2.0 \text{ max}} = 1.3 \times 10^3 \text{ m} \left(\frac{3.281 \text{ ft}}{1 \text{ m}}\right) \left(\frac{1 \text{ yard}}{3 \text{ ft}}\right) = 1.4 \times 10^3 \text{ yards}$$

Since  $\Delta x_{2.0 \text{ max}}$  is considerably greater than the 500 yards she must still run, she can easily meet the requirment of running 1.0 miles in 12 minutes.

## 2.65 We solve Part (b) of this problem first.

(b) When the either ball reaches the ground, its displacement from the balcony is  $\Delta y = -19.6$  m (taking upward as positive). The initial velocities of the two balls were  $v_{01} = -14.7$  m/s and  $v_{02} = +14.7$  m/s, so  $v_0^2$  has the value of  $(14.7 \text{ m/s})^2$  for either ball. Also, a = -g for each ball, giving the downward velocity of either ball when it reaches the ground as

$$v_{\text{either}} = -\sqrt{v_0^2 + 2a} \Delta y = -\sqrt{14.5 \text{ m/s}^2 + 2} -9.80 \text{ m/s}^2 -19.6 \text{ m} = -24.5 \text{ m/s}$$

(a) The time for either ball to reach the ground (and hence achieve the velocity computed above) is given by

$$t = \frac{\frac{v_{\text{either}} - v_0}{\text{ball}}}{a} = \frac{-24.5 \text{ m/s} - v_0}{-g} = \frac{24.5 \text{ m/s} + v_0}{9.80 \text{ m/s}^2}$$

where  $v_0$  is the initial velocity of the particular ball of interest.

For ball 1,  $v_0 = -14.7 \text{ m/s}$ , giving

$$t_1 = \frac{24.5 \text{ m/s} - 14.7 \text{ m/s}}{9.80 \text{ m/s}^2} = 1.00 \text{ s}$$

For ball 2,  $v_0 = +14.7 \text{ m/s}$ , and

$$t_2 = \frac{24.5 \text{ m/s} + 14.7 \text{ m/s}}{9.80 \text{ m/s}^2} = 4.00 \text{ s}$$

The difference in the time of flight for the two balls is seen to be

 $\Delta t = t_2 - t_1 = 4.00 - 1.00 \text{ s} = 3.00 \text{ s}$ 

(c) At t = 0.800 s, the displacement of each ball from the balcony (at height h above ground) is

$$y_1 - h = v_{01}t - \frac{1}{2}gt^2 = -14.7 \text{ m/s} \quad 0.800 \text{ s} - 4.90 \text{ m/s}^2 \quad 0.800 \text{ s}^2$$

$$y_2 - h = v_{02}t - \frac{1}{2}gt^2 = +14.7 \text{ m/s} \quad 0.800 \text{ s} - 4.90 \text{ m/s}^2 \quad 0.800 \text{ s}^2$$

These give the altitudes of the two balls at t = 0.800 s as  $y_1 = h - 14.9$  m and  $y_2 = h + 8.62$  m. Therefore the distance separating the two balls at this time is

 $d = y_2 - y_1 = h + 8.62 \text{ m} - h - 14.9 \text{ m} = 23.5 \text{ m}$ 

2.66 (a) While in the air, both balls have acceleration  $a_1 = a_2 = -g$  (where upward is taken as positive). Ball 1 (thrown downward) has initial velocity  $v_{01} = -v_0$ , while ball 2 (thrown upward) has initial velocity  $v_{02} = +v_0$ . Taking y = 0 at ground level, the initial *y*-coordinate of each ball is  $y_{01} = y_{02} = +h$ . Applying  $\Delta y = y - y_i = v_i t + \frac{1}{2} a t^2$  to each ball gives their *y*-coordinates at time *t* as

Ball 1: 
$$y_1 - h = -vyt + \frac{1}{2} -g t^2$$
 or  $y_1 = h - v_0t - \frac{1}{2}gt^2$ 

Ball 2: 
$$y_2 - h = +v_0t + \frac{1}{2} -g t^2$$
 or  $y_2 = h + v_0t - \frac{1}{2}gt^2$ 

(b) At ground level, y = 0. Thus, we equate each of the equations found above to zero and use the quadratic formula to solve for the times when each ball reaches the ground. This gives the following:

