PROBLEM SOLUTIONS

12.1 (a) The work done *on the gas* is

$$W = -P\ \Delta V = -\ 1.50 \times 10^5\ \text{Pa}\quad 0.250\ \text{m}^2 - 0.750\ \text{m}^2 = \boxed{7.50 \times 10^4\ \text{J}}$$

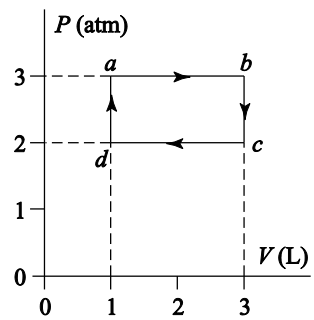
(b) The work done by the gas on the environment is

$$W_{\text{env}} = +P\ \Delta V = 1.50 \times 10^5\ \text{Pa}\quad 0.250\ \text{m}^2 - 0.750\ \text{m}^2 = \boxed{-7.50 \times 10^4\ \text{J}}$$

(c) The work done on the gas is the negative of the work the gas does on the environment because of $\boxed{\text{Newton's third law}}$. When the environment exerts a force on the gas, the gas exerts an equal magnitude force in the opposite direction on the environment.

12.2 (a)

$$W_{ab} = P_a\ V_b - V_a$$
$$= \left[3\ 1.013 \times 10^5\ \text{Pa}\right] 3.0\ \text{L} - 1.0\ \text{L}\left(\frac{10^{-3}\ \text{m}^3}{1\ \text{L}}\right)$$
$$= \boxed{610\ \text{J}}$$



(b) $W_{bc} = P\ V_c - V_b = \boxed{0}$

(c)

$$W_{cd} = P_c\ V_d - V_c$$
$$= \left[2\ 1.013 \times 10^5\ \text{Pa}\right] 1.0\ \text{L} - 3.0\ \text{L}\left(\frac{10^{-3}\ \text{m}^3}{1\ \text{L}}\right)$$
$$= \boxed{-410\ \text{J}}$$

(d) $W_{da} = P\ V_a - V_d = \boxed{0}$

(e) $W_{\text{net}} = W_{ab} + W_{bc} + W_{cd} + W_{da} = +610\ \text{J} + 0 - 410\ \text{J} + 0 = \boxed{+\ 200\ \text{J}}$

12.3 The constant pressure is $P = 1.5\ \text{atm}\quad 1.013 \times 10^5\ \text{Pa/atm} = 1.52 \times 10^5\ \text{Pa}$ and the work done on the gas is $W = -P\ \Delta V$.

(a) $\Delta V = +4.0\ \text{m}^3$ and

$$W = -P\ \Delta V = -\ 1.52 \times 10^5\ \text{Pa}\quad +4.0\ \text{m}^3 = \boxed{-\ 6.1 \times 10^5\ \text{J}}$$

(b) $\Delta V = -3.0\ \text{m}^3$, so

$$W = -P\ \Delta V = -\ 1.52 \times 10^5\ \text{Pa}\quad -3.0\ \text{m}^3 = \boxed{+4.6 \times 10^5\ \text{J}}$$

12.4 (a) The work done *by* the gas on the projectile is given by the area under the curve in the *PV* diagram. This is

$$W_{\text{by gas}} = \text{triangular area} + \text{rectangular area}$$

$$= \frac{1}{2}\ P_0 - P_f\quad V_f - V_0\ + P_f\ V_f - V_0\ = \frac{1}{2}\ P_0 + P_f\quad V_f - V_0$$

$$= \frac{1}{2}\left[\ 11 + 1.0\ \times 10^5\ \text{Pa}\right]\left[\ 40.0 - 8.0\ \ \text{cm}^3\right]\left(\frac{1\ \text{m}^3}{10^6\ \text{cm}^3}\right) = 19\ \text{J}$$

From the work-energy theorem, $W = \Delta KE = \frac{1}{2}m\text{v}^2 - 0$ where W is the work done on the projectile by the gas. Thus, the speed of the emerging projectile is

$$\text{v} = \sqrt{\frac{2\,W}{m}} = \sqrt{\frac{2\ \ 19\ \text{J}}{40.0 \times 10^{-3}\ \text{kg}}} = \boxed{31\ \ \text{m/s}}$$

