## **PROBLEM SOLUTIONS**

1.1 Substituting dimensions into the given equation  $T = 2\pi \sqrt{\ell/g}$ , recognizing that  $2\pi$  is a dimensionless constant, we have

$$T = \sqrt{\frac{\ell}{g}}$$
 or  $T = \sqrt{\frac{L}{L/T^2}} = \sqrt{T^2} = T$ 

Thus, the dimensions are consistent .

1.2 (a) From  $x = Bt^2$ , we find that  $B = \frac{x}{t^2}$  Thus, B has units of

$$[B] = \frac{[x]}{[t^2]} = \boxed{\frac{\mathrm{L}}{\mathrm{T}^2}}$$

(b) If  $x = A \sin(2\pi ft)$ , then [A] = [x]/sin  $(2\pi ft)$ ]

But the sine of an angle is a dimensionless ratio.

Therefore, [A] = [x] =**L** 

**1.3** (a) The units of volume, area and height are:

 $[V] = L^3$ ,  $[A] = L^2$ , and [h] = L

We then observe that or  $L^3 = L^2L$  or [V] = [A][h]

Thus, the equation is V = Ah is dimensionally correct

(b)  $V_{\text{cylinder}} = \pi R^2 h = \pi R^2 h = Ah$ , where  $A = \pi R^2$ 

 $V_{\text{rectangular box}} = \ell w h = \ell w h = Ah$ , where  $A = \ell w = length \times width$ 

1.4 In the equation 
$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + \sqrt{mgh}$$
,  $[mv^2] = [mv_0^2] = M\left(\frac{L}{T}\right)^2 = \frac{ML^2}{T^2}$ 

while  $\left[\sqrt{mgh}\right] = \sqrt{M\left(\frac{L}{T^2}\right)L} = \frac{M^{\frac{1}{2}L}}{T}$ . Thus, the equation is dimensionally incorrect.

In 
$$v = v_0 + at^2$$
,  $[v] = [v_0] = \frac{L}{T}$  but  $[at^2] = [a][t^2] = \left(\frac{L}{T^2}\right) T^2 = L$ 

Hence, this equation is dimensionally incorrect

In the equation  $ma = v^2$ , we see that  $[ma] = [m][a] = M\left(\frac{L}{T^2}\right) = \frac{ML}{T^2}$ 

while  $[v^2] = \left(\frac{L}{T}\right)^2 = \frac{L^2}{T^2}$ 

Therefore, this equation is also dimensionally incorrect.

1.5 From the universal gravitation law, the constant G is  $G = Fr^2/Mm$ . Its units are then

$$G = \frac{F[r^2]}{M m} = \frac{(kg \cdot m/s^2)(m^2)}{kg \cdot kg} = \frac{m^3}{kg \cdot s^2}$$

1.6 (a) Solving  $KE = p^2/2m$  for the momentum, p, gives  $p = \sqrt{2m \ KE}$  where the numeral 2 is a dimensionless constant. Dimensional analysis gives the units of momentum as:

$$p = \sqrt{m \ KE} = \sqrt{M \ M \cdot L^2/T^2} = \sqrt{M^2 \cdot L^2/T^2} = M \ L/T$$

Therefore, in the SI system, the units of momentum are  $\left| kg \cdot m/s \right|$  .

(b) Note that the units of force are kg  $\cdot$  m/s<sup>2</sup> or [F] = M  $\cdot$  L/T<sup>2</sup>. Then, observe that

$$F \quad t = \mathbf{M} \cdot \mathbf{L}/\mathbf{T}^2 \quad \mathbf{T} = \mathbf{M} \quad \mathbf{L}/\mathbf{T} = p$$

From this, it follows that force times time is proportional to momentum: Ft = p. (See the impulse–momentum theorem in Chapter 6,  $F \cdot \Delta t = \Delta p$ , which says that a constant force *F* 

times a duration of time  $\Delta t$  equals the change in momentum,  $\Delta p$ .)

Blindly adding the two lengths, we get 228.76 cm. However, 135.3 cm has only one decimal place.
 Therefore, only one decimal place accuracy is possible in the sum, and the answer must be rounded to 228.8 cm.

$$A = \ell w = \begin{bmatrix} 21.3 \pm 0.2 \text{ cm} \end{bmatrix} \begin{bmatrix} 9.8 \pm 0.1 \text{ cm} \end{bmatrix}$$

Multiplying out this product of binominals gives

$$A = \begin{bmatrix} 21.3 & 9.8 \\ \pm & 21.3 & 0.1 \\ \pm & 0.2 & 9.8 \\ \pm & 0.2 & 0.1 \end{bmatrix} \text{ cm}^2$$

The first term gives the best value of the area. The second and third terms add to give the uncertainty and the fourth term is negligible in comparison to the other terms. The area and its uncertainty are found to be

$$A = \boxed{209 \text{ cm}^2 \pm 4 \text{ cm}^2}$$

- 1.9 (a)  $78.9 \pm 0.2$  has 3 significant figures with the uncertainty in the tenths position.
  - (b)  $3.788 \times 10^9$  has 4 significant figures
  - (c)  $2.46 \times 10^{-6}$  has 3 significant figures

(d)  $0.003 \ 2 = 3.2 \times 10^{-3}$  has 2 significant figures. The two zeros were originally included only to position the decimal.