Ball 1: 
$$0 = h - v_0 t_1 - \frac{1}{2} g t_1^2 \rightarrow g t_1^2 + 2v_0 t_1 + -2h = 0$$

so 
$$t_1 = \frac{-2v_0 \pm \sqrt{2v_0^2 - 4g} - 2h}{2g} = -\frac{v_0}{g} \pm \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Using only the *positive* solution gives

$$t_1 = -\frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Ball 2: 
$$0 = h + v_0 t_2 - \frac{1}{2} g t_2^2 \rightarrow g t_2^2 + -2v_0 t_2 + -2h = 0$$

and

$$t_2 = \frac{-2v_0 \pm \sqrt{-2v_0^2 - 4g - 2h}}{2g} = +\frac{v_0}{g} \pm \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Again, using only the *positive* solution

$$t_2 = \frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Thus, the difference in the times of flight of the two balls is

$$\Delta t = t_2 - t_1 = \frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}} - \left(-\frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}\right) = \boxed{\frac{2v_0}{g}}$$

(c) Realizing that the balls are going *downward* v < 0 as they near the ground, we use  $v_f^2 = v_i^2 + 2a \Delta y$  with  $\Delta y = -h$  to find the velocity of each ball just before it strikes the ground:

Ball 1: 
$$v_{1f} = -\sqrt{v_{1i}^2 + 2a_1 - h} = -\sqrt{-v_0^2 + 2 - g} - h = \boxed{-\sqrt{v_0^2 + 2gh}}$$

Ball 2: 
$$v_{2f} = -\sqrt{v_{2i}^2 + 2a_2 - h} = -\sqrt{+v_0^2 + 2 - g} - h = \boxed{-\sqrt{v_0^2 + 2gh}}$$

(d) While both balls are still in the air, the distance separating them is

$$d = y_2 - y_1 = \left(h + v_0 t - \frac{1}{2}gt^2\right) - \left(h - v_0 t - \frac{1}{2}gt^2\right) = 2v_0 t$$

2.67 (a) The first ball is dropped from rest  $(v_{01} = 0)$  from the height *h* of the window. Thus,  $v_f^2 = v_0^2 + 2a \Delta y$  gives the speed of this ball as it reaches the ground (and hence the initial velocity of the second ball) as  $\left|v_f\right| = \sqrt{v_{01}^2 + 2a_1 \Delta y_1} = \sqrt{0 + 2 - g} - h = \sqrt{2gh}$ When ball 2 is thrown upward at the same time that ball 1 is dropped, their *y*-coordinates at time *t* during the flights are given by  $y - y_o = v_0 t + \frac{1}{2}at^2$  as

Ball 1: 
$$y_1 - h = 0 t + \frac{1}{2} - g t^2$$
 or  $y_1 = h - \frac{1}{2}gt^2$ 

Ball 2: 
$$y_2 - 0 = \sqrt{2gh} t + \frac{1}{2} - g t^2$$
 or  $y_2 = \sqrt{2gh} t - \frac{1}{2}gt^2$ 

When the two balls pass,  $y_1 = y_2$ , or

$$h - \frac{1}{2}gt^2 = \sqrt{2gh} - \frac{1}{2}gt^2$$

giving

$$t = \frac{h}{\sqrt{2gh}} = \sqrt{\frac{h}{2g}} = \sqrt{\frac{28.7 \text{ m}}{2 9.80 \text{ m/s}^2}} = \boxed{1.21 \text{ s}}$$

(b) When the balls meet,

$$t = \sqrt{\frac{h}{2g}}$$

and

$$y_1 = h - \frac{1}{2}g\left(\sqrt{\frac{h}{2g}}\right)^2 = h - \frac{h}{4} = \frac{3h}{4}$$

Thus, the distance below the window where this event occurs is

$$d = h - y_1 = h - \frac{3h}{4} = \frac{h}{4} = \frac{28.7 \text{ m}}{4} = \boxed{7.18 \text{ m}}$$

**2.68** We do not know either the initial velocity nor the final velocity (that is, velocity just before impact) for the truck. What we do know is that the truck skids 62.4 m in 4.20 s while accelerating at  $-5.60 \text{ m/s}^2$ .

We have  $v = v_0 + at$  and  $\Delta x = \overline{v} \cdot t = [(v + v_0)/2] \cdot t$ . Applied to the motion of the truck, these yield

$$v - v_0 = at$$
 or  $v - v_0 = -5.60 \text{ m/s}^2 4.20 \text{ s} = -23.5 \text{ m/s}$  [1]

and

$$v + v_0 = \frac{2 \Delta x}{t} = \frac{2 62.4 \text{ m}}{4.20 \text{ s}} = 29.7 \text{ m/s}$$
 [2]

Adding equations [1] and [2] gives the velocity just before impact as

$$2v = -23.5 + 29.7$$
 m/s, or  $v = 3.10$  m/s

2.69 When released from rest ( $v_0 = 0$ ), the bill falls freely with a downward acceleration due to gravity (a = -g = -9.80 m/s<sup>2</sup>). Thus, the magnitude of its downward displacement during David's 0.2 s reaction time will be

$$\left|\Delta y\right| = \left|v_0 t + \frac{1}{2}at^2\right| = \left|0 + \frac{1}{2}\right| -9.80 \text{ m/s}^2 \quad 0.2 \text{ s}^2\right| = 0.2 \text{ m} = 20 \text{ cm}$$

This is over twice the distance from the center of the bill to its top edge (≈ 8 cm), so David will be unsuccessful.