(b) The air in front of the projectile would exert a retarding force of

$$F_r = P_{\text{air}}A = \ 1.0 \times 10^5\ \text{Pa}\ \left[\ 1.0\ \text{cm}^2\quad 1\ \text{m}^2/10^4\ \text{cm}^2\ \right] = 10\ \text{N}$$

on the projectile as it moves down the launch tube. The energy spent overcoming this retarding force would be

$$W_{\text{spent}} = F_r \cdot s = \ 10\ \text{N}\quad 0.32\ \text{m}\ = 3.2\ \text{J}$$

and the needed fraction is

$$\frac{W_{\text{spent}}}{W} = \frac{3.2\ \text{J}}{19\ \text{J}} = \boxed{0.17}$$

12.5 In each case, the work done *on* the gas is given by the negative of the area under the path on the *PV* diagram. Along those parts of the path where volume is constant, no work is done. Note that $1\ \text{atm} = 1.013 \times 10^5\ \text{Pa}$ and $1\ \text{Liter} = 10^{-3}\ \text{m}^3$.

(a) $W_{IAF} = W_{IA} + W_{AF} = -P_I \ V_A - V_I \ + 0$

$$= -\left[4.00 \ 1.013 \times 10^5 \text{ Pa}\right]\left[4.00 - 2.00 \ \times 10^{-3} \text{ m}^3\right] = \boxed{-810 \text{ J}}$$

(b) $W_{IF} = -$ triangular area $-$ rectangular area

$$= -\frac{1}{2} \ P_I - P_B \ \ V_F - V_B \ - P_B \ V_F - V_B \ = -\frac{1}{2} \ P_I + P_B \ \ V_F - V_B$$

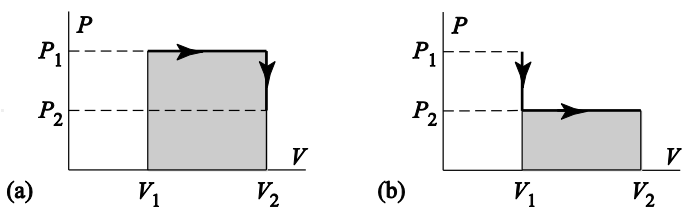
$$= -\frac{1}{2}\left[4.00 + 1.00 \ \ 1.013 \times 10^5 \text{ Pa}\right] 4.00 - 2.00 \ \times 10^{-3} \text{ m}^3$$

$$= \boxed{-507 \text{ J}}$$

(c) $W_{IBF} = W_{IB} + W_{BF} = 0 - P_B \ V_F - V_I$

$$= -\left[1.00 \ 1.013 \times 10^5 \text{ Pa}\right]\left[4.00 - 2.00 \ \times 10^{-3} \text{ m}^3\right] = \boxed{-203 \text{ J}}$$

12.6 The sketches for (a) and (b) are shown below:



(c) As seen from the areas under the paths in the *PV* diagrams above, the higher pressure during the expansion phase of the process results in more work done *by* the gas in (a) than in (b).

12.7 With $P_f = P_i = P$, the ideal gas law gives $P_f V_f - P_i V_i = P \ \Delta V \ = nR \ \Delta T$, so the work done by the gas is

$$W_{\text{env}} = +P \ \Delta V \ = nR \ \Delta T \ = \left(\frac{m}{M_{\text{He}}}\right) R \ T_f - T_i$$

If $W_{\text{env}} = 20.0 \text{ J}$, the mass of helium in the gas sample is

$$m = \frac{W_{\text{env}} \ M_{\text{He}}}{R \ T_f - T_i} = \frac{20.0 \text{ J} \ \ 4.00 \text{ g/mol}}{8.31 \text{ J/mol} \cdot \text{K} \ \ 373 \text{ K} - 273 \text{ K}} = 0.0963 \text{ g} = \boxed{96.3 \text{ mg}}$$

12.8 (a) The initial absolute temperature is $T_i = 1.50 \times 10^2 + 273.15 = 423 \text{ K}$, and the ideal gas law with

$P_f = P_i = 2.00 \text{ atm}$ yields

$$\frac{\cancel{P}\cancel{V}_f}{\cancel{P}\cancel{V}_i} = \frac{\cancel{nR}T_f}{\cancel{nR}T_i} \qquad \text{or} \qquad T_f = \left(\frac{V_f}{V_i}\right)T_i = \frac{2}{3}\ 423 \text{ k} = \boxed{282 \text{ K}}$$

