PROBLEM SOLUTIONS

13.1 (a) Taking to the right as positive, the spring force acting on the block at the instant of release is

$$F_s = -kx_i = -\ 130\ \text{N/m}\quad +0.13\ \text{m}\quad = -17\ \text{N} \quad \text{or} \quad \boxed{17\ \text{N to the left}}$$

(b) At this instant, the acceleration is

$$a = \frac{F_s}{m} = \frac{-17\ \text{N}}{0.60\ \text{kg}} = -28\ \text{m/s}^2 \quad \text{or} \quad \boxed{a = 28\ \text{m/s}^2\ \text{to the left}}$$

13.2 When the object comes to equilibrium (at distance y_0 *below* the unstretched position of the end of the spring), $\Sigma F_y = -k\ \ -y_0\ \ - mg = 0$ and the force constant is

$$k = \frac{mg}{y_0} = \frac{4.25\ \text{kg}\quad 9.80\ \text{m/s}^2}{2.62 \times 10^{-2}\ \text{m}} = 1.59 \times 10^3\ \text{N} = \boxed{1.59\ \text{kN}}$$

13.3 (a) Since the collision is perfectly elastic, the ball will rebound to the height of 4.00 m before coming to rest momentarily. It will then repeat this motion over and over again with a regular period.

(b) From $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, with $v_{0y} = 0$, the time required for the ball to reach the ground is

$$t = \sqrt{\frac{2\ \ \Delta y}{a_y}} = \sqrt{\frac{2\ \ -4.00\ \text{m}}{-9.80\ \text{m/s}^2}} = 0.904\ \text{s}$$

This is one-half of the time for a complete cycle of the motion. Thus, the period is $T = \boxed{1.81\ \text{s}}$.

(c) $\boxed{\text{No}}$. The net force acting on the object is a constant given by $F = -mg$ (except when it is contact with the ground). This is not in the form of Hooke's law.

13.4 (a) The spring constant is

$$k = \frac{|F_s|}{x} = \frac{mg}{x} = \frac{50 \text{ N}}{5.0 \times 10^{-2} \text{ m}} = 1.0 \times 10^3 \text{ N/m}$$

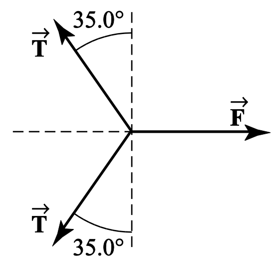
$$F = |F_s| = kx = 1.0 \times 10^3 \text{ N/m} \quad 0.11 \text{ m} = \boxed{1.1 \times 10^2 \text{ N}}$$

(b) The graph will be a $\boxed{\text{straight line passing through the origin}}$ with a slope equal to $k = 1.0 \times 10^3 \text{ N/m}$.

13.5 When the system is in equilibrium, the tension in the spring $F = k|x|$ must equal the weight of the object. Thus, $k|x| = mg$ giving

$$m = \frac{k|x|}{g} = \frac{47.5 \text{ N} \quad 5.00 \times 10^{-2} \text{ m}}{9.80 \text{ m/s}^2} = \boxed{0.242 \text{ kg}}$$

13.6 (a) The free-body diagram of the point in the center of the string is given at the right. From this, we see that



$$\Sigma F_x = 0 \Rightarrow F - 2T \sin 35.0° = 0$$

or $$T = \frac{F}{2 \sin 35.0°} = \frac{375 \text{ N}}{2 \sin 35.0°} = \boxed{327 \text{ N}}$$

(b) Since the bow requires an applied horizontal force of 375 N to hold the string at 35.0° from the vertical, the tension in the spring must be 375 N when the spring is stretched 30.0 cm. Thus, the spring constant is

$$k = \frac{F}{x} = \frac{375 \text{ N}}{0.300 \text{ m}} = \boxed{1.25 \times 10^3 \text{ N/m}}$$

13.7 (a) When the block comes to equilibrium, $\Sigma F_y = -ky_0 - mg = 0$ giving

$$y_0 = -\frac{mg}{k} = -\frac{10.0 \text{ kg} \quad 9.80 \text{ m/s}^2}{475 \text{ N/m}} = -0.206 \text{ m}$$

or the equilibrium position is $\boxed{0.206 \text{ m}}$ *below* the unstretched position of the lower end of the spring.

(b) When the elevator (and everything in it) has an upward acceleration of $a = 2.00 \text{ m/s}^2$, applying Newton's second law to the block gives

$$\Sigma F_y = -k\left(y_0 + y\right) - mg = ma_y \qquad \text{or} \qquad \Sigma F_y = \left(-ky_0 - mg\right) - ky = ma_y$$

where $y = 0$ at the equilibrium position of the block. Since $-ky_0 - mg = 0$ [see part (a)], this becomes $-ky = ma$ and the new position of the block is

$$y = \frac{ma_y}{-k} = \frac{\left(10.0 \text{ kg}\right)\left(+2.00 \text{ m/s}^2\right)}{-475 \text{ N/m}} = -4.21 \times 10^{-2} \text{ m} = \boxed{-4.21 \text{ cm}}$$

or 4.21 cm below the equilibrium position.

(c) When the cable breaks, the elevator and its contents will be in free-fall with $a_y = -g$. The new "equilibrium" position of the block is found from $\Sigma F_y = -ky_0' - mg = m\left(-g\right)$, which yields $y_0' = 0$. When the cable snapped, the block was at rest relative to the elevator at distance $y_0 + y = 0.206 \text{ m} + 0.042\,1 \text{ m} = 0.248 \text{ m}$ below the new "equilibrium" position. Thus, while the elevator is in free-fall, the block will oscillate with $\boxed{\text{amplitude} = 0.248 \text{ m}}$ about the new "equilibrium" position, which is the unstretched position of the spring's lower end.

13.8 (a) When the gun is fired, the elastic potential energy initially stored in the spring is transformed into kinetic energy of the projectile. Thus, it is necessary to have

$$\frac{1}{2}kx_0^2 = \frac{1}{2}mv_0^2 \qquad \text{or} \qquad k = \frac{mv_0^2}{x_0^2} = \frac{\left(3.00 \times 10^{-3} \text{ kg}\right)\left(45.0 \text{ m/s}\right)^2}{\left(8.00 \times 10^{-2} \text{ m}\right)^2} = \boxed{949 \text{ N/m}}$$

(b) The magnitude of the force required to compress the spring 8.00 cm and load the gun is

$$F_s = k\left|x\right| = \left(949 \text{ N/m}\right)\left(8.00 \times 10^{-2} \text{ m}\right) = \boxed{75.9 \text{ N}}$$

13.9 (a) Assume the rubber bands obey Hooke's law. Then, the force constant of each band is

$$k = \frac{F_s}{x} = \frac{15 \text{ N}}{1.0 \times 10^{-2} \text{ m}} = 1.5 \times 10^3 \text{ N/m}$$

Thus, when both bands are stretched 0.20 m, the total elastic potential energy is

$$PE_s = 2\left(\frac{1}{2}kx^2\right) = \left(1.5 \times 10^3 \text{ N/m}\right)\left(0.20 \text{ m}\right)^2 = \boxed{60 \text{ J}}$$

(b) Conservation of mechanical energy gives $\left(KE + PE_s\right)_f = \left(KE + PE_s\right)_i$, or

$$\frac{1}{2}mv^2 + 0 = 0 + 60 \text{ J}, \text{ so } v = \sqrt{\frac{2\left(60 \text{ J}\right)}{50 \times 10^{-3} \text{ kg}}} = \boxed{49 \text{ m/s}}$$

13.10 (a) $k = \dfrac{F_{\text{max}}}{x_{\text{max}}} = \dfrac{230 \text{ N}}{0.400 \text{ m}} = \boxed{575 \text{ N/m}}$

(b) $\textit{work done} = PE_s = \dfrac{1}{2}kx^2 = \dfrac{1}{2}\left(575 \text{ N/m}\right)\left(0.400\right)^2 = \boxed{46.0 \text{ J}}$

13.11 From conservation of mechanical energy,

$$\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i \text{ or } 0 + mgh_f + 0 = 0 + 0 + \frac{1}{2}kx_i^2$$

giving

$$k = \frac{2\,mgh_f}{x_i^2} = \frac{2\left(0.100 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)\left(0.600 \text{ m}\right)}{\left(2.00 \times 10^{-2} \text{ m}\right)^2} = \boxed{2.94 \times 10^3 \text{ N/m}}$$

13.12 Conservation of mechanical energy, $(KE + PE_g + PE_s)_f = (KE + PE_g + PE_s)_i$, gives

$\frac{1}{2}mv_i^2 + 0 + 0 = 0 + 0 + \frac{1}{2}kx_f^2$, or

$$v_i = \sqrt{\frac{k}{m}}\, x_i = \sqrt{\frac{5.00 \times 10^6 \text{ N/m}}{1000 \text{ kg}}} \left(3.16 \times 10^{-2} \text{ m}\right) = \boxed{2.23 \text{ m/s}}$$

13.13 An unknown quantity of mechanical energy is converted into internal energy during the collision. Thus, we apply conservation of momentum from just before to just after the collision and obtain $mv_i + M(0) = (M + m)V$, or the speed of the block and embedded bullet just after collision is

$$V = \left(\frac{m}{M + m}\right) v_i = \left(\frac{10.0 \times 10^{-3} \text{ kg}}{2.01 \text{ kg}}\right) \left(300 \text{ m/s}\right) = 1.49 \text{ m/s}$$

Now, we use conservation of mechanical energy from just after collision until the block comes to rest. This gives $0 + \frac{1}{2}kx_f^2 = \frac{1}{2}\left(M + m\right)V^2$, or

$$x_f = V\sqrt{\frac{M + m}{k}} = \left(1.49 \text{ m/s}\right)\sqrt{\frac{2.01 \text{ kg}}{19.6 \text{ N/m}}} = \boxed{0.477 \text{ m}}$$

13.14 (a) At either of the turning points, $x = \pm A$, the constant total energy of the system is momentarily stored as elastic potential energy in the spring. Thus, $\boxed{E = kA^2/2}$.