2.70 (a) The velocity with which the first stone hits the water is

$$v_1 = -\sqrt{v_{01}^2 + 2a \Delta y} = -\sqrt{\left(-2.00 \frac{m}{s}\right)^2 + 2\left(-9.80 \frac{m}{s^2}\right)} -50.0 m = -31.4 \frac{m}{s}$$

The time for this stone to hit the water is

$$t_1 = \frac{v_1 - v_{01}}{a} = \frac{\left[-31.4 \text{ m/s} - -2.00 \text{ m/s}\right]}{-9.80 \text{ m/s}^2} = \boxed{3.00 \text{ s}}$$

(b) Since they hit simultaneously, the second stone which is released 1.00 s later will hit the water after an flight time of 2.00 s. Thus,

$$v_{02} = \frac{\Delta y - at_2^2/2}{t_2} = \frac{-50.0 \text{ m} - -9.80 \text{ m/s}^2}{2.00 \text{ s}} = \frac{-15.2 \text{ m/s}}{-15.2 \text{ m/s}}$$

(c) From part (a), the final velocity of the first stone is  $v_1 = -31.4 \text{ m/s}$ . The final velocity of the second stone is

$$v_2 = v_{02} + at_2 = -15.2 \text{ m/s} + -9.80 \text{ m/s}^2 \quad 2.00 \text{ s} = -34.8 \text{ m/s}$$

**2.71** (a) The sled's displacement,  $\Delta x_1$ , while accelerating at  $a_1 = +40$  ft/s<sup>2</sup> for time  $t_1$  is

$$\Delta x_1 = 0 \ t_1 + \frac{1}{2} a_1 t_1^2 = 20 \ \text{ft/s}^2 \ t_1^2 \qquad \text{or} \qquad \Delta x_1 = 20 \ \text{ft/s}^2 \ t_1^2 \qquad [1]$$

At the end of time  $t_1$ , the sled had achieved a velocity of

$$v = v_0 + a_1 t_1 = 0 + 40 \text{ ft/s}^2 t_1$$
 or  $v = 40 \text{ ft/s}^2 t_1$  [2]

The displacement of the sled while moving at constant velocity v for time  $t_2$  is

$$\Delta x_2 = vt_2 = \begin{bmatrix} 40 \text{ ft/s}^2 & t_1 \end{bmatrix} t_2$$
 or  $\Delta x_2 = 40 \text{ ft/s}^2 & t_1 t_2$  [3]

It is known that  $\Delta x_1 + \Delta x_2 = 17500$  ft, and substitutions from Equations [1] and [3] give

20 ft/s<sup>2</sup> 
$$t_1^2 + 40$$
 ft/s<sup>2</sup>  $t_1 t_2 = 17500$  ft or  $t_1^2 + 2t_1 t_2 = 875$  s<sup>2</sup> [4]

Also, it is known that

$$t_1 + t_2 = 90 \,\mathrm{s}$$
 [5]

Solving Equations [4] and [5] simultaneously yields

$$t_1^2 + 2t_1 \ 90 \ s - t_1 = 875 \ s^2$$
 or  $t_1^2 + -180 \ s \ t_1 + 875 \ s^2 = 0$ 

The quadratic formula then gives

$$t_1 = \frac{-180 \text{ s} \pm \sqrt{-180 \text{ s}^2 - 4.1 \text{ } 875 \text{ s}^2}}{2.1}$$

with solutions  $t_1 = 5.00$  s and  $t_2 = 90$  s -5.0 s = 85 s or  $t_1 = 175$  s and  $t_2 = -85$  s.

Since it is necessary that  $t_2 > 0$ , the valid solutions are  $t_1 = 5.0$  s and  $t_2 = 85$  s.

(b) From Equation [2] above,  $v = 40 \text{ ft/s}^2 t_1 = 40 \text{ ft/s}^2 5.0 \text{ s} = 200 \text{ ft/s}$ .

(c) The displacement  $\Delta x_3$  of the sled as it comes to rest (with acceleration  $a_3 = -20$  ft/s<sup>2</sup>) is

$$\Delta x_3 = \frac{0 - v^2}{2a_3} = \frac{-200 \text{ ft/s}^2}{2 - 20 \text{ ft/s}^2} = 1\,000 \text{ ft}$$

Thus, the total displacement for the trip (measured from the starting point) is

$$\Delta x_{total} = \Delta x_1 + \Delta x_2 + \Delta x_3 = 17500 \text{ ft} + 1000 \text{ ft} = 18500 \text{ ft}$$

(d) The time required to come to rest from velocity v (with acceleration  $a_3$ ) is

$$t_3 = \frac{0 - v}{a_3} = \frac{-200 \text{ ft/s}}{-20 \text{ ft/s}^2} = \boxed{10 \text{ s}}$$

so the duration of the entire trip is  $t_{total} = t_1 + t_2 + t_3 = 5.0 \text{ s} + 85 \text{ s} + 10 \text{ s} = 100 \text{ s}$ .