(b) The work done on the gas is

$$W = -P\ \Delta V = -P\left(-\frac{V_i}{3}\right) = \frac{\cancel{P}}{3}\left(\frac{nRT_i}{\cancel{P}}\right) = \frac{nRT_i}{3}$$

or

$$W = \frac{1 \text{ mol}\quad 8.31 \text{ J/mol}\cdot\text{K}\quad 423 \text{ K}}{3} = 1.17 \times 10^3 \text{ J} = \boxed{1.17 \text{ kJ}}$$

12.9 (a) From the ideal gas law, $nR = PV_f/T_f = PV_i/T_i$. With pressure constant this gives

$$T_f = T_i\left(\frac{V_f}{V_i}\right) = 273 \text{ K}\quad 4 = \boxed{1.09 \times 10^3 \text{ K}}$$

(b) The work done on the gas is

$$W = -P\ \Delta V = -\ PV_f - PV_i = -nR\ T_f - T_i$$

$$= -\ 1.00 \text{ mol}\quad 8.31 \text{ J/mol}\cdot\text{K}\quad 1\,092 \text{ J} - 273 \text{ K}$$

$$= -6.81 \times 10^3 \text{ J} = \boxed{-\ 6.81 \text{ kJ}}$$

12.10 (a) The work done *on* the fluid is the negative since $V_f > V_i$ of the area under the curve on the PV diagram. Thus,

$$W_{if} = -\quad 6.00 \times 10^6 \text{ Pa}\quad 2.00 - 1.00\quad \text{m}^3$$
$$+\ \frac{1}{2}\left[\ 6.00 - 2.00\ \times 10^6 \text{ Pa}\right] 2.00 - 1.00\quad \text{m}^3$$
$$+\ 2.00 \times 10^6 \text{ Pa}\quad 4.00 - 2.00\quad \text{m}^3$$

$$W_{if} = -1.20 \times 10^7 \text{ J} = \boxed{-\ 12.0 \text{ MJ}}$$

(b) When the system follows the process curve in the reverse direction with $V_f < V_i$, the work done *on* the fluid equals the area under the process curve, which is the negative of that computed in (a). Thus,

$$W_{fi} = -W_{if} = \boxed{+12.0 \text{ MJ}}$$

12.11 From kinetic theory, the average kinetic energy per molecule is

$$\overline{KE}_{\text{molecule}} = \frac{3}{2} k_B T = \frac{3}{2}\left(\frac{R}{N_A}\right) T$$

For a monatomic ideal gas containing N molecules, the total energy associated with random molecular motions is

$$U = N \cdot \overline{KE}_{\text{molecule}} = \frac{3}{2}\left(\frac{N}{N_A}\right) RT = \frac{3}{2} nRT$$

Since $PV = nRT$ for an ideal gas, the internal energy of a monatomic ideal gas is found to be given by $\boxed{U = (3/2)PV}$.

12.12 (a) The initial absolute temperature is $T_i = 20.0° + 273.15 = 293 \text{ K}$, so the initial pressure is

$$P_i = \frac{nRT_i}{V_i} = \frac{10.0 \text{ mol} \quad 8.31 \text{ J/mol} \cdot \text{K} \quad 293 \text{ K}}{0.300 \text{ m}^3} = \boxed{8.12 \times 10^4 \text{ K}}$$

(b) For a monatomic ideal gas, the internal energy is $U = 3nRT/2$. Thus,

$$U_i = \frac{3}{2} nRT_i = \frac{3}{2} \; 10.0 \text{ mol} \quad 8.31 \text{ J/mol} \cdot \text{K} \quad 293 \text{ K} = \boxed{3.65 \times 10^4 \text{ J}}$$

(c) The work done on the gas in this isobaric expansion is

$$W = -P \; \Delta V = - \; 8.12 \times 10^4 \text{ Pa} \quad 1.000 \text{ m}^3 - 0.300 \text{ m}^3 = \boxed{-5.68 \times 10^4 \text{ J}}$$