**1.10**  $c = 2.997 \ 924 \ 58 \times 10^8 \ \text{m/s}$ 

- (a) Rounded to 3 significant figures:  $c = 3.00 \times 10^8$  m/s
- (b) Rounded to 5 significant figures:  $c = 2.997.9 \times 10^8 \text{ m/s}$
- (c) Rounded to 7 significant figures:  $c = 2.997.925 \times 10^8 \text{ m/s}$
- 1.11 Observe that the length l = 5.62 cm, the width w = 6.35 cm, and the height h = 2.78 cm all contain 3 significant figures. Thus, any product of these quantities should contain 3 significant figures.

(a)  $\ell w = 5.62 \text{ cm} 6.35 \text{ cm} = 35.7 \text{ cm}^2$ 

(b) 
$$V = \ell w h = 35.7 \text{ cm}^2 2.78 \text{ cm} = 99.2 \text{ cm}^3$$

(c) 
$$wh = 6.35 \text{ cm} 2.78 \text{ cm} = 17.7 \text{ cm}^2$$

$$V = wh \ \ell = 17.7 \ \text{cm}^2 \ 5.62 \ \text{cm} = 99.5 \ \text{cm}^3$$

(d) In the rounding process, small amounts are either added to or subtracted from an answer to satisfy the rules of significant figures. For a given rounding, different small adjustments are made, introducing a certain amount of randomness in the last significant digit of the final answer.

**1.12** (a) 
$$A = \pi r^2 = \pi \ 10.5 \text{ m} \pm 0.2 \text{ m}^2 = \pi \left[ \ 10.5 \text{ m}^2 \pm 2 \ 10.5 \text{ m} \ 0.2 \text{ m} + 0.2 \text{ m}^2 \right]$$

Recognize that the last term in the brackets is insignificant in comparison to the other two. Thus, we have

$$A = \pi \left[ 110 \text{ m}^2 \pm 4.2 \text{ m}^2 \right] = \boxed{346 \text{ m}^2 \pm 13 \text{ m}^2}$$
  
(b) 
$$C = 2\pi r = 2\pi \ 10.5 \text{ m} \pm 0.2 \text{ m} = \boxed{66.0 \text{ m} \pm 1.3 \text{ m}}$$

**1.13** (a) The sum is rounded to 797 because 756 in the terms to be added has no positions beyond the decimal.

(b)  $0.003 \ 2 \times 356.3 = (3.2 \times 10^{-3}) \times 356.3 = 1.140 \ 16$  must be rounded to 1.1 because  $3.2 \times 10^{-3}$  has only two significant figures.

(c)  $5.620 \times \pi$  must be rounded to 17.66 because 5.620 has only four significant figures.

1.14 (a) Answer limited to three significant figures because of the accuracy of the denominator

$$2.437 \times 10^{4}$$
 6.521 1 × 10<sup>9</sup> / 5.37 × 10<sup>4</sup> = 2.959 4 × 10<sup>9</sup> = 2.96 × 10<sup>9</sup>

(b) Answer limited to the four significant figure accuracy of two of the operands

$$3.14159 \times 10^2$$
  $27.01 \times 10^4$  /  $1234 \times 10^6$  =  $6.8764 \times 10^{-2}$  =  $6.876 \times 10^{-2}$ 

1.15 
$$d = 250\ 000\ \text{mi}\ \left(\frac{5\ 280\ \text{ft}}{1.000\ \text{mi}}\right)\left(\frac{1\ \text{fathom}}{6\ \text{ft}}\right) = \boxed{2 \times 10^{\ 8}\ \text{fathoms}}$$

The answer is limited to one significant figure because of the accuracy to which the conversion from fathoms to feet is given.

1.16 
$$c = \left(3.00 \times 10^8 \ \frac{\text{m}}{\text{s}}\right) \left(\frac{1 \ \text{km}}{10^3 \ \text{m}}\right) \left(\frac{1 \ \text{mi}}{1.609 \ \text{km}}\right) \left(\frac{1 \ \text{furlong}}{0.125 \ \text{mi}}\right) = 1.49 \times 10^6 \ \frac{\text{furlongs}}{\text{s}}$$

and 
$$c = \left(1.49 \times 10^6 \quad \frac{\text{furlongs}}{\text{\$}}\right) \left(\frac{8.64 \times 10^4 \text{\$}}{1 \text{ day}}\right) \left(\frac{14.0 \text{ days}}{1 \text{ fortnight}}\right) \left(\frac{1 \text{ megafurlong}}{10^6 \text{ furlongs}}\right)$$

yielding  $c = 1.80 \times 10^6$  megafurlongs/fortnight

1.17 6.00 firkins = 6.00 firkins 
$$\left(\frac{9 \text{ gal}}{1 \text{ firkin}}\right) \left(\frac{3.786 \text{ K}}{1 \text{ gal}}\right) \left(\frac{10^3 \text{ em}^3}{1 \text{ K}}\right) \left(\frac{1 \text{ m}^3}{10^6 \text{ em}^3}\right) = 0.204 \text{ m}^3$$

**1.18** (a) 
$$\ell = 348 \text{ mi} \left(\frac{1.609 \text{ km}}{1.000 \text{ mi}}\right) = 5.60 \times 10^2 \text{ km} = 5.60 \times 10^5 \text{ m} = 5.60 \times 10^7 \text{ cm}$$