(b) When the object is distance x from the equilibrium position, the elastic potential energy is $PE_s = kx^2/2$ and the kinetic energy is $KE = mv^2/2$. At the position where $KE = 2PE_s$, is it necessary that

$$\frac{1}{2}mv^2 = 2\left(\frac{1}{2}kx^2\right) \qquad \text{or} \qquad \boxed{\frac{1}{2}mv^2 = kx^2}$$

(c) When $KE = 2PE$, conservation of energy gives $E = KE + PE_s = 2\left(PE_s\right) + PE_s = 3PE_s$, or

$$\frac{1}{2}kA^2 = 3\left(\frac{1}{2}kx^2\right) \quad \Rightarrow \quad x = \pm\sqrt{\frac{\cancel{k}A^2/\cancel{2}}{3\cancel{k}/\cancel{2}}} \qquad \text{or} \qquad \boxed{x = \pm\frac{A}{\sqrt{3}}}$$

13.15 (a) At maximum displacement from equilibrium, all of the energy is in the form of elastic potential energy, giving $E = kx_{max}^2/2$, and

$$k = \frac{2E}{x_{max}^2} = \frac{2(47.0\text{ J})}{(0.240\text{ m})^2} = \boxed{1.63 \times 10^3\text{ N/m}}$$

(b) At the equilibrium position ($x = 0$), the spring is momentarily in its relaxed state and $PE_s = 0$, so all of the energy is in the form of kinetic energy. This gives

$$KE\big|_{x=0} = \frac{1}{2}mv_{max}^2 = E = \boxed{47.0\text{ J}}$$

(c) If, at the equilibrium position, $v = v_{max} = 3.45$ m/s, the mass of the block is

$$m = \frac{2E}{v_{max}^2} = \frac{2(47.0\text{ J})}{(3.45\text{ m/s})^2} = 7.90\text{ kg}$$

(d) At any position, the constant total energy is, $E = KE + PE_s = mv^2/2 + kx^2/2$, so at $x = 0.160$ m

$$v = \sqrt{\frac{2E - kx^2}{m}} = \sqrt{\frac{2(47.0\text{ J}) - (1.63 \times 10^3\text{ N/m})(0.160\text{ m})^2}{7.90\text{ kg}}} = \boxed{2.57\text{ m/s}}$$

(e) At $x = 0.160$ m, where $v = 2.57$ m/s, the kinetic energy is

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(7.90\text{ kg})(2.57\text{ m/s})^2 = \boxed{26.1\text{ J}}$$

(f) At $x = 0.160$ m, where $KE = 26.1$ J, the elastic potential energy is

$$PE_s = E - KE = 47.0\text{ J} - 26.1\text{ J} = \boxed{20.9\text{ J}}$$

or alternately,

$$PE_s = \frac{1}{2}kx^2 = \frac{1}{2}(1.63 \times 10^3\text{ N/m})(0.160\text{ m})^2 = \boxed{20.9\text{ J}}$$

(g) At the first turning point (for which $x < 0$ since the block started from rest at $x = +0.240$ m and has passed through the equilibrium at $x = 0$) all of the remaining energy is in the form of elastic potential energy, so

$$\frac{1}{2}kx^2 = E - E_{loss} = 47.0 \text{ J} - 14.0 \text{ J} = 33.0 \text{ J}$$

and

$$x = -\sqrt{\frac{2(33.0 \text{ J})}{k}} = -\sqrt{\frac{2(33.0 \text{ J})}{1.63 \times 10^3 \text{ N/m}}} = \boxed{-0.201 \text{ m}}$$

13.16 (a) $F = k|x| = (83.8 \text{ N/m})(5.46 \times 10^{-2} \text{ m}) = \boxed{4.58 \text{ N}}$

(b) $E = PE_s = \frac{1}{2}kx^2 = \frac{1}{2}(83.8 \text{ N/m})(5.46 \times 10^{-2} \text{ m})^2 = \boxed{0.125 \text{ J}}$

(c) While the block was held stationary at $x = 5.46$ cm, $\Sigma F_x = -F_s + F = 0$, or the spring force was equal in magnitude and oppositely directed to the applied force. When the applied force is suddenly removed, there is a net force $F_s = 4.58$ N directed toward the equilibrium position acting on the block. This gives the block an acceleration having magnitude

$$|a| = \frac{F_s}{m} = \frac{4.58 \text{ N}}{0.250 \text{ kg}} = \boxed{18.3 \text{ m/s}^2}$$

(d) At the equilibrium position, $PE_s = 0$, so the block has kinetic energy $KE = E = 0.125$ J and speed

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.125 \text{ J})}{0.250 \text{ kg}}} = \boxed{1.00 \text{ m/s}}$$

(e) If the surface was rough, the block would spend energy overcoming a retarding friction force as it moved toward the equilibrium position, causing it to arrive at that position with $\boxed{\text{a lower speed}}$ than that computed above. Computing a number value for this lower speed requires $\boxed{\text{knowledge of the coefficient of friction}}$ between the block and surface.

13.17 From conservation of mechanical energy,

$$\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$$

we have $\frac{1}{2}mv^2 + 0 + \frac{1}{2}kx^2 = 0 + 0 + \frac{1}{2}kA^2$, or

$$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$$

(a) The speed is a maximum at the equilibrium position, $x = 0$.

$$v_{max} = \sqrt{\frac{k}{m}A^2} = \sqrt{\frac{19.6\text{ N/m}}{0.40\text{ kg}}\left(0.040\text{ m}\right)^2} = \boxed{0.28\text{ m/s}}$$

(b) When $x = -0.015$ m,

$$v = \sqrt{\frac{19.6\text{ N/m}}{0.40\text{ kg}}\left[\left(0.040\text{ m}\right)^2 - \left(-0.015\text{ m}\right)^2\right]} = \boxed{0.26\text{ m/s}}$$

(c) When $x = +0.015$ m,

$$v = \sqrt{\frac{19.6\text{ N/m}}{0.40\text{ kg}}\left[\left(0.040\text{ m}\right)^2 - \left(+0.015\text{ m}\right)^2\right]} = \boxed{0.26\text{ m/s}}$$

(d) If $v = \frac{1}{2}v_{max}$, then $\sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \frac{1}{2}\sqrt{\frac{k}{m}A^2}$

This gives $A^2 - x^2 = A^2/4$, or $x = A\sqrt{3}/2 = \left(4.0\text{ cm}\right)\sqrt{3}/2 = \boxed{3.5\text{ cm}}$.

13.18 (a) $KE = 0$ at $x = A$, so $E = KE + PE_s = 0 + \frac{1}{2}kA^2$, or the total energy is

$$E = \frac{1}{2}kA^2 = \frac{1}{2}\left(250 \text{ N/m}\right)\left(0.035 \text{ m}\right)^2 = \boxed{0.15 \text{ J}}$$

(b) The maximum speed occurs at the equilibrium position where $PE_s = 0$. Thus, $E = \frac{1}{2}mv_{max}^2$, or

$$v_{max} = \sqrt{\frac{2E}{m}} = A\sqrt{\frac{k}{m}} = \left(0.035 \text{ m}\right)\sqrt{\frac{250 \text{ N/m}}{0.50 \text{ kg}}} = \boxed{0.78 \text{ m/s}}$$

(c) The acceleration is $a = \Sigma F/m = -kx/m$. Thus, $a = a_{max}$ at $x = -x_{max} = -A$.

$$a_{max} = \frac{-k\left(-A\right)}{m} = \left(\frac{k}{m}\right)A = \left(\frac{250 \text{ N/m}}{0.50 \text{ kg}}\right)\left(0.035 \text{ m}\right) = \boxed{18 \text{ m/s}^2}$$

13.19 The maximum speed occurs at the equilibrium position and is

$$v_{max} = \sqrt{\frac{k}{m}}\,A$$

Thus,

$$m = \frac{kA^2}{v_{max}^2} = \frac{\left(16.0 \text{ N/m}\right)\left(0.200 \text{ m}\right)^2}{\left(0.400 \text{ m/s}\right)^2} = 4.00 \text{ kg},$$

and

$$F_g = mg = \left(4.00 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right) = \boxed{39.2 \text{ N}}$$

13.20 $$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \sqrt{\left(\frac{10.0 \text{ N/m}}{50.0 \times 10^{-3} \text{ kg}}\right)\left[\left(0.250 \text{ m}\right)^2 - \left(0.125 \text{ m}\right)^2\right]} = \boxed{3.06 \text{ m/s}}$$

13.21 (a) The motion is simple harmonic because the tire is rotating with constant velocity and you are looking at the uniform circular motion of the “bump” projected on a plane perpendicular to the tire.