(d) $$T_f = \frac{P_f V_f}{nR} = \frac{8.12 \times 10^4 \text{ Pa} \quad 1.00 \text{ m}^3}{10.0 \text{ mol} \quad 8.31 \text{ J/mol} \cdot \text{K}} = \boxed{977 \text{ K}}$$

(e) $U_f = \frac{3}{2} nRT_f = \frac{3}{2}\ 10.0 \text{ mol}\ \ 8.31 \text{ J/mol} \cdot \text{K}\ \ 977 \text{ K} = \boxed{1.22 \times 10^5 \text{ J}}$

(f) $\Delta U = U_f - U_i = 1.22 \times 10^5 \text{ J} - 3.65 \times 10^4 \text{ J} = \boxed{8.55 \times 10^4 \text{ J}}$

(g) $\Delta U - W = 8.55 \times 10^4 \text{ J} - -5.68 \times 10^4 \text{ J} = \boxed{+1.42 \times 10^5 \text{ J}}$

(h) Since $\Delta U - W > 0$, the increase in the internal energy of the gas exceeds the energy transferred to the gas by work. Thus, $\boxed{\text{additional energy must be transferred to the gas by heat.}}$

(i) The additional energy that must be transferred to the gas by heat is

$\Delta U - W = \boxed{+1.42 \times 10^5 \text{ J}}$ [See part (g) above]

(j) The suggested relationship is $\boxed{Q = \Delta U - W}$, which is a statement of the first law of thermodynamics.

12.13 (a) Along the direct path *IF*, the work done on the gas is

$$W = -\ \textit{area under curve}$$

$$= -\left[\ 1.00 \text{ atm}\ \ 4.00 \text{ L} - 2.00 \text{ L}\ + \frac{1}{2}\ 4.00 \text{ atm} - 1.00 \text{ atm}\ \ 4.00 \text{ L} - 2.00 \text{ L}\ \right]$$

$$W = -\ 5.00 \text{ atm} \cdot \text{L} \left(\frac{1.013 \times 10^5 \text{ Pa}}{1 \text{ atm}}\right)\left(\frac{10^{-3} \text{ m}^3}{1 \text{ L}}\right) = -506.5 \text{ J}$$

Thus, $\Delta U = Q + W = 418 \text{ J} - 506.5 \text{ J} = \boxed{-88.5 \text{ J}}$.

(b) Along path *IAF*, the work done on the gas is

$$W = -\ 4.00 \text{ atm}\ \ 4.00 \text{ L} - 2.00 \text{ L} \left(\frac{1.013 \times 10^5 \text{ Pa}}{1 \text{ atm}}\right)\left(\frac{10^{-3} \text{ m}^3}{1 \text{ L}}\right) = -810 \text{ J}$$

From the first law, $Q = \Delta U - W = -88.5 \text{ J} - -810 \text{ J} = \boxed{722 \text{ J}}$.

12.14 From the first law,

$$Q = \Delta U - W = -500\text{ J} - 220\text{ J} = \boxed{-720\text{ J}}$$

The negative sign in the result means that energy is transferred *from* the system by heat.

12.15 (a)

$$W = -P\ \Delta V = -\ 0.800\text{ atm}\quad -7.00\text{ L}\left(\frac{1.013\times 10^5\text{ Pa}}{1\text{ atm}}\right)\left(\frac{10^{-3}\text{ m}^3}{1\text{ L}}\right) = \boxed{567\text{ J}}$$

(b)

$$\Delta U = Q + W = -400\text{ J}+567\text{ J} = \boxed{167\text{ J}}$$

12.16 The work done on the gas is the negative (since $V_f > V_i$) of the area under the curve on the PV diagram, or

$$W = -\left[P_0\ \ 2V_0 - V_0\ \ + \frac{1}{2}\ 2P_0 - P_0\ \ 2V_0 - V_0\right] = -\frac{3}{2}P_0V_0$$

From the result of problem 11,

$$\Delta U = \frac{3}{2}P_fV_f - \frac{3}{2}P_iV_i = \frac{3}{2}\ 2P_0\ \ 2V_0\ - \frac{3}{2}P_0V_0 = \frac{9}{2}P_0V_0$$

Thus, from the first law,

$$Q = \Delta U - W = \frac{9}{2}P_0V_0 - \left(-\frac{3}{2}P_0V_0\right) = \boxed{6P_0V_0}$$