(b) 
$$h = 1.612 \text{ ft} \left(\frac{1.609 \text{ km}}{5.280 \text{ ft}}\right) = \boxed{0.491.2 \text{ km}} = \boxed{491.2 \text{ m}} = \boxed{4.912 \times 10^4 \text{ cm}}$$

(c) 
$$h = 20\ 320\ \text{ft}\ \left(\frac{1.609\ \text{km}}{5\ 280\ \text{ft}}\right) = 6.192\ \text{km} = 6.192\ \times 10^3\ \text{m} = 6.192\ \times 10^5\ \text{cm}$$

(d) 
$$d = 8\ 200\ \text{ft}\left(\frac{1.609\ \text{km}}{5\ 280\ \text{ft}}\right) = \boxed{2.499\ \text{km}} = \boxed{2.499\ \times 10^3\ \text{m}} = \boxed{2.499\ \times 10^5\ \text{cm}}$$

In (a), the answer is limited to three significant figures because of the accuracy of the original data value, 348 miles. In (b), (c), and (d), the answers are limited to four significant figures because of the accuracy to which the kilometers-to-feet conversion factor is given.

**1.19** Note that 1 m = 3.281 ft, so  $(1 \text{ m})^2 = (3.281 \text{ ft})^2 \text{ or } 1 \text{ m}^2 = (3.281)^2 \text{ ft}^2$ 

$$A = \ell w = 100 \text{ ft} \quad 150 \text{ ft} = 1.50 \times 10^4 \text{ ft}^2 \left(\frac{1 \text{ m}^2}{3.281^2 \text{ ft}^2}\right) = \boxed{1.39 \times 10^3 \text{ m}^2}$$

1.20 Age of Earth = 
$$1 \times 10^{17} \text{ s} \left(\frac{1 \text{ yr}}{365.242 \text{ days}}\right) \left(\frac{1 \text{ day}}{24 \text{ h}}\right) \left(\frac{1 \text{ h}}{3 600 \text{ s}}\right) = \boxed{3 \times 10^9 \text{ yr}}$$

**1.21** Distance to Proxima Centauri = 
$$4 \times 10^{16}$$
 m  $\left(\frac{3.281 \text{ ft}}{1 \text{ m}}\right) = 1 \times 10^{17} \text{ ft}$ 

1.22 
$$rate = \left(\frac{1}{32} \quad \frac{\mathrm{in}}{\mathrm{day}}\right) \left(\frac{1}{24} \frac{\mathrm{day}}{\mathrm{M}}\right) \left(\frac{1}{3} \frac{\mathrm{M}}{600 \mathrm{s}}\right) \left(\frac{2.54}{1.00 \mathrm{in}}\right) \left(\frac{10^9 \mathrm{nm}}{10^2 \mathrm{cm}}\right) = 9.2 \mathrm{nm/s}$$

This means that the proteins are assembled at a rate of many layers of atoms each second!

1.23 
$$c = \left(3.00 \times 10^8 \ \frac{\text{h}}{\text{s}}\right) \left(\frac{3\ 600-\text{s}}{1\ \text{h}}\right) \left(\frac{1\ \text{km}}{10^3\ \text{h}}\right) \left(\frac{1\ \text{mi}}{1.609\ \text{km}}\right) = \boxed{6.71 \times 10^8\ \text{mi/h}}$$

Volume of house = 50.0 ft 26 ft 8.0 ft 
$$\left(\frac{2.832 \times 10^{-2} \text{ m}^3}{1 \text{ ft}^3}\right)$$
  
=  $2.9 \times 10^2 \text{ m}^3$  =  $2.9 \times 10^2 \text{ m}^3 \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3$  =  $2.9 \times 10^8 \text{ cm}^3$ 

1.25 Volume = 
$$(5.0 \text{ acre At}) \left[ \left( \frac{1 \text{ m}}{3.281 \text{ ft}} \right) \left[ \left( \frac{43 560 \text{ ft}^2}{1 \text{ acre}} \right) \left( \frac{1 \text{ m}}{3.281 \text{ ft}} \right)^2 \right] = (3.08 \times 10^4 \text{ m}^3)$$

Volume of pyramid = 
$$\frac{1}{3}$$
 area of base height  
=  $\frac{1}{3} \begin{bmatrix} 13.0 \text{ acres } 43\ 560\ \text{ft}^2/\text{acre } \end{bmatrix} 481\ \text{ft} = 9.08 \times 10^7\ \text{ft}^3$   
=  $9.08 \times 10^7\ \text{ft}^3\ \left(\frac{2.832 \times 10^{-2}\ \text{m}^3}{1\ \text{ft}^3}\right) = \boxed{2.57 \times 10^6\ \text{m}^3}$ 

1.27 Volume of cube =  $L^3 = 1$  quart (Where L = length of one side of the cube.)

Thus, 
$$L^3 = \left( \text{quart} \right) \left( \frac{1 \text{ gallon}}{4 \text{ guarts}} \right) \left( \frac{3.786 \text{ liter}}{1 \text{ gallon}} \right) \left( \frac{1 000 \text{ cm}^3}{1 \text{ liter}} \right) = 946 \text{ cm}^3$$

and  $L = \sqrt[3]{946 \text{ cm}^3} = 9.82 \text{ cm}$ 

**1.28** number of pounds = (number of burgers)(weight/burger)

$$= (5 \times 10^{10} \text{ burgers})(0.25 \text{ Ib/burgers}) = 1.25 \times 10^{10} \text{ Ib, or } \sim 10^{10} \text{ Ib}$$

Number of head of cattle = (weight needed)/(weight per head)

=( $1.25 \times 10^{10}$  Ib)/(300 Ib/head) =  $4.17 \times 10^{7}$  head, or  $\sim 10^{7}$  head

Assumptions are 0.25 lb of meat per burger and a net of 300 lb of meat per head of cattle.