(b) Note that the tangential speed of a point on the rim of a rolling tire is the same as the translational speed of the axle. Thus, $v_t = v_{car} = 3.00 \text{ m/s}$ and the angular velocity of the tire is

$$\omega = \frac{v_t}{r} = \frac{3.00 \text{ m/s}}{0.300 \text{ m}} = 10.0 \text{ rad/s}$$

Therefore, the period of the motion is

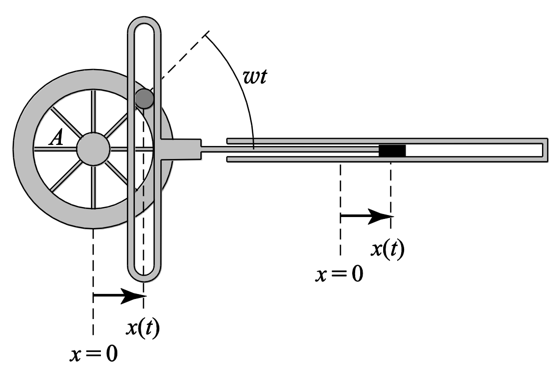
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{10.0 \text{ rad/s}} = \boxed{0.628 \text{ s}}$$

13.22 (a) $v_t = \dfrac{2\pi r}{T} = \dfrac{2\pi\ 0.200 \text{ m}}{2.00 \text{ s}} = \boxed{0.628 \text{ m/s}}$

(b) $f = \dfrac{1}{T} = \dfrac{1}{2.00 \text{ s}} = \boxed{0.500 \text{ Hz}}$

(c) $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{2.00 \text{ s}} = \boxed{3.14 \text{ rad/s}}$

13.23 The angle of the crank pin is $\theta = \omega t$. Its x-coordinate is $x = A\cos\theta = A\cos\omega t$ where A is the distance from the center of the wheel to the crank pin. This is of the correct form to describe simple harmonic motion. Hence, one must conclude that the motion is indeed simple harmonic.



13.24 The period of vibration for an object-spring system is $T = 2\pi\sqrt{m/k}$. Thus, if T = 0.223 s and m = 35.4 g = 35.4 × 10^{-3} kg, the force constant of the spring is

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2\ 35.4 \times 10^{-3} \text{ kg}}{0.223 \text{ s}^{\ 2}} = \boxed{28.1 \text{ N/m}}$$

13.25 The spring constant is found from

$$k = \frac{F_s}{x} = \frac{mg}{x} = \frac{0.010\ \text{kg}\quad 9.80\ \text{m/s}^2}{3.9 \times 10^{-2}\ \text{m}} = 2.5\ \text{N/m}$$

When the object attached to the spring has mass m = 25 g, the period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.025\ \text{kg}}{2.5\ \text{N/m}}} = \boxed{0.63\ \text{s}}$$

13.26 The springs compress 0.80 cm when supporting an additional load of m = 320 kg. Thus, the spring constant is

$$k = \frac{mg}{x} = \frac{320\ \text{kg}\quad 9.80\ \text{m/s}^2}{0.80 \times 10^{-2}\ \text{m}} = 3.9 \times 10^5\ \text{N/m}$$

When the empty car, $M = 2.0 \times 10^3$ kg, oscillates on the springs, the frequency will be

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{3.9 \times 10^5\ \text{N/m}}{2.0 \times 10^3\ \text{kg}}} = \boxed{2.2\ \text{Hz}}$$

13.27 (a) The period of oscillation is $T = 2\pi\sqrt{m/k}$ where k is the spring constant and m is the mass of the object attached to the end of the spring. Hence,

$$T = 2\pi\sqrt{\frac{0.250\ \text{kg}}{9.5\ \text{N/m}}} = \boxed{1.0\ \text{s}}$$

(b) If the cart is released from rest when it is 4.5 cm from the equilibrium position, the amplitude of oscillation will be $A = 4.5$ cm $= 4.5 \times 10^{-2}$ m. The maximum speed is then given by

$$v_{\text{max}} = A\omega = A\sqrt{\frac{k}{m}} = 4.5 \times 10^{-2}\ \text{m}\ \sqrt{\frac{9.5\ \text{N/m}}{0.250\ \text{kg}}} = \boxed{0.28\ \text{m/s}}$$

(c) When the cart is 14 cm from the left end of the track, it has a displacement of x = 14 cm – 12 cm = 2.0 cm from the equilibrium position. The speed of the cart at this distance from equilibrium is

$$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \sqrt{\frac{9.5\ \text{N/m}}{0.250\ \text{kg}}\left[\left(0.045\ \text{m}\right)^2 - \left(0.020\ \text{m}\right)^2\right]} = \boxed{0.25\ \text{m/s}}$$

13.28 The general expression for the position as a function of time for an object undergoing simple harmonic motion with $x = 0$ at $t = 0$ is $x = A\sin\left(\omega t\right)$. Thus, if $x = \left(5.2\ \text{cm}\right)\sin\left(8.0\pi \cdot t\right)$, we have that the amplitude is $A = 5.2$ cm and the angular frequency is $\omega = 8.0\pi\ \text{rad/s}$.

(a) The period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{8.0\pi\ \text{s}^{-1}} = \boxed{0.25\ \text{s}}$$

(b) The frequency of motion is

$$f = \frac{1}{T} = \frac{1}{0.25\ \text{s}} = 4.0\ \text{s}^{-1} = \boxed{4.0\ \text{Hz}}$$

(c) As discussed above, the amplitude of the motion is $\boxed{A = 5.2\ \text{cm}}$

(d) *Note:* For this part, your calculator should be set to operate in *radians mode*.

If $x = 2.6$ cm, then

$$\omega t = \sin^{-1}\left(\frac{x}{A}\right) = \sin^{-1}\left(\frac{2.6\ \text{cm}}{5.2\ \text{cm}}\right) = \sin^{-1}\left(0.50\right) = 0.52\ \text{radians}$$

and

$$t = \frac{0.52\ \text{rad}}{\omega} = \frac{0.52\ \text{rad}}{8.0\pi\ \text{rad/s}} = 2.1\times 10^{-2}\ \text{s} = 21\times 10^{-3}\ \text{s} = \boxed{21\ \text{ms}}$$

13.29 (a) At the equilibrium position, the total energy of the system is in the form of kinetic energy and $mv_{\max}^2/2 = E$, so the maximum speed is

$$v_{max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\ 5.83\ \text{J}}{0.326\ \text{kg}}} = \boxed{5.98\ \text{m/s}}$$

(b) The period of an object-spring system is $T = 2\pi\sqrt{m/k}$, so the force constant of the spring is

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2\ 0.326\ \text{kg}}{0.250\ \text{s}\ ^2} = \boxed{206\ \text{N/m}}$$

(c) At the turning points, $x = \pm A$, the total energy of the system is in the form of elastic potential energy, or $E = kA^2/2$, giving the amplitude as

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2\ 5.83\ \text{J}}{206\ \text{N/m}}} = \boxed{0.238\ \text{m}}$$

13.30 For a system executing simple harmonic motion, the total energy may be written as $E = KE + PE_s = \frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$, where A is the amplitude and v_{max} is the speed at the equilibrium position. Observe from this expression, that we may write $v_{max}^2 = kA^2/m$.