12.17 (a) The change in the volume occupied by the gas is

$$\Delta V = V_f - V_i = A\ \ L_f - L_i\ = 0.150\text{ m}^2\quad -0.200\text{ m}\ = -3.00\times 10^{-2}\text{ m}^3$$

and the work done by the gas is

$$W_{\text{by gas}} = +P\ \ \Delta V = 6\ 000\text{ Pa}\quad -3.00\times 10^{-2}\text{ m}^3 = \boxed{-180\text{ J}}$$

(b) The first law of thermodynamics is $\Delta U = Q_{\text{input}} + W_{\text{on gas}} = -Q_{\text{output}} - W_{\text{by gas}}$. Thus, if $\Delta U = -8.00\text{ J}$, the energy transferred out of the system by heat is

$$Q_{\text{output}} = -\Delta U - W_{\text{by gas}} = -\ -8.00\text{ J}\ -\ -180\text{ J} = \boxed{+188\text{ J}}$$

12.18 The work done on the gas is the negative since $V_f > V_i$ of the area under the curve on the PV diagram, so

$$W = -\left[P_0 \quad V_0 - 2V_0 \quad + \frac{1}{2} \quad 2P_0 - P_0 \quad V_0 - 2V_0 \right] = +\frac{3}{2} P_0 V_0 \text{, or } \boxed{W > 0}$$

From the result of problem 11,

$$\Delta U = \frac{3}{2} P_f V_f - \frac{3}{2} P_i V_i = \frac{3}{2} \quad 2P_0 \quad V_0 \quad - \frac{3}{2} \quad P_0 \quad 2V_0 \quad = \boxed{0}$$

Then, from the first law, $Q = \Delta U - W = 0 - \frac{3}{2} P_0 V_0 = -\frac{3}{2} P_0 V_0$, or $\boxed{Q < 0}$.

12.19 (a) The change in the internal energy of an ideal gas is $\Delta U = nC_v \quad \Delta T$. Thus, for this isothermal process, $\boxed{\Delta U = 0}$.

(b) From the first law of thermodynamics, the energy transfer by heat is $Q = \Delta U - W$. Therefore, if the work done on the gas in this isothermal process is $W = 75 \text{ J}$, we have

$$Q = \Delta U - W = 0 - 75 \text{ J} = \boxed{-75 \text{ J}}$$

12.20 (a) Since the process is adiabatic, the energy transfer by heat is $\boxed{Q = 0}$.

(b) If the work done on the gas is $W = +125 \text{ J}$, the first law of thermodynamics gives

$$\Delta U = Q + W = 0 + 125 \text{ J} = \boxed{125 \text{ J}}$$

12.21 (a) In an isothermal process involving an ideal gas, the work done on the gas is $W = -W_{\text{env}} = -nRT \ln \; V_f / V_i$. But, when temperature is constant, the ideal gas law gives $P_i V_i = P_f V_f = nRT$ and we may write the work done on the gas as

$$W = -P_i V_i \ln\left(\frac{V_f}{V_i}\right) = - \; 1.00 \times 10^5 \text{ Pa} \quad 0.500 \text{ m}^3 \quad \ln\left(\frac{1.25 \text{ m}^3}{0.500 \text{ m}^3}\right) = \boxed{-4.58 \times 10^4 \text{ J}}$$

(b) The change in the internal energy of an ideal gas is $\Delta U = nC_v \ \Delta T$, and for an isothermal process, we have $\Delta U = 0$. Thus, from the first law of thermodynamics, the energy transfer by heat in this isothermal expansion is

$$Q = \Delta U - W = 0 - \ -4.58 \times 10^4 \text{ J} \ = \boxed{+4.58 \times 10^4 \text{ J}}$$

(c) $\boxed{\Delta U = 0}$ [See part (b) above.]