**1.29** Consider a room that is 12 ft square with an 8.0 ft high ceiling. Recognizing that 1 m = 3.281 ft, so  $(1 \text{ m})^3 = (3.281 \text{ ft})^3$  or  $1 \text{ m}^3 = (3.281)^3 \text{ ft}^3$ , the volume of this room is

$$V_{\text{room}} = 12 \text{ ft} \quad 12 \text{ ft} \quad 8.0 \text{ ft} \left[ \frac{1 \text{ m}^3}{3.281^3 \text{ ft}^3} \right] = 33 \text{ m}^3$$

A ping pong ball has a radius of about 2.0 cm, so its volume is

$$V_{\text{ball}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \ 2.0 \times 10^{-2} \ \text{m}^3 = 3.4 \times 10^{-5} \ \text{m}^3$$

The number of balls that would easily fit into the room is therefore

$$n = \frac{V_{\text{room}}}{V_{\text{ball}}} = \frac{33 \text{ m}^3}{3.4 \times 10^{-5} \text{ m}^3} = 9.7 \times 10^5 \text{ or } 10^6$$

1.30 We assume that the average person catches a cold twice a year and is sick an average of 7 days (or 1 week) each time. Thus, on average, each person is sick for 2 weeks out of each year (52 weeks). The probability that a particular person will be sick at any given time equals the percentage of time that person is sick, or

probability of sickness = 
$$\frac{2 \text{ weeks}}{52 \text{ weeks}} = \frac{1}{26}$$

The population of the Earth is approximately 6 billion. The number of people expected to have a cold on any given day is then

Number sick = population probability of sickness = 
$$6 \times 10^9 \left(\frac{1}{26}\right) = \boxed{2 \times 10^8}$$

1.31 (a) Assume that a typical intestinal tract has a length of about 7 m and average diameter of 4 cm. The estimated total intestinal volume is then

$$V_{\text{total}} = A\ell = \left(\frac{\pi d^2}{4}\right)\ell = \frac{\pi \ 0.04 \ \text{m}^2}{4} \ 7 \ \text{m} = 0.009 \ \text{m}^3$$

The approximate volume occupied by a single bacteria is

$$V_{\text{bacteria}} \sim typical length scale^3 = 10^{-6} \text{ m}^3 = 10^{-18} \text{ m}^3$$

If it is assumed that bacteria occupy one hundredth of the total intestinal volume, the estimate of the number of microorganisms in the human intestinal tract is

$$n = \frac{V_{\text{total}}/100}{V_{\text{bacteria}}} = \frac{0.009 \text{ m}^3 / 100}{10^{-18} \text{ m}^3} = 9 \times 10^{13} \text{ or } n \sim 10^{14}$$

(b) The large value of the number of bacteria estimated to exist in the intestinal tract means that they are probably not dangerous. Intestinal bacteria help digest food and provide important nutrients.

Humans and

bacteria enjoy a mutually beneficial symbiotic relationship.

1.32 A blade of grass is  $\sim 1/4$  inch wide, so we might expect each blade of grass to require at least 1/16 in<sup>2</sup> = 4.3  $\times 10^{-4}$  ft<sup>2</sup>. Since, 1 acre = 43 560 ft<sup>2</sup>, the number of blades of grass to be expected on a quarter-acre plot of land is about

$$n = \frac{\text{total area}}{\text{area per blade}} = \frac{0.25 \text{ acre} \quad 43 560 \text{ ft}^2/\text{acre}}{4.3 \times 10^{-4} \text{ ft}^2/\text{blade}}$$

=  $2.5 \times 10^7$  blades, or  $\sim 10^7$  blades

**1.33** A reasonable guess for the diameter of a tire might be 3 ft, with a circumference ( $C = 2\pi r = \pi D$  = distance travels per revolution) of about 9 ft. Thus, the total number of revolutions the tire might make is

$$n = \frac{\text{total distance traveled}}{\text{distance per revolution}} = \frac{50\ 000\ \text{mi}\ 5\ 280\ \text{ft/mi}}{9\ \text{ft/rev}} = 3 \times 10^7\ \text{rev, or}\ \boxed{\sim 10^7\ \text{rev}}$$

**1.34** Answers to this problem will vary, dependent on the assumptions one makes. This solution assumes that bacteria and other prokaryotes occupy approximately one ten-millionth of  $(10^{-7})$  of the Earth's volume, and that the density of a prokaryote, like the density of the human body, is approximately equal to that of water  $(10^3 \text{ kg/m}^3)$ .