(a) If $v = \frac{1}{2}v_{max}$, then $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}mv_{max}^2$ becomes

$$\cancel{\frac{1}{2}}m\left(\frac{v_{max}^2}{4}\right) + \cancel{\frac{1}{2}}kx^2 = \cancel{\frac{1}{2}}mv_{max}^2$$

and gives

$$x^2 = \frac{3}{4}\left(\frac{m}{k}\right)v_{max}^2 = \frac{3}{4}\left(\frac{m}{k}\right)\left[\frac{k}{m}A^2\right] = \frac{3A^2}{4}$$

or

$$\boxed{x = \pm\frac{A\sqrt{3}}{2}}$$

(b) If the elastic potential energy is $PE_s = E/2$, we have

$$\frac{1}{2}kx^2 = \frac{1}{2}\left(\frac{1}{2}kA^2\right) \qquad \text{or} \qquad x^2 = \frac{A^2}{2} \qquad \text{and} \qquad \boxed{x = \pm\frac{A}{\sqrt{2}}}$$

13.31 (a) At $t = 3.50$ s,

$$F = -kx = -\left(5.00\ \frac{\text{N}}{\text{m}}\right)(3.00\ \text{m})\cos\left[\left(1.58\ \frac{\text{rad}}{\text{s}}\right)(3.50\ \text{s})\right] = -11.0\ \text{N},$$

or $F = \boxed{11.0\ \text{N directed to the left}}$

(b) The angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5.00\ \text{N/m}}{2.00\ \text{kg}}} = 1.58\ \text{rad/s}$$

and the period of oscillation is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.58\ \text{rad/s}} = 3.97\ \text{s}.$$

Hence the number of oscillations made in 3.50 s is

$$N = \frac{\Delta t}{T} = \frac{3.50\ \text{s}}{3.97\ \text{s}} = \boxed{0.881}$$

13.32 (a) $k = \dfrac{F}{x} = \dfrac{7.50\ \text{N}}{3.00 \times 10^{-2}\ \text{m}} = \boxed{250\ \text{N/m}}$

(b) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{250\ \text{N/m}}{0.500\ \text{kg}}} = \boxed{22.4\ \text{rad/s}}$

$$f = \frac{\omega}{2\pi} = \frac{22.4\ \text{rad/s}}{2\pi} = \boxed{3.56\ \text{Hz}}$$

$$T = \frac{1}{f} = \frac{1}{3.56\ \text{Hz}} = \boxed{0.281\ \text{s}}$$

(c) At $t = 0$, $v = 0$ and $x = 5.00 \times 10^{-2}$ m, so the total energy of the oscillator is

$$E = KE + PE_s = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$= 0 + \frac{1}{2}\left(250\ \text{N/m}\right)\left(5.00 \times 10^{-2}\ \text{m}\right)^2 = \boxed{0.313\ \text{J}}$$

(d) When $x = A$, $v = 0$ so $E = KE + PE_s = 0 + \frac{1}{2}kA^2$.

Thus,

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2\left(0.313\ \text{J}\right)}{250\ \text{N/m}}} = 5.00 \times 10^{-2}\ \text{m} = \boxed{5.00\ \text{cm}}$$

(e) At $x = 0$, $KE = \frac{1}{2}mv_{\text{max}}^2 = E$, or

$$v_{\text{max}} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\left(0.313\ \text{J}\right)}{0.500\ \text{kg}}} = \boxed{1.12\ \text{m/s}}$$

$$a_{\text{max}} = \frac{F_{\text{max}}}{m} = \frac{kA}{m} = \frac{\left(250\ \text{N/m}\right)\left(5.00 \times 10^{-2}\ \text{m}\right)}{0.500\ \text{kg}} = \boxed{25.0\ \text{m/s}^2}$$

Note: To solve parts (f) and (g), your calculator should be set in ***radians mode***.

(f) At $t = 0.500$ s, Equation 13.14a gives the displacement as

$$x = A\cos \ \omega t \ = A\cos \ t\sqrt{k/m} \ = \ 5.00\ \text{cm} \ \cos\left[\ 0.500\ \text{s} \ \sqrt{\frac{250\ \text{N/m}}{0.500\ \text{kg}}}\right] = \boxed{0.919\ \text{cm}}$$

(g) From Equation 13.14b, the velocity at $t = 0.500$ s is

$$v = -A\omega \sin \ \omega t \ = -A\sqrt{k/m}\sin \ t\sqrt{k/m}$$

$$= - \ 5.00 \times 10^{-2}\ \text{m} \ \sqrt{\frac{250\ \text{N/m}}{0.500\ \text{kg}}}\sin\left[\ 0.500\ \text{s} \ \sqrt{\frac{250\ \text{N/m}}{0.500\ \text{kg}}}\right] = \boxed{+1.10\ \text{m}}$$

and from Equation 13.14c, the acceleration at this time is

$$a = -A\omega^2 \cos \ \omega t \ = -A \ k/m \ \cos \ t\sqrt{k/m}$$

$$= - \ 5.00 \times 10^{-2}\ \text{m} \ \left(\frac{250\ \text{N/m}}{0.500\ \text{kg}}\right)\cos\left[\ 0.500\ \text{s} \ \sqrt{\frac{250\ \text{N/m}}{0.500\ \text{kg}}}\right] = \boxed{-4.59\ \text{m/s}^2}$$

13.33 From Equation 13.6,

$$v = \pm\sqrt{\frac{k}{m}\ A^2 - x^2} \ = \pm\sqrt{\omega^2 \ A^2 - x^2}$$

Hence,

$$v = \pm\,\omega\sqrt{A^2 - A^2\cos^2 \ \omega t} \ = \pm\,\omega A\sqrt{1 - \cos^2 \ \omega t} \ = \boxed{\pm\,\omega A \sin \ \omega t}$$

From Equation 13.2,

$$a = -\frac{k}{m}x = -\,\omega^2\left[\,A\cos \ \omega t \ \right] = \boxed{-\,\omega^2 A\cos \ \omega t}$$

13.34 (a) The height of the tower is almost the same as the length of the pendulum. From $T = 2\pi\sqrt{L/g}$, we obtain

$$L = \frac{g\,T^2}{4\,\pi^2} = \frac{9.80\ \text{m/s}^2 \quad 15.5\ \text{s}^{\ 2}}{4\,\pi^2} = \boxed{59.6\ \text{m}}$$

(b) On the Moon, where $g = 1.67$ m/s^2, the period will be

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{59.6\ \text{m}}{1.67\ \text{m/s}^2}} = \boxed{37.5\ \text{s}}$$

13.35 The period of a pendulum is the time for one complete oscillation and is given by $T = 2\pi\sqrt{\ell/g}$, where ℓ is the length of the pendulum.

(a) $$T = \frac{3.00\ \text{min}}{120\ \text{oscillations}}\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \boxed{1.50\ \text{s}}$$

(b) The length of the pendulum is

$$\ell = g\left(\frac{T^2}{4\pi^2}\right) = 9.80\ \text{m/s}^2\left(\frac{1.50\ \text{s}^{\ 2}}{4\pi^2}\right) = \boxed{0.559\ \text{m}}$$

13.36 The period in Tokyo is $T_T = 2\pi\sqrt{L_T / g_T}$ and the period in Cambridge is $T_C = 2\pi\sqrt{L_C / g_C}$. We know that $T_T = T_C = 2.000$ s, from which, we see that

$$\frac{L_T}{g_T} = \frac{L_C}{g_C}, \text{ or } \frac{g_C}{g_T} = \frac{L_C}{L_T} = \frac{0.994\ 2}{0.992\ 7} = \boxed{1.001\ 5}$$

13.37 (a) The period of the pendulum is $T = 2\pi\sqrt{L/g}$. Thus, on the Moon where the free-fall acceleration is smaller, the period will be longer and the clock will run $\boxed{\text{slow}}$.

(b) The ratio of the pendulum's period on the Moon to that on Earth is

$$\frac{T_{Moon}}{T_{Earth}} = \frac{2\pi\sqrt{L/g_{Moon}}}{2\pi\sqrt{L/g_{Earth}}} = \sqrt{\frac{g_{Earth}}{g_{Moon}}} = \sqrt{\frac{9.80}{1.63}} = 2.45$$

Hence, the pendulum of the clock on Earth makes 2.45 "ticks" while the clock on the Moon is making 1.00 "tick". After the Earth clock has ticked off 24.0 h and again reads 12:00 midnight, the Moon clock will have ticked off 24.0 h/2.45 = 9.80 h and will read $\boxed{9:48\ \text{AM}}$.

13.38 (a) The lower temperature will cause the pendulum to contract. The shorter length will produce a smaller period, so the clock will run faster or $\boxed{\text{gain time}}$.

(b) The period of the pendulum is $T_0 = 2\pi\sqrt{L/g}$ at 20°C, and at –5.0°C it is $T = 2\pi\sqrt{L/g}$. The ratio of these periods is $T_0/T = \sqrt{L_0/L}$.

From Chapter 10, the length at –5.0°C is $L = L_0 + \alpha_{\text{Al}}L_0(\Delta T)$, so

$$\frac{L_0}{L} = \frac{1}{1+\alpha_{\text{Al}}(\Delta T)} = \frac{1}{1+\left[24\times10^{-6}\,(°\text{C})^{-1}\right](-5.0°\text{C}-20°\text{C})} = \frac{1}{0.999\,4} = 1.000\,6$$

This gives

$$\frac{T_0}{T} = \sqrt{\frac{L_0}{L}} = \sqrt{1.000\,6} = 1.000\,3$$

Thus in one hour (3 600 s), the chilled pendulum will gain $(1.000\,3-1)(3\,600\text{ s}) = \boxed{1.1\text{ s}}$.

13.39 (a) From $T = 2\pi\sqrt{L/g}$, the length of a pendulum with period T is $L = gT^2/4\pi^2$.