2.72 (a) From 
$$\Delta y = v_0 t + \frac{1}{2}at^2$$
 with  $v_0 = 0$ , we have

$$t = \sqrt{\frac{2 \Delta y}{a}} = \sqrt{\frac{2 - 23 \text{ m}}{-9.80 \text{ m/s}^2}} = \boxed{2.2 \text{ s}}$$

(b) The final velocity is  $v = 0 + -9.80 \text{ m/s}^2 \text{ m/s}^2 - 2.2 \text{ s} = -21 \text{ m/s}$ .

(c) The time it takes for the sound of the impact to reach the spectator is

$$t_{sound} = \frac{\Delta y}{v_{sound}} = \frac{23 \text{ m}}{340 \text{ m/s}} = 6.8 \times 10^{-2} \text{ s}$$

so the total elapsed time is  $t_{\text{total}} = 2.2 \text{ s} + 6.8 \times 10^{-2} \text{ s} \approx 2.3 \text{ s}$ 

2.73 (a) Since the sound has constant velocity, the distance it traveled is

$$\Delta x = v_{sound} t = 1100 \text{ ft/s} 5.0 \text{ s} = 5.5 \times 10^3 \text{ ft}$$

(b) The plane travels this distance in a time of 5.0 s + 10 s = 15 s, so its velocity must be

$$v_{plane} = \frac{\Delta x}{t} = \frac{5.5 \times 10^3 \text{ ft}}{15 \text{ s}} = \boxed{3.7 \times 10^2 \text{ ft/s}}$$

(c) The time the light took to reach the observer was

$$t_{light} = \frac{\Delta x}{v_{light}} = \frac{5.5 \times 10^3 \text{ ft}}{3.00 \times 10^8 \text{ m/s}} \left(\frac{1 \text{ m/s}}{3.281 \text{ ft/s}}\right) = 5.6 \times 10^{-6} \text{ s}$$

During this time the plane would only travel a distance of 0.002 ft

2.74 The distance the glider moves during the time  $\Delta t_d$  is given by  $\Delta x = \ell = v_0 \Delta t_d + \frac{1}{2}a \Delta t_d^2$ , where  $v_0$  is the glider's velocity when the flag first enters the photogate and *a* is the glider's acceleration. Thus, the average velocity is

$$v_{d} = \frac{\ell}{\Delta t_{d}} = \frac{v_{0} \ \Delta t_{d} \ + \frac{1}{2} a \ \Delta t_{d}^{2}}{\Delta t_{d}} = v_{0} + \frac{1}{2} a \ \Delta t_{d}$$

(a) The glider's velocity when it is halfway through the photogate in space i.e., when  $\Delta x = l/2$  is found from

$$v^{2} = v_{0}^{2} + 2a \ \Delta x \text{ as}$$
$$v_{1} = \sqrt{v_{0}^{2} + 2a \ \ell/2} = \sqrt{v_{0}^{2} + a\ell} = \sqrt{v_{0}^{2} + a\left[v_{d} \ \Delta t_{d}\right]} = \sqrt{v_{0}^{2} + av_{d} \ \Delta t_{d}}$$

Note that this is not equal to  $v_d$  unless a = 0, in which case  $v_1 = v_d = v_0$ .

(b) The speed  $v_2$  when the glider is halfway through the photogate in time (i.e., when the elapsed time is  $t_2 = \Delta t_d/2$ ) is given by  $v = v_0 + at$  as

$$v = v_0 + at_2 = v_0 + a \Delta t_d / 2 = v_0 + \frac{1}{2} a \Delta t_d$$

which is equal to  $v_d$  for all possible values of  $v_0 = a$ .

**2.75** The time required for the stunt man to fall 3.00 m, starting from rest, is found from  $\Delta y = v_0 t + \frac{1}{2} a t^2$  as

$$-3.00 \text{ m} = 0 + \frac{1}{2} -9.80 \text{ m/s}^2 t^2 \qquad \text{so} \qquad t = \sqrt{\frac{2 \ 3.00 \text{ m}}{9.80 \text{ m/s}^2}} = 0.782 \text{ s}$$

(a) With the horse moving with constant velocity of 10.0 m/s, the horizontal distance is

$$\Delta x = v_{horse} t = 10.0 \text{ m/s} \quad 0.782 \text{ s} = 7.82 \text{ m}$$

(b) The required time is t = 0.782 s as calculated above.