(a)

estimated number = 
$$n = \frac{V_{\text{total}}}{V_{\text{single}}} \sim \frac{10^{-7} V_{\text{Earth}}}{V_{\text{single}}} \sim \frac{10^{-7} R_{\text{Earth}}^3}{length scale^3} \sim \frac{10^{-7} 10^6 \text{ m}^3}{10^{-6} \text{ m}^3} \sim \frac{10^{29}}{10^{29}}$$

(b)

$$m_{\text{total}} = density \ total \ volume \ \sim \ \rho_{\text{water}} \left( nV_{\text{single}}_{\text{perokaryote}} \right) = \left( 10^3 \ \frac{\text{kg}}{\text{m}^3} \right) \ 10^{29} \ 10^{-6} \ \text{m}^3 \ \sim \ \boxed{10^{14} \ \text{kg}}$$

(c) The very large mass of prokaryotes implies they are important to the biosphere. They are responsible for

fixing carbon, producing oxygen, and breaking up pollutants, among many other biological roles. Humans depend on them!

1.35 The x coordinate is found as 
$$x = r \cos \theta = 2.5 \text{ m} \cos 35^\circ = 2.0 \text{ m}$$

and the y coordinate  $y = r \sin \theta = 2.5 \text{ m} \sin 35^\circ = 1.4 \text{ m}$ 

**1.36** The *x* distance out to the fly is 2.0 m and the *y* distance up to the fly is 1.0 m. Thus, we can use the Pythagorean theorem to find the distance from the origin to the fly as,

$$d = \sqrt{x^2 + y^2} = \sqrt{2.0 \text{ m}^2 + 1.0 \text{ m}^2} = 2.2 \text{ m}$$

**1.37** The distance from the origin to the fly is r in polar coordinates, and this was found to be 2.2 m in Problem 36. The angle  $\theta$  is the angle between r and the horizontal reference line (the x axis in this case). Thus, the angle can be found as

$$\tan \theta = \frac{y}{x} = \frac{1.0 \text{ m}}{2.0 \text{ m}} = 0.50$$
 and  $\theta = \tan^{-1} 0.50 = 27^{\circ}$ 

The polar coordinates are  $r = 2.2 \text{ m and } \theta = 27^{\circ}$ .

**1.38** The *x* distance between the two points is  $|\Delta x| = |x_2 - x_1| = |-3.0 \text{ cm} - 5.0 \text{ cm}| = 8.0 \text{ cm}$  and the y distance between them is  $|\Delta y| = |y_2 - y_1| = |3.0 \text{ cm} - 4.0 \text{ cm}| = 1.0 \text{ cm}$ . The distance between them is found from the Pythagorean theorem:

$$d = \sqrt{|\Delta x|^2 + |\Delta y|^2} = \sqrt{8.0 \text{ cm}^2 + 1.0 \text{ cm}^2} = \sqrt{65 \text{ cm}^2} = 8.1 \text{ cm}$$

**1.39** Refer to the figure given in Problem 1.40 below. The Cartesian coordinates for the two given points are

$$x_1 = r_1 \cos \theta_1 = 2.00 \text{ m} \cos 50.0^\circ = 1.29 \text{ m}$$
$$x_2 = r_2 \cos \theta_2 = 5.00 \text{ m} \cos -50.0^\circ = 3.21 \text{ m}$$
$$y_1 = r_1 \sin \theta_1 = 2.00 \text{ m} \sin 50.0^\circ = 1.53 \text{ m}$$
$$y_2 = r_2 \sin \theta_2 = 5.00 \text{ m} \sin -50.0^\circ = -3.83 \text{ m}$$

The distance between the two points is then:

$$\Delta s = \sqrt{\Delta x^{2} + \Delta y^{2}} = \sqrt{1.29 \text{ m} - 3.21 \text{ m}^{2} + 1.53 \text{ m} + 3.83 \text{ m}^{2}} = 5.69 \text{ m}$$

1.40 Consider the figure shown at the right. The Cartesian coordinates for the two points are:

$$\begin{array}{ll} x_1 = r_1 \cos \theta_1 & x_2 = r_2 \cos \theta_2 \\ y_1 = r_1 \sin \theta_1 & y_2 = r_2 \sin \theta_2 \end{array}$$

The distance between the two points is the length of the hypotenuse of the shaded triangle and is given by

$$\Delta s = \sqrt{\Delta x^{2} + \Delta y^{2}} = \sqrt{x_{1} - x_{2}^{2} + y_{1} - y_{2}^{2}}$$



$$\Delta s = \sqrt{r_1^2 \cos^2 \theta_1 + r_2^2 \cos^2 \theta_2 - 2r_1r_2 \cos \theta_1 \cos \theta_2 + r_1^2 \sin^2 \theta_1 + r_2^2 \sin^2 \theta_2 - 2r_1r_2 \sin \theta_1 \sin \theta_2}$$
  
=  $\sqrt{r_1^2 \cos^2 \theta_1 + \sin^2 \theta_1 + r_2^2 \cos^2 \theta_2 + \sin^2 \theta_2 - 2r_1r_2 \cos \theta_1 \cos \theta_2 + \sin \theta_2 \sin \theta_2}$ 

y

 $(x_1, y_1)$ 

 $\Delta x = x_1$ 

 $\Delta y = y_1 - y_2$ 

 $-x_{2}$ 

x

Δs¦

 $(x_2, y_2)$ 

0

 $\theta_1$ 

$$= \sqrt{r_1^2 \cos^2 \theta_1 + \sin^2 \theta_1 + r_2^2 \cos^2 \theta_2 + \sin^2 \theta_2 - 2r_1r_2 \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2}$$

Applying the identities  $\cos^2 \theta + \sin^2 \theta = 1$  and  $\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 = \cos \theta_1 - \theta_2$ , this reduces to

$$\Delta s = \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2} = \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos \theta_1 - \theta_2}$$