On Earth, with $T = 1.0$ s,

$$L = \frac{(9.80\text{ m/s}^2)(1.0\text{ s})^2}{4\pi^2} = 0.25\text{ m} = \boxed{25\text{ cm}}$$

If $T = 1.0$ s on Mars,

$$L = \frac{(3.7\text{ m/s}^2)(1.0\text{ s})^2}{4\pi^2} = 0.094\text{ m} = \boxed{9.4\text{ cm}}$$

(b) The period of an object on a spring is $T = 2\pi\sqrt{m/k}$, which is independent of the local free-fall acceleration. Thus, the same mass will work on Earth and on Mars. This mass is

$$m = \frac{kT^2}{4\pi^2} = \frac{(10\ \text{N/m})(1.0\ \text{s})^2}{4\pi^2} = \boxed{0.25\ \text{kg}}$$

13.40 The apparent free-fall acceleration is the vector sum of the actual free-fall acceleration and the negative of the elevator's acceleration. To see this, consider an object that is hanging from a vertical string in the elevator **and appears to be at rest to the elevator passengers**. These passengers believe the tension in the string is the negative of the object's weight, or $\vec{\mathbf{T}} = -m\,\vec{\mathbf{g}}_{app}$ where $\vec{\mathbf{g}}_{app}$ is the apparent free-fall acceleration in the elevator.

An observer located outside the elevator applies Newton's second law to this object by writing $\Sigma\vec{\mathbf{F}} = \vec{\mathbf{T}} + m\,\vec{\mathbf{g}} = m\vec{\mathbf{a}}_e$ where $\vec{\mathbf{a}}_e$ is the acceleration of the elevator and all its contents. Thus, $\vec{\mathbf{T}} = m\left(\vec{\mathbf{a}}_e - \vec{\mathbf{g}}\right) = -m\,\vec{\mathbf{g}}_{app}$, which gives $\vec{\mathbf{g}}_{app} = \vec{\mathbf{g}} - \vec{\mathbf{a}}_e$.

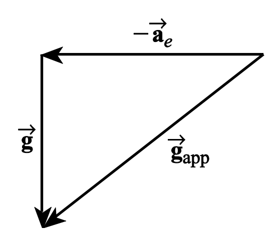
(a) If we choose downward as the positive direction, then $\vec{\mathbf{a}}_e = -5.00\ \text{m/s}^2$ in this case and $\vec{\mathbf{g}}_{app} = (9.80 + 5.00)\ \text{m/s}^2 = +14.8\ \text{m/s}^2$ (downward). The period of the pendulum is

$$T = 2\pi\sqrt{\frac{L}{g_{app}}} = 2\pi\sqrt{\frac{5.00\ \text{m}}{14.8\ \text{m/s}^2}} = \boxed{3.65\ \text{s}}$$

(b) Again choosing downward as positive, $\vec{\mathbf{a}}_e = 5.00\ \text{m/s}^2$ and $\vec{\mathbf{g}}_{app} = (9.80 - 5.00)\ \text{m/s}^2 = +4.80\ \text{m/s}^2$ (downward) in this case. The period is now given by

$$T = 2\pi\sqrt{\frac{L}{g_{app}}} = 2\pi\sqrt{\frac{5.00\ \text{m}}{4.80\ \text{m/s}^2}} = \boxed{6.41\ \text{s}}$$

(c) If $\vec{\mathbf{a}}_e = 5.00\ \text{m/s}^2$ horizontally, the vector sum $\vec{\mathbf{g}}_{app} = \vec{\mathbf{g}} - \vec{\mathbf{a}}_e$ is as shown in the sketch at the right. The magnitude is



$$g_{app} = \sqrt{(5.00\ \text{m/s}^2)^2 + (9.80\ \text{m/s}^2)^2} = 11.0\ \text{m/s}^2$$

and the period of the pendulum is

$$T = 2\pi\sqrt{\frac{L}{g_{app}}} = 2\pi\sqrt{\frac{5.00\ \text{m}}{11.0\ \text{m/s}^2}} = \boxed{4.24\ \text{s}}$$

13.41 (a) The distance from the bottom of a trough to the top of a crest is twice the amplitude of the wave. Thus, $2A = 8.26$ cm and

$$\boxed{A = 4.13 \text{ cm}}$$

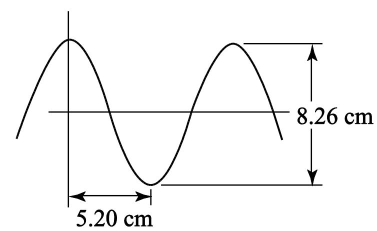


Figure P13.41

(b) The horizontal distance from a crest to a trough is a half wavelength. Hence,

$$\frac{\lambda}{2} = 5.20 \text{ cm and } \boxed{\lambda = 10.4 \text{ cm}}$$

(c) The period is

$$T = \frac{1}{f} = \frac{1}{18.0 \text{ s}^{-1}} = \boxed{5.56 \times 10^{-2} \text{ s}}$$

(d) The wave speed is $v = \lambda f = \left(10.4 \text{ cm}\right)\left(18.0 \text{ s}^{-1}\right) = \boxed{187 \text{ cm/s} = 1.87 \text{ m/s}}$

13.42 (a) The amplitude is the magnitude of the maximum displacement from equilibrium (at $x = 0$). Thus,

$$\boxed{A = 2.00 \text{ cm}}$$

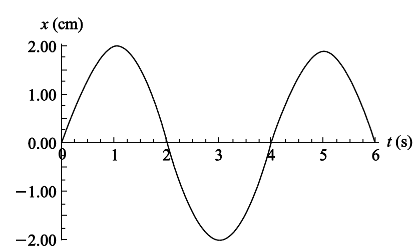


Figure P13.42

(b) The period is the time for one full cycle of the motion. Therefore, $\boxed{T = 4.00 \text{ s}}$

(c) The period may be written as $T = 2\pi/\omega$, so the angular frequency is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4.00 \text{ s}} = \boxed{\frac{\pi}{2} \text{ rad/s}}$$

(d) The total energy may be expressed as $E = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$. Thus, $v_{\max} = A\sqrt{k/m}$, and since $\omega = \sqrt{k/m}$, this becomes $v_{\max} = \omega A$ and yields

$$v_{\max} = \omega A = \left(\frac{\pi}{2}\ \text{rad/s}\right)(2.00\ \text{cm}) = \boxed{\pi\ \text{cm/s}}$$

(e) The spring exerts maximum force, $|F| = k|x|$, when the object is at maximum distance from equilibrium, i.e., at $|x| = A = 2.00\ \text{cm}$. Thus, the maximum acceleration of the object is

$$a_{\max} = \frac{|F_{\max}|}{m} = \frac{kA}{m} = \omega^2 A = \left(\frac{\pi}{2}\ \text{rad/s}\right)^2 (2.00\ \text{cm}) = \boxed{4.93\ \text{cm/s}^2}$$

(f) The general equation for position as a function of time for an object undergoing simple harmonic motion with $t = 0$ when $x = 0$ is $x = A \sin(\omega t)$. For this oscillator, this becomes

$$\boxed{x = (2.00\ \text{cm})\sin\left(\frac{\pi}{2}t\right)}$$

13.43 (a) The period and the frequency are reciprocals of each other. Therefore,

$$T = \frac{1}{f} = \frac{1}{101.9\ \text{MHz}} = \frac{1}{101.9 \times 10^6\ \text{s}^{-1}} = 9.81 \times 10^{-9}\ \text{s} = \boxed{9.81\ \text{ns}}$$

(b) $$\lambda = \frac{v}{f} = \frac{3.00 \times 10^8\ \text{m/s}}{101.9 \times 10^6\ \text{s}^{-1}} = \boxed{2.94\ \text{m}}$$

13.44 (a) The frequency of a transverse wave is the number of crests that pass a given point each second. Thus, if 5.00 crests pass in 14.0 seconds, the frequency is

$$f = \frac{5.00}{14.0\ \text{s}} = 0.357\ \text{s}^{-1} = \boxed{0.357\ \text{Hz}}$$

(b) The wavelength of the wave is the distance between successive maxima or successive minima. Thus, $\lambda = 2.76 \text{ m}$ and the wave speed is

$$v = \lambda f = (2.76 \text{ m})(0.357 \text{ s}^{-1}) = \boxed{0.985 \text{ m/s}}$$

13.45 The speed of the wave is

$$v = \frac{\Delta x}{\Delta t} = \frac{425 \text{ cm}}{10.0 \text{ s}} = 42.5 \text{ cm/s}$$

and the frequency is

$$f = \frac{40.0 \text{ vib}}{30.0 \text{ s}} = 1.33 \text{ Hz}$$

Thus,

$$\lambda = \frac{v}{f} = \frac{42.5 \text{ cm/s}}{1.33 \text{ Hz}} = \boxed{31.9 \text{ cm}}$$

13.46 From $v = \lambda f$, the wavelength (and size of smallest detectable insect) is

$$\lambda = \frac{v}{f} = \frac{340 \text{ m/s}}{60.0 \times 10^3 \text{ Hz}} = 5.67 \times 10^{-3} \text{ m} = \boxed{5.67 \text{ mm}}$$

13.47 The frequency of the wave (that is, the number of crests passing the cork each second) is $f = 2.00 \text{ s}^{-1}$ and the wavelength (distance between successive crests) is $\lambda = 8.50 \text{ cm}$. Thus, the wave speed is

$$v = \lambda f = (8.50 \text{ cm})(2.00 \text{ s}^{-1}) = 17.0 \text{ cm/s} = 0.170 \text{ m/s}$$

and the time required for the ripples to travel 10.0 m over the surface of the water is

$$\Delta t = \frac{\Delta x}{v} = \frac{10.0 \text{ m}}{0.170 \text{ m/s}} = \boxed{58.8 \text{ s}}$$