12.22 (a) From the ideal gas law, $P_i V_i = nRT_i$ and $P_f V_f = nRT_f$. Thus, if $P_i = P_f = P$, subtracting these two expressions gives $PV_f - PV_i = nRT_f - nRT_i$, or

$$\boxed{P \ \Delta V \ = nR \ \Delta T}$$

(b) For a monatomic, ideal gas containing N gas atoms, the internal energy is $U = N \ \frac{1}{2} m\overline{v^2} \ = \ nN_A \ \ \frac{3}{2} k_B T \ = \frac{3}{2} nRT$. Thus, the change in internal energy of this gas in a thermodynamic process is $\Delta U = \frac{3}{2} nR \ \Delta T$. But, using the result of part (a) above, we have, for an isobaric process involving an monatomic ideal gas,

$$\Delta U = \frac{3}{2} nR \ \Delta T \ = \frac{3}{2} P \ \Delta V \ = \boxed{\frac{3}{2} W_{\text{env}}}$$

(c) We recall that the work done on the gas is $W = -W_{\text{env}}$, and use the first law of thermodynamics to find that the energy transferred to the gas by heat to be

$$Q = \Delta U - W = \Delta U + W_{\text{env}} = \frac{3}{2} W_{\text{env}} + W_{\text{env}} \qquad \text{or} \qquad \boxed{Q = \frac{5}{2} W_{\text{env}}}$$

(d) In an isobaric *expansion* $\Rightarrow \ \Delta V > 0$, the work done on the environment is $W_{\text{env}} = P \ \Delta V \ > 0$. Thus, from the result of part (c) above, the energy transfer as heat is $Q > 0$, meaning that the energy flow is *into* the gas. Therefore, it is

$$\boxed{\text{impossible for the gas to exhaust thermal energy in an isobaric expansion}}$$

12.23 (a) $W = -P(\Delta V)$

$$= -(1.013 \times 10^5\,\text{Pa})\left[1.09\ \text{cm}^3 - 1.00\,\text{cm}^3\left(\frac{1\,\text{m}^3}{10^6\ \text{cm}^3}\right)\right] = \boxed{-9.12 \times 10^{-3}\,\text{J}}$$

(b) To freeze the water, the required energy transfer by heat is

$$Q = -mL_f = -\ 1.00 \times 10^{-3}\ \text{kg}\quad 3.33 \times 10^5\ \text{J/kg} = -333\ \text{J}$$

The first law then gives

$$\Delta U = Q + W = -333\ \text{J} - 9.12 \times 10^{-3}\ \text{J} = \boxed{-\ 333\ \text{J}}$$

12.24 Volume is constant in process BC, so $\boxed{W_{BC} = 0}$. Given that $\boxed{Q_{BC} < 0}$, the first law shows that $\Delta U_{BC} = Q_{BC} + W_{BC} = Q_{BC} + 0$. Thus, $\boxed{\Delta U_{BC} < 0}$.

For process CA, $\Delta V_{CA} = V_A - V_C < 0$, so $W = -P\ \Delta V$ shows that $\boxed{W_{CA} > 0}$. Then, given that $\boxed{\Delta U_{CA} < 0}$, the first law gives $Q_{CA} = \Delta U_{CA} - W_{CA}$ and $\boxed{Q_{CA} < 0}$.

In process AB, the work done on the system is $W = -$ area under curve AB where

$$\text{area under curve } AB = P_A\ V_B - V_A + \frac{1}{2}\ P_B - P_A\ \ V_B - V_A > 0$$

Hence, $\boxed{W_{AB} < 0}$. For the cyclic process, $\Delta U = \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA} = 0$, so,

$\Delta U_{AB} = -\ \Delta U_{BC} + \Delta U_{CA}$. This gives $\boxed{\Delta U_{AB} > 0}$, since both ΔU_{BC} and ΔU_{CA} are negative.

Finally, from the first law, $Q = \Delta U - W$ shows that $\boxed{Q_{AB} > 0}$ since both ΔU_{AB} and $-\ \text{W}_{AB}$ are positive.