1.41 (a) With 
$$a = 6.00$$
 and  $b$  being two sides of this right  
triangle having hypotenuse  $C = 9.00$  m, the Pythagorean  
theorem gives the unknown side as  
 $b = \sqrt{c^2 - a^2} = \sqrt{9.00 \text{ m}^2 - 6.00 \text{ m}^2} = 6.71 \text{ m}$   
(b)  $\tan \theta = \frac{a}{b} = \frac{6.00 \text{ m}}{6.71 \text{ m}} = 0.894$  (c)  $\sin \phi = \frac{b}{c} = \frac{6.71 \text{ m}}{9.00 \text{ m}} = 0.746$   
1.42 From the diagram,  $\cos (75.0^\circ) = d/L$   
Thus,

$$d = L \cos 75.0^\circ = 9.00 \text{ m} \cos 75.0^\circ = |2.33 \text{ m}|$$

**1.43** The circumference of the fountain is,  $C = 2\pi r$ , so the radius is

$$r = \frac{C}{2\pi} = \frac{15.0 \text{ m}}{2\pi} = 2.39 \text{ m}$$

Thus,  $\tan 55.0^\circ = \frac{h}{r} = \frac{h}{2.39 \text{ m}}$  which gives

$$h = (2.39 \text{ m}) \tan (55.0^{\circ}) = 3.41 \text{ m}$$



1.44 (a) 
$$\sin \theta = \frac{\text{side opposite}}{\text{hypotenuse}}$$
 so, side opposite (3.00 m) (sin 30.0°) = 1.50 m

(b) 
$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$
 so, adjacent side (3.00 m) (cos 30.0°) = 2.60 m

**1.45** (a) The side opposite 
$$\theta = \overline{3.00}$$
 (b) The side adjacent to  $\phi = \overline{3.00}$ 

(c) 
$$\cos \theta = \frac{4.00}{5.00} = \boxed{0.800}$$
 (d)  $\sin \phi = \frac{4.00}{5.00} = \boxed{0.800}$ 

(e) 
$$\tan \phi = \frac{4.00}{3.00} = \boxed{1.33}$$

$$c = \sqrt{5.00 \text{ m}^2 + 7.00 \text{ m}^2} = 8.60 \text{ m}$$



1.47 From the diagram given in Problem 1.46 above, it is seen that

$$\tan \theta = \left[ \frac{5.00}{7.00} \right] = 0.714 \text{ and } \theta = \tan^{-1} 0.714 = \boxed{35.5^{\circ}}$$

**1.48** (a) and (b) See the figure given below:



(c) Applying the definition of the tangent function to the large right triangle containing the 12.0° angle gives:

$$y/x = \tan 12.0^{\circ}$$
 [1]

Also, applying the definition of the tangent function to the smaller right triangle containing the 14.0° angle gives:

$$\frac{y}{x - 1.00 \text{ km}} = \tan 14.0^{\circ}$$
 [2]

(d) From equation [1] above, observe that:  $y/x = \tan 12.0^{\circ}$ 

Substituting this result into equation [2] gives:  $\frac{y \cdot \tan 12.0^{\circ}}{y - 1.00 \text{ km } \tan 12.0^{\circ}} = \tan 14.0^{\circ}$ 

Then, solving for the height of the mountain, y, yields

$$y = \frac{1.00 \text{ km } \tan 12.0^{\circ} \tan 14.0^{\circ}}{\tan 14.0^{\circ} - \tan 12.0^{\circ}} = 1.44 \text{ km} = \boxed{1.44 \times 10^3 \text{ m}}$$

**1.49** Using the sketch at the right:

$$\frac{w}{100 \text{ m}} = \tan 35.0$$
°, or  
 $w = 100 \text{ m} \tan 35.0^\circ = 70.0 \text{ m}$ 







Applying the definition of the tangent function to the large right triangle containing the angle  $\theta$  in the figure, one obtains:

$$y/x = \tan \theta$$
 [1]

Also, applying the definition of the tangent function to the small right triangle containing the angle  $\phi$  gives:

$$\frac{y}{x-d} = \tan\phi$$
<sup>[2]</sup>

Solving equation [1] for *x* and substituting the result into equation [2] yields:

$$\frac{y}{y/\tan\theta - d} = \tan\phi$$
 or  $\frac{y\cdot\tan\theta}{y - d\cdot\tan\theta} = \tan\phi$ 

The last result simplifies to

 $y \cdot \tan \theta = y \cdot \tan \phi - d \cdot \tan \theta \cdot \tan \phi$ 

Solving for *y*:  $y \tan \theta - \tan \phi = -d \cdot \tan \theta \cdot \tan \phi$  or

$$y = -\frac{d \cdot \tan \theta \cdot \tan \phi}{\tan \theta - \tan \phi} = \frac{d \cdot \tan \theta \cdot \tan \phi}{\tan \phi - \tan \theta}$$

1.51 (a) Given that a  $\propto F/M$ , we have  $F \propto ma$  Therefore, the units of force are those of ma,

$$[F] = [ma] = [m][a] = M L/T^2 = M L T^{-2}$$

(b) 
$$[F] = M\left(\frac{L}{T^2}\right) = \frac{M \cdot L}{T^2}$$
 so newton  $= \boxed{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}}$ 