13.48 (a) When the boat is at rest in the water, the speed of the wave relative to the boat is the same as the speed of the wave relative to the water, $v = 4.0\ \text{m/s}$. The frequency detected in this case is

$$f = \frac{v}{\lambda} = \frac{4.0\ \text{m/s}}{20\ \text{m}} = \boxed{0.20\ \text{Hz}}$$

(b) Taking eastward as positive, $\vec{\mathbf{v}}_{wave,boat} = \vec{\mathbf{v}}_{wave,water} - \vec{\mathbf{v}}_{boat,water}$ (see the discussion of relative velocity in Chapter 3 of the textbook) gives

$$\vec{\mathbf{v}}_{wave,boat} = +4.0\ \text{m/s} - (-1.0\ \text{m/s}) = +5.0\ \text{m/s} \quad \text{and} \quad v_{boat,wave} = \left|\vec{\mathbf{v}}_{wave,boat}\right| = 5.0\ \text{m/s}$$

Thus,

$$f = \frac{v_{boat,wave}}{\lambda} = \frac{5.0\ \text{m/s}}{20\ \text{m}} = \boxed{0.25\ \text{Hz}}$$

13.49 The down and back distance is 4.00 m + 4.00 m = 8.00 m.

The speed is then

$$v = \frac{d_{total}}{t} = \frac{4(8.00\ \text{m})}{0.800\ \text{s}} = 40.0\ \text{m/s} = \sqrt{F/\mu}$$

Now,

$$\mu = \frac{m}{L} = \frac{0.200\ \text{kg}}{4.00\ \text{m}} = 5.00 \times 10^{-2}\ \text{kg/m}$$

so

$$F = \mu v^2 = (5.00 \times 10^{-2}\ \text{kg/m})(40.0\ \text{m/s})^2 = \boxed{80.0\ \text{N}}$$

13.50 The speed of the wave is

$$v = \frac{\Delta x}{\Delta t} = \frac{20.0\ \text{m}}{0.800\ \text{s}} = 25.0\ \text{m/s}$$

and the mass per unit length of the rope is $\mu = m / L = 0.350\ \text{kg/m}$. Thus, from $v = \sqrt{F/\mu}$, we obtain

$$F = v^2\mu = \left(25.0\ \text{m/s}\right)^2 \left(0.350\ \text{kg/m}\right) = \boxed{219\ \text{N}}$$

13.51 (a) The speed of transverse waves in the cord is $v = \sqrt{F/\mu}$, where $\mu = m/L$ is the mass per unit length. With the tension in the cord being $F = 12.0$ N, the wave speed is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{FL}{m}} = \sqrt{\frac{\left(12.0\ \text{N}\right)\left(6.30\ \text{m}\right)}{0.150\ \text{kg}}} = \boxed{22.4\ \text{m/s}}$$

(b) The time to travel the length of the cord is

$$\Delta t = \frac{L}{v} = \frac{6.30\ \text{m}}{22.4\ \text{m/s}} = \boxed{0.281\ \text{s}}$$

13.52 (a) In making 6 round trips, the pulse travels the length of the line 12 times for a total distance of 144 m. The speed of the pulse is then

$$v = \frac{\Delta x}{\Delta t} = \frac{12L}{\Delta t} = \frac{12\left(12.0\ \text{m}\right)}{2.96\ \text{s}} = \boxed{48.6\ \text{m/s}}$$

(b) The speed of transverse waves in the line is $v = \sqrt{F/\mu}$, so the tension in the line is

$$F = \mu v^2 = \left(\frac{m}{L}\right)v^2 = \left(\frac{0.375\ \text{kg}}{12.0\ \text{m}}\right)\left(48.6\ \text{m/s}\right)^2 = \boxed{73.8\ \text{N}}$$

13.53 (a) The mass per unit length is

$$\mu = \frac{m}{L} = \frac{0.0600\ \text{kg}}{5.00\ \text{m}} = 0.0120\ \text{kg/m}$$

From $v = \sqrt{F/\mu}$, the required tension in the string is

$$F = v^2\mu = \left(50.0 \text{ m/s}\right)^2 \left(0.0120 \text{ kg/m}\right) = \boxed{30.0 \text{ N}},$$

(b) $$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{8.00 \text{ N}}{0.0120 \text{ kg/m}}} = \boxed{25.8 \text{ m/s}}$$

13.54 The mass per unit length of the wire is

$$\mu = \frac{m}{L} = \frac{4.00 \times 10^{-3} \text{ kg}}{1.60 \text{ m}} = 2.50 \times 10^{-3} \text{ kg/m}$$

and the speed of the pulse is

$$v = \frac{L}{\Delta t} = \frac{1.60 \text{ m}}{0.0361 \text{ s}} = 44.3 \text{ m/s}$$

Thus, the tension in the wire is

$$F = v^2\mu = \left(44.3 \text{ m/s}\right)^2 \left(2.50 \times 10^{-3} \text{ kg/m}\right) = 4.91 \text{ N}$$

But, the tension in the wire is the weight of a 3.00-kg object on the Moon. Hence, the local free-fall acceleration is

$$g = \frac{F}{m} = \frac{4.91 \text{ N}}{3.00 \text{ kg}} = \boxed{1.64 \text{ m/s}^2}$$

13.55 The period of the pendulum is $T = 2\pi\sqrt{L/g}$, so the length of the string is

$$L = \frac{gT^2}{4\pi^2} = \frac{\left(9.80 \text{ m/s}^2\right)\left(2.00 \text{ s}\right)^2}{4\pi^2} = 0.993 \text{ m}$$

Then mass per unit length of the string is then

$$\mu = \frac{m}{L} = \frac{0.060\ 0\ \text{kg}}{0.993\ \text{m}} = 0.060\ 4\ \frac{\text{kg}}{\text{m}}$$

When the pendulum is vertical and stationary, the tension in the string is

$$F = M_{ball}g = (5.00\ \text{kg})(9.80\ \text{m/s}^2) = 49.0\ \text{N}$$

and the speed of transverse waves in it is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{49.0\ \text{N}}{0.060\ 4\ \text{kg/m}}} = \boxed{28.5\ \text{m/s}}$$

13.56 If $\mu_1 = m_1/L$ is the mass per unit length for the first string, then $\mu_2 = m_2/L = m_1/2L = \mu_1/2$ is that of the second string. Thus, with $F_2 = F_1 = F$, the speed of waves in the second string is

$$v_2 = \sqrt{\frac{F}{\mu_2}} = \sqrt{\frac{2F}{\mu_1}} = \sqrt{2}\left(\sqrt{\frac{F}{\mu_1}}\right) = \sqrt{2}\,v_1 = \sqrt{2}\,(5.00\ \text{m/s}) = \boxed{7.07\ \text{m/s}}$$

13.57 (a) The tension in the string is $F = mg = (3.0\ \text{kg})(9.80\ \text{m/s}^2) = 29\ \text{N}$. Then, from $v = \sqrt{F/\mu}$, the mass per unit length is

$$\mu = \frac{F}{v^2} = \frac{29\ \text{N}}{(24\ \text{m/s})^2} = \boxed{0.051\ \text{kg/m}}$$

(b) When $m = 2.00$ kg, the tension is

$$F = mg = (2.0\ \text{kg})(9.80\ \text{m/s}^2) = 20\ \text{N}$$

and the speed of transverse waves in the string is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{20\ \text{N}}{0.051\ \text{kg/m}}} = \boxed{20\ \text{m/s}}$$

13.58 If the tension in the wire is F, the tensile stress is $Stress = F/A$, so the speed of transverse waves in the wire may be written as

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{A \cdot Stress}{m/L}} = \sqrt{\frac{Stress}{m/(A \cdot L)}}$$

But, $A \cdot L = V = \text{volume}$, so $m/\left(A \cdot L\right) = \rho = \text{density}$. Thus, $v = \sqrt{\frac{Stress}{\rho}}$.

When the stress is at its maximum, the speed of waves in the wire is

$$v_{\text{max}} = \sqrt{\frac{(Stress)_{\text{max}}}{\rho}} = \sqrt{\frac{2.70 \times 10^9\ \text{Pa}}{7.86 \times 10^3\ \text{kg/m}^3}} = \boxed{586\ \text{m/s}}$$

13.59 (a) The speed of transverse waves in the line is $v = \sqrt{F/\mu}$, with $\mu = m/L$ being the mass per unit length. Therefore,

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{FL}{m}} = \sqrt{\frac{\left(12.5\ \text{N}\right)\left(38.0\ \text{m}\right)}{2.65\ \text{kg}}} = \boxed{13.4\ \text{m/s}}$$

(b) The worker could throw an object, such as a snowball, at one end of the line to set up a pulse, and use a stopwatch to measure the time it takes a pulse to travel the length of the line. From this measurement, the wo rker would have an estimate of the wave speed, which in turn can be used to estimate the tension.

13.60 (a) In making n round trips along the length of the line, the total distance traveled by the pulse is $\Delta x = n\left(2L\right) = 2nL$. The wave speed is then

$$v = \frac{\Delta x}{t} = \boxed{\frac{2nL}{t}}$$

(b) From $v = \sqrt{F/\mu}$ as the speed of transverse waves in the line, the tension is

$$F = \mu v^2 = \left(\frac{M}{L}\right)\left(\frac{2nL}{t}\right)^2 = \left(\frac{M}{L}\right)\left(\frac{4n^2L^2}{t^2}\right) = \boxed{\frac{4n^2ML^2}{t^2}}$$

13.61 (a) $\boxed{\text{Constructive interference}}$ produces the maximum amplitude

$$A'_{\text{max}} = A_1 + A_2 = \boxed{0.50 \text{ m}}$$

(b) $\boxed{\text{Destructive interference}}$ produces the minimum amplitude

$$A'_{\text{min}} = A_1 - A_2 = \boxed{0.10 \text{ m}}$$

13.62 We are given that $x = A\cos(\omega t) = (0.25 \text{ m})\cos(0.4\pi\, t)$.

(a) By inspection, the amplitude is seen to be $A = \boxed{0.25 \text{ m}}$

(b) The angular frequency is $\omega = 0.4\pi \text{ rad/s}$. But $\omega = \sqrt{k/m}$, so the spring constant is

$$k = m\omega^2 = (0.30 \text{ kg})(0.4\pi \text{ rad/s})^2 = \boxed{0.47 \text{ N/m}}$$

(c) *Note:* Your calculator must be in <u>radians mode</u> for part (c).

At $t = 0.30$ s, $x = \left(0.25 \text{ m}\right)\cos\left[\left(0.4\pi \text{ rad/ s}\right)\left(0.30 \text{ s}\right)\right] = \boxed{0.23 \text{ m}}$

(d) From conservation of mechanical energy, the speed at displacement x is given by $v = \omega\sqrt{A^2 - x^2}$. Thus, at $t = 0.30$ s, when $x = 0.23$ m, the speed is

$$v = \left(0.4\pi \text{ rad/s}\right)\sqrt{(0.25 \text{ m})^2 - (0.23 \text{ m})^2} = \boxed{0.12 \text{ m/s}}$$

13.63 (a) The period of a vibrating object-spring system is $T = 2\pi/\omega = 2\pi\sqrt{m/k}$, so the spring constant is

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2\left(2.00\text{ kg}\right)}{\left(0.600\text{ s}\right)^2} = \boxed{219\text{ N/m}}$$

(b) If $T = 1.05$ s for mass m_2, this mass is

$$m_2 = \frac{kT^2}{4\pi^2} = \frac{(219\text{ N/m})(1.05\text{ s})^2}{4\pi^2} = \boxed{6.12\text{ kg}}$$

13.64 (a) The period is the reciprocal of the frequency, or

$$T = \frac{1}{f} = \frac{1}{196\text{ s}^{-1}} = 5.10\times10^{-3}\text{ s} = \boxed{5.10\text{ ms}}$$

(b) $\lambda = \dfrac{v_{sound}}{f} = \dfrac{343\text{ m/s}}{196\text{ s}^{-1}} = \boxed{1.75\text{ m}}$

13.65 (a) The period of a simple pendulum is $T = 2\pi\sqrt{\ell/g}$, so the period of the first system is

$$T_1 = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{0.700\text{ m}}{9.80\text{ m/s}^2}} = \boxed{1.68\text{ s}}$$

(b) The period of mass-spring system is $T = 2\pi\sqrt{m/k}$, so if the period of the second system is $T_2 = T_1$, then $2\pi\sqrt{m/k} = 2\pi\sqrt{1/g}$ and the spring constant is

$$k = \frac{mg}{\ell} = \frac{\left(1.20\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{0.700\text{ m}} = \boxed{16.8\text{ N/m}}$$

13.66 Since the spring is “light”, we neglect any small amount of energy lost in the collision with the spring, and apply conservation of mechanical energy from when the block first starts until it comes to rest again. This gives

$$\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i\text{, or } 0 + 0 + \frac{1}{2}kx_{\max}^2 = 0 + 0 + mgh_i$$

Thus,

$$x_{\max} = \sqrt{\frac{2mgh_i}{k}} = \sqrt{\frac{2(0.500\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})}{20.0\text{ N/m}}} = \boxed{0.990\text{ m}}$$

13.67 Choosing $PE_g = 0$ at the initial height of the block, conservation of mechanical energy gives

$(KE + PE_g + PE_s)_f = (KE + PE_g + PE_s)_i$, or

$$\frac{1}{2}mv^2 + mg(-x) + \frac{1}{2}kx^2 = 0,$$

where v is the speed of the block after falling distance x.

(a) When $v = 0$, the non-zero solution to the energy equation from above gives

$$\frac{1}{2}kx_{\max}^2 = mgx_{\max} \qquad \text{or} \qquad k = \frac{2\,mg}{x_{\max}} = \frac{2(3.00\text{ kg})(9.80\text{ m/s}^2)}{0.100\text{ m}} = \boxed{588\text{ N/m}}$$

(b) When $x = 5.00\text{ cm} = 0.050\,0\text{ m}$, the energy equation gives

$$v = \sqrt{2gx - \frac{kx^2}{m}},$$

or

$$v = \sqrt{2(9.80\text{ m/s}^2)(0.050\,0\text{ m}) - \frac{(588\text{ N/m})(0.050\,0\text{ m})^2}{3.00\text{ kg}}} = \boxed{0.700\text{ m/s}}$$

13.68 (a) We apply conservation of mechanical energy from *just after* the collision until the block comes to rest.

$$(KE + PE_s)_f = (KE + PE_s)_i \text{ gives } 0 + \frac{1}{2}kx_f^2 = \frac{1}{2}MV^2 + 0$$

or the speed of the block just after the collision is

$$V = \sqrt{\frac{kx_f^2}{M}} = \sqrt{\frac{(900\ \text{N/m})(0.050\ 0\ \text{m})^2}{1.00\ \text{kg}}} = 1.50\ \text{m/s}$$

Now, we apply conservation of momentum from just before impact to immediately after the collision. This gives

$$m\left(v_{bullet}\right)_i + 0 = m\left(v_{bullet}\right)_f + MV$$

or

$$\left(v_{bullet}\right)_f = \left(v_{bullet}\right)_i - \left(\frac{M}{m}\right)V$$

$$= 400\ \text{m/s} - \left(\frac{1.00\ \text{kg}}{5.00 \times 10^{-3}\ \text{kg}}\right)(1.5\ \text{m/s}) = \boxed{100\ \text{m/s}}$$

(b) The mechanical energy converted into internal energy during the collision is

$$\Delta E = KE_i - \Sigma KE_f = \frac{1}{2}m\left(v_{bullet}\right)_i^2 - \frac{1}{2}m\left(v_{bullet}\right)_f^2 - \frac{1}{2}MV^2$$

or

$$\Delta E = \frac{1}{2}\left(5.00 \times 10^{-3}\ \text{kg}\right)\left[(400\ \text{m/s})^2 - (100\ \text{m/s})^2\right] - \frac{1}{2}(1.00\ \text{kg})(1.50\ \text{m/s})^2$$

$$\Delta E = \boxed{374\ \text{J}}$$

13.69 Choose $PE_g = 0$ when the blocks start from rest. Then, using conservation of mechanical energy from when the blocks are released until the spring returns to its unstretched length gives

$$\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i \text{, or}$$

$$\frac{1}{2}(m_1 + m_2)v_f^2 + (m_1 g x \sin 40° - m_2 g x) + 0 = 0 + 0 + \frac{1}{2}k x^2$$

$$\frac{1}{2}\left[\ 25+30\ \ \text{kg}\right]v_f^2 + \ 25\text{ kg}\ \ 9.80\ \text{m/s}^2\ \left[\ 0.200\text{ m}\ \sin 40°\right]$$

$$-\ \ 30\text{ kg}\ \ 9.80\ \text{m/s}^2\ \ 0.200\text{ m}\ \ = \frac{1}{2}\ 200\ \text{N/m}\ \ 0.200\text{ m}\ ^2$$

yielding $v_f = \boxed{1.1\ \text{m/s}}$

13.70 (a) When the gun is fired, the energy initially stored as elastic potential energy in the spring is transformed into kinetic energy of the bullet. Assuming no loss of energy, we have $\frac{1}{2}mv^2 = \frac{1}{2}kx_i^2$, or

$$v = x_i\sqrt{\frac{k}{m}} = \ 0.200\text{ m}\ \sqrt{\frac{9.80\ \text{N/m}}{1.00\times 10^{-3}\ \text{kg}}} = \boxed{19.8\ \text{m/s}}$$