**1.52** (a) 
$$1 \frac{\text{mi}}{\text{h}} = \left(1 \frac{\text{mi}}{\text{h}}\right) \left(\frac{1.609 \text{ km}}{1 \text{ mi}}\right) = 1.609 \frac{\text{km}}{\text{h}}$$

(b) 
$$v_{\text{max}} = 55 \frac{\text{mi}}{\text{h}} = \left(55 \frac{\text{mi}}{\text{h}}\right) \left(\frac{1.609 \text{ km/h}}{1 \text{ mi/h}}\right) = \boxed{88 \frac{\text{km}}{\text{h}}}$$

(c) 
$$\Delta v_{\text{max}} = 65 \ \frac{\text{mi}}{\text{h}} - 55 \ \frac{\text{mi}}{\text{h}} = \left(10 \ \frac{\text{mi}}{\text{h}}\right) \left(\frac{1.609 \ \text{km/h}}{1 \ \text{mi/h}}\right) = \left[16 \ \frac{\text{km}}{\text{h}}\right]$$

**1.53** (a) Since  $1 \text{ m} = 10^2 \text{ cm}$ , then  $1 \text{ m}^3 = 1 \text{ m}^3 = 10^2 \text{ cm}^3 = 10^2^3 \text{ cm}^3 = 10^6 \text{ cm}^3$ , giving

mass = density volume = 
$$\left(\frac{1.0 \times 10^{-3} \text{ kg}}{1.0 \text{ cm}^3}\right) 1.0 \text{ m}^3$$
  
=  $\left(1.0 \times 10^{-3} \frac{\text{kg}}{\text{cm}^3}\right) 1.0 \text{ m}^3 \left(\frac{10^6 \text{ cm}^3}{1 \text{ m}^3}\right) = \boxed{1.0 \times 10^3 \text{ kg}}$ 

(b) As rough calculation, treat as if 100% water.

cell: mass = density × volume = 
$$\left(10^3 \ \frac{\text{kg}}{\text{m}^3}\right) \frac{4}{3} \pi \ 0.50 \times 10^{-6} \text{ m}^3 = \boxed{5.2 \times 10^{-16} \text{ kg}}$$

kidney:

mass = density × volume = 
$$\rho\left(\frac{4}{3}\pi r^3\right) = \left(10^3 \frac{\text{kg}}{\text{m}^3}\right)\frac{4}{3}\pi 4.0 \times 10^{-2} \text{ m}^3 = \boxed{0.27 \text{ kg}}$$

fly: mass = density × volume = density  $\pi r^2 h$ 

$$= \left(10^3 \ \frac{\text{kg}}{\text{m}^3}\right) \pi \ 1.0 \times 10^{-3} \text{ m}^2 \ 4.0 \times 10^{-3} \text{ m} = \boxed{1.3 \times 10^{-5} \text{ kg}}$$

## **1.54** Assume an average of 1 can per person each week and a population of 300 million.

number cans/year = 
$$\left(\frac{\text{number cans/person}}{\text{week}}\right)$$
 population weeks/year  
 $\approx \left(1 \frac{\text{can/person}}{\text{week}}\right) 3 \times 10^8$  people 52 weeks/yr  
 $\approx 2 \times 10^{10} \text{ cans/yr}, \text{ or } \boxed{\sim 10^{10} \text{ cans/yr}}$ 

number of tons = weight/can number cans /year

$$\approx \left[ \left( 0.5 \ \frac{\text{oz}}{\text{can}} \right) \left( \frac{1 \text{ lb}}{16 \text{ oz}} \right) \left( \frac{1 \text{ ton}}{2 \text{ 000 lb}} \right) \right] \left( 2 \times 10^{10} \ \frac{\text{can}}{\text{yr}} \right)$$
$$\approx 3 \times 10^5 \text{ ton/yr, or } \boxed{\sim 10^5 \text{ ton/yr}}$$

Assumes an average weight of 0.5 oz of aluminum per can.

**1.55** The term *s* has dimensions of L, *a* has dimension of  $LT^{-2}$ , and *t* has dimensions of T. Therefore, the equaton,  $s = k a^m t^n$  with *k* being dimensionless, has dimensions of

 $L = (LT^{-2})^m (T)^n$  or  $L^1T^0 = L^m T^{n-2m}$ 

The powers of L and T must be the same on each side of the equation. Therefore,  $L^1 = L^m$ and  $\underline{m = 1}$ 

Likewise, equating powers of T, we see that n-2m = 0, or n = 2m = 2.

Dimensional analysis cannot determine the value of  $k_{i}$ , a dimensionless constant.

**1.56** Assume the tub measures 1.3 m by 0.5 m by 0.3 m

(a) It then has a volume  $V = 0.2 \text{ m}^3$  and contains a mass of water

$$m_{\text{water}} = \rho_{\text{water}} V_{\text{tub}} = 10^3 \text{ kg/m}^3 \quad 0.2 \text{ m}^3 = 200 \text{ kg, or} \sim 10^2 \text{ kg}$$

(b) Pennies are now mostly zinc, but consider copper pennies filling 80% of the volume of the tub. Their mass is

$$m_{\text{pennies}} = \rho_{\text{copper}} \ 0.8V_{\text{tub}} = 8.93 \times 10^3 \text{ kg/m}^3 \ \left[ 0.80 \ 0.2 \ \text{m}^3 \ \right] = 1 \ 400 \ \text{kg}, \text{ or } \ \overline{-10^3 \ \text{kg}}$$

1.57 The volume of oil equals 
$$V = \frac{\text{mass}}{\text{density}} = \frac{9.00 \times 10^{-7} \text{ kg}}{918 \text{ kg/m}^3} = 9.80 \times 10^{-10} \text{ m}^3$$

If the slick is a circle of radius r and thickness equal to the diameter, d, of a molecule,

 $V = d \pi r^2$  = thickness of slick area of oil slick , where r = 0.418 m

Thus, 
$$d = \frac{F}{\pi r^2} = \frac{9.80 \times 10^{-10} \text{ m}^3}{\pi \ 0.418 \text{ m}^2} = 1.78 \times 10^{-9} \text{ m}, \text{ or } \boxed{\sim 10^{-9} \text{ m}}$$

**1.58** (a) For a sphere,  $A = 4\pi R^2$ . In this case, the radius of the second sphere is twice that of the first, or  $R_2 = 2R_1$ .

Hence, 
$$\frac{A_2}{A_1} = \frac{4\pi R_2^2}{4\pi R_1^2} = \frac{R_2^2}{R_1^2} = \frac{2R_1^2}{R_1^2} = \boxed{\frac{4}{4\pi R_1^2}}$$

(b) For a sphere, the volume is  $V = \frac{4}{3}\pi R^3$ 

Thus, 
$$\frac{V_2}{V_1} = \frac{4/3 \ \pi R_2^3}{4/3 \ \pi R_1^3} = \frac{R_2^3}{R_1^3} = \frac{2R_1^3}{R_1^3} = \boxed{8}$$

**1.59** The number of tuners is found by dividing the number of residents of the city by the number of residents serviced by one tuner. We shall assume 1 tuner per 10,000 residents and a population of 8 million. Thus, number of tuners = *population frequency of occurrence* =  $8 \times 10^6$  residents  $\left(\frac{1 \text{ tuner}}{10^4 \text{ residents}}\right) = 8 \times 10^2$  tuners or  $\sim 10^3$  tuners

**1.60** (a) The amount paid per year would be

annual amount = 
$$\left(1\ 000\ \frac{\text{dollars}}{\text{s}}\right)\left(\frac{8.64 \times 10^4 \text{ s}}{1.00 \text{ day}}\right)\left(\frac{365.25 \text{ days}}{\text{yr}}\right) = 3.16 \times 10^{10}\ \frac{\text{dollars}}{\text{yr}}$$

Therefore, it would take  $\frac{9 \times 10^{12} \text{ dollars}}{3.16 \times 10^{10} \text{ dollars/yr}} = 3 \times 10^2 \text{ yr}, \text{ or } [\sim 10^2 \text{ yr}]$ 

(b) The circumference of the Earth at the equator is

 $C = 2\pi r = 2\pi \ 6.378 \times 10^6 \ \text{m} = 4.007 \times 10^7 \ \text{m}$ 

The length of one dollar bill is 0.155 m so that the length of nine trillion bills is

 $\ell = \left(0.155 \ \frac{\text{m}}{\text{dollar}}\right) 9 \times 10^{12} \text{ dollars} = 1 \times 10^{12} \text{ m}$ . Thus, the nine trillion dollars would

encircle the Earth

$$n = \frac{\ell}{C} = \frac{1 \times 10^{12} \text{ m}}{4.007 \times 10^7 \text{ m}} = 2 \times 10^4, \text{ or } \boxed{\sim 10^4 \text{ times}}$$

**1.61** (a) 
$$1 \text{ yr} = \left( \sum_{i=1}^{\infty} \sqrt{\frac{365.2 \text{ days}}{1 \sum_{i=1}^{\infty}}} \right) \left( \frac{8.64 \times 10^4 \text{ s}}{1 \text{ day}} \right) = \boxed{3.16 \times 10^7 \text{ s}}$$

(b) Consider a segment of the surface of the moon which has an area of  $1 \text{ m}^2$  and a depth of 1 m. When filled

with meteorites, each having a diameter  $10^{-6}$  m, the number of meteorites along each edge of this box is

$$n = \frac{\text{length of an edge}}{\text{meteorite diameter}} = \frac{1 \text{ m}}{10^{-6} \text{ m}} = 10^{6}$$

The total number of meteorites in the filled box is then

$$N = n^3 = 10^{6-3} = 10^{18}$$

At the rate of 1 meteorite per second, the time to fill the box is

$$t = 10^{18} \text{ s} = 10^{18} \text{ s} \left(\frac{1 \text{ y}}{3.16 \times 10^7 \text{ s}}\right) = 3 \times 10^{10} \text{ yr}, \text{ or } \sim 10^{10} \text{ yr}$$

1.62 We will assume that, on average, 1 ball will be lost per hitter, that there will be about 10 hitters per inning, a game has 9 innings, and the team plays 81 home games per season. Our estimate of the number of game balls needed per season is then

number of balls needed = number lost per hitter number hitters/game home games/year

$$= 1 \text{ ball per hitter } \left[ \left( 10 \frac{\text{hitters}}{\text{inning}} \right) \left( 9 \frac{\text{innings}}{\text{game}} \right) \right] \left( 81 \frac{\text{games}}{\text{year}} \right)$$
$$= 7 300 \frac{\text{balls}}{\text{year}} \text{ or } \left[ \sim 10^4 \frac{\text{balls}}{\text{year}} \right]$$