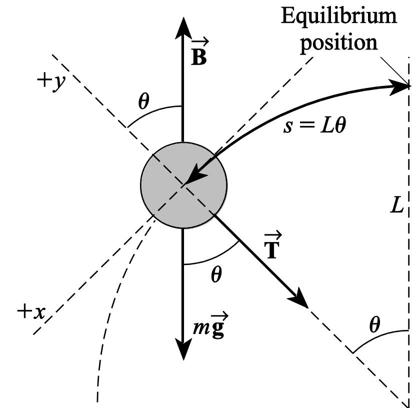
(b) From $\Delta y = v_{0y}t + \frac{1}{2}a_yt^2$, the time required for the pellet to drop 1.00 m to the floor, starting with $v_{0y} = 0$, is

$$t = \sqrt{\frac{2\ \ \Delta y}{a_y}} = \sqrt{\frac{2\ \ -1.00\text{ m}}{-9.80\ \text{m/s}^2}} = 0.452\text{ s}$$

The range (horizontal distance traveled during the flight) is then

$$\Delta x = v_{0x}t = \ 19.8\ \text{m/s}\ \ 0.452\text{ s}\ \ = \boxed{8.94\text{ m}}$$

13.71 The free-body diagram at the right shows the forces acting on the balloon when it is displaced distance $s = L\theta$ along the circular arc it follows. The net force tangential to this path is



$$F_{net} = \Sigma F_x = -B\sin\theta + mg\sin\theta = -\left(B - mg\right)\sin\theta$$

For small angles, $\sin\theta \approx \theta = s/L$

Also, $mg = \left(\rho_{\text{He}}V\right)g$

and the buoyant force is $B = \left(\rho_{\text{air}}V\right)g$. Thus, the net restoring force acting on the balloon is

$$F_{net} \approx -\left[\frac{\left(\rho_{air} - \rho_{\text{He}}\right)Vg}{L}\right]s$$

Observe that this is in the form of Hooke's law, $F = -k\,s$, with $k = \left(\rho_{air} - \rho_{\text{He}}\right)Vg/L$

Thus, the motion will be simple harmonic and the period is given by

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\rho_{\text{He}}V}{\left(\rho_{air} - \rho_{\text{He}}\right)Vg/L}} = 2\pi\sqrt{\left(\frac{\rho_{\text{He}}}{\rho_{air} - \rho_{\text{He}}}\right)\frac{L}{g}}$$

This yields

$$T = 2\pi\sqrt{\left(\frac{0.180}{1.29 - 0.180}\right)\frac{3.00\text{ m}}{9.80\text{ m/s}^2}} = \boxed{1.40\text{ s}}$$

13.72 (a) When the block is given some small upward displacement, the net restoring force exerted on it by the rubber bands is

$$F_{net} = \Sigma F_y = -2F\sin\theta\text{, where }\tan\theta = \frac{y}{L}$$

For small displacements, the angle θ will be very small. Then $\sin\theta \approx \tan\theta = \dfrac{y}{L}$, and the net restoring force is

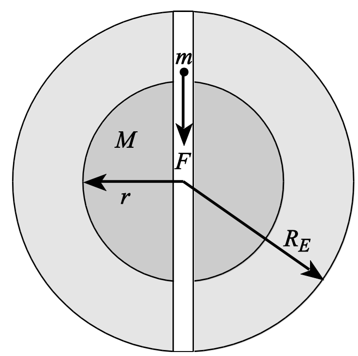
$$F_{net} = -2F\left(\frac{y}{L}\right) = \boxed{-\left(\frac{2F}{L}\right)y}$$

(b) The net restoring force found in part (a) is in the form of Hooke's law $F = -ky$, with $k = 2F/L$. Thus, the motion will be simple harmonic, and the angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \boxed{\sqrt{\frac{2F}{mL}}}$$

13.73 Newton's law of gravitation is

$$F = -\frac{GMm}{r^2}, \quad \text{where } M = \rho\left(\frac{4}{3}\pi r^3\right)$$



Thus,

$$\boxed{F = -\left(\frac{4}{3}\pi\rho G m\right) r}$$

which is of Hooke's law form, $F = -k\,r$, with

$$\boxed{k = \frac{4}{3}\pi\rho G m}$$

13.74 The inner tip of the wing is attached to the end of the spring and always moves with the same speed as the end of the vibrating spring. Thus, its maximum speed is

$$v_{inner,\ \max} = v_{spring,\ \max} = A\sqrt{\frac{k}{m}} = 0.20\ \text{cm}\sqrt{\frac{4.7\times10^{-4}\ \text{N/m}}{0.30\times10^{-3}\ \text{kg}}} = 0.25\ \text{cm/s}$$

Treating the wing as a rigid bar, all points in the wing have the same angular velocity at any instant in time. As the wing rocks on the fulcrum, the inner tip and outer tips follow circular paths of different radii. Since the angular velocities of the tips are always equal, we may write

$$\omega = \frac{v_{outer}}{r_{outer}} = \frac{v_{inner}}{r_{inner}}$$

The maximum speed of the outer tip is then

$$v_{outer,\ \max} = \left(\frac{r_{outer}}{r_{inner}}\right) v_{inner,\ \max} = \left(\frac{15.0\text{ mm}}{3.00\text{ mm}}\right)\left(0.25\text{ cm/s}\right) = \boxed{1.3\text{ cm/s}}$$

13.75 (a) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{500\text{ N/m}}{2.00\text{ kg}}} = \boxed{15.8\text{ rad/s}}$

(b) Apply Newton's second law to the block while the elevator is accelerating:

$$\Sigma F_y = F_s - mg = ma_y$$

With $F_s = kx$ and $a_y = g/3$, this gives $kx = m\left(g + g/3\right)$, or

$$x = \frac{4\,mg}{3k} = \frac{4\left(2.00\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{3\left(500\text{ N/m}\right)} = 5.23 \times 10^{-2}\text{ m} = \boxed{5.23\text{ cm}}$$

13.76 (a) Note that as the spring passes through the vertical position, the object is moving in a circular arc of radius $L - y_f$. Also, observe that the y-coordinate of the object at this point must be negative $\left(y_f < 0\right)$ so the spring is stretched and exerting an upward tension force of magnitude greater than the object's weight. This is necessary so the object experiences a net force toward the pivot to supply the needed centripetal acceleration in this position. This is summarized by Newton's second law applied to the object at this point, stating

$$\boxed{\Sigma F_y = \frac{mv^2}{L - y_f} = -ky_f - mg}$$

(b) Conservation of energy requires that $E = KE_i + PE_{g,i} + PE_{s,i} = KE_f + PE_{g,f} + PE_{s,f}$, or

$E = 0 + mgL + 0 = \frac{1}{2}mv^2 + mgy_f + \frac{1}{2}ky_f^2$, reducing to $\boxed{2mg\left(L - y_f\right) = mv^2 + ky_f^2}$

(c) From the result of part (a), observe that $mv^2 = -(L - y_f)(ky_f + mg)$. Substituting this into the result from part (b) gives $2mg(L - y_f) = -(L - y_f)(ky_f + mg) + ky_f^2$. After expanding and regrouping terms, this becomes $(2k)y_f^2 + (3mg - kL)y_f + (-3mgL) = 0$, which is a quadratic equation $ay_f^2 + by_f + c = 0$, with

$$a = 2k = 2\left(1\,250\ \text{N/m}\right) = 2.50 \times 10^3\ \text{N/m}$$

$$b = 3mg - kL = 3\left(5.00\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) - \left(1\,250\ \text{N/m}\right)\left(1.50\ \text{m}\right) = -1.73 \times 10^3\ \text{N}$$

and

$$c = -3mgL = -3\left(5.00\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\left(1.50\ \text{m}\right) = -221\ \text{N}\cdot\text{m}$$

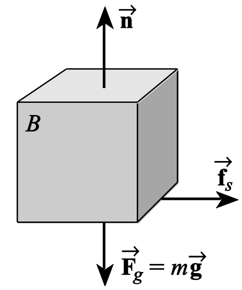
Applying the quadratic formula, keeping only the negative solution [see the discussion in part (a)] gives

$$y_f = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-\left(-1.73 \times 10^3\right) - \sqrt{\left(-1.73 \times 10^3\right)^2 - 4\left(2.50 \times 10^3\right)\left(-221\right)}}{2\left(2.50 \times 10^3\right)}$$

or $\boxed{y_f = -0.110\ \text{m}}$

(d) Because the length of this pendulum varies and is longer throughout its motion than a simple pendulum of length L, $\boxed{\text{its period will be longer}}$ than that of a simple pendulum.

13.77 The maximum acceleration of the oscillating system is



$$a_{\max} = \omega^2 A = (2\pi f)^2 A$$

The friction force exerted between the two blocks must be capable of accelerating block *B* at this rate. When block *B* is on the verge of slipping,

$f_s = (f_s)_{\max} = \mu_s n = \mu_s mg = ma_{\max}$ and we must have

$$a_{\max} = (2\pi f)^2 A = \mu_s g$$

Thus,

$$A = \frac{\mu_s g}{(2\pi f)^2} = \frac{(0.600)(9.80\ \text{m/s}^2)}{\left[2\pi(1.50\ \text{Hz})\right]^2} = 6.62 \times 10^{-2}\ \text{m} = \boxed{6.62\ \text{cm}}$$