12.25 (a) The original volume of the aluminum is

$$V_0 = \frac{m}{\rho} = \frac{5.0\ \text{kg}}{2.70 \times 10^3\ \text{kg/m}^3} = 1.85 \times 10^{-3}\ \text{m}^3$$

and the change in volume is $\Delta V = \beta V_0\ \Delta T = 3\alpha\ V_0\ \Delta T$, or

$$\Delta V = 3\left[24 \times 10^{-6}\ \ ^\circ\text{C}^{-1}\right] 1.85 \times 10^{-3}\ \ 70^\circ\text{C} = 9.3 \times 10^{-6}\ \text{m}^3$$

The work done *by* the aluminum is then

$$W_{\text{by system}} = +P\ \Delta V = 1.013 \times 10^5\ \text{Pa}\quad 9.3 \times 10^{-6}\ \text{m}^3 = \boxed{0.95\ \text{J}}$$

(b) The energy transferred by heat to the aluminum is

$$Q = mc_{\text{Al}}\ \Delta T = 5.0\ \text{kg}\quad 900\ \text{J/kg}\cdot°\text{C}\quad 70°\text{C} = \boxed{3.2 \times 10^5\ \text{J}}$$

(c) The work done on the aluminum is $W = -W_{\text{by system}} = -0.95\ \text{J}$, so the first law gives

$$\Delta U = Q + W = 3.2 \times 10^5\ \text{J} - 0.95\ \text{J} = \boxed{3.2 \times 10^5\ \text{J}}$$

12.26 (a) The work done on the gas in each process is the negative of the area under the process curve on the *PV* diagram.

For path *IAF*, $W_{IAF} = W_{IA} + W_{AF} = 0 + W_{AF}$, or

$$W_{IAF} = -\left[1.50\ \text{atm}\left(1.013 \times 10^5\ \frac{\text{Pa}}{\text{atm}}\right)\right]\left[0.500\ \text{L}\left(\frac{10^{-3}\ \text{m}^3}{1\ \text{L}}\right)\right]$$
$$= \boxed{-76.0\ \text{J}}$$

For path *IBF*, $W_{IBF} = W_{IB} + W_{BF} = W_{IB} + 0$, or

$$W_{IBF} = -\left[2.00\ \text{atm}\left(1.013 \times 10^5\ \frac{\text{Pa}}{\text{atm}}\right)\right]\left[0.500\ \text{L}\left(\frac{10^{-3}\ \text{m}^3}{1\ \text{L}}\right)\right]$$
$$= \boxed{-101\ \text{J}}$$

For path *IF*, $W_{IF} = W_{AF} - \textit{triangular area}$, or

$$W_{IF} = -76.0\ \text{J} - \frac{1}{2}\left[0.500\ \text{m}^3\left(1.013 \times 10^5\ \frac{\text{Pa}}{\text{atm}}\right)\right]\left[0.500\ \text{L}\left(\frac{10^{-3}\ \text{m}^3}{1\ \text{L}}\right)\right]$$
$$= \boxed{-88.7\ \text{J}}$$

(b) Using the first law, with $\Delta U = U_F - U_A = 180 - 91.0\ \text{J} = 89.0\ \text{J}$, for each process gives

$$Q_{IAF} = \Delta U - W_{IAF} = 89.0\ \text{J} - -76.0\ \text{J} = \boxed{165\ \text{J}}$$

$$Q_{IBF} = \Delta U - W_{IBF} = 89.0\text{ J} - -101\text{ J} = \boxed{190\text{ J}}$$

$$Q_{IF} = \Delta U - W_{IF} = 89.0\text{ J} - -88.7\text{ J} = \boxed{178\text{ J}}$$

12.27 (a) For adiabatic processes in ideal gases, $P_f V_f^\gamma = P_i V_i^\gamma = \text{constant}$. From the ideal gas law, $P = nRT/V$, so the above expression becomes

$$\left(\frac{\cancel{nR}T_f}{V_f}\right)V_f^\gamma = \left(\frac{\cancel{nR}T_i}{V_i}\right)V_i^\gamma \qquad \text{or} \qquad T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$$

which can be summarized as $\boxed{TV^{\gamma-1} = C}$, where C is a constant.

(b) The current radius of the universe is assumed to be $r_f = 1.4 \times 10^{26}\text{ m}$ and the temperature is $T_f = 2.7\text{ K}$. Since $\gamma = 1.67 = 5/3$ for monatomic ideal gases, the temperature T_i of the universe when its radius was $r_i = 2.0\text{ cm} = 2.0 \times 10^{-2}\text{ m}$ must have been

$$T_i = T_f\left(\frac{V_f^{\gamma-1}}{V_i^{\gamma-1}}\right) = T_f\left(\frac{V_f}{V_i}\right)^{\gamma-1} = T_f\left(\frac{\cancel{\frac{4}{3}\pi} r_f^3}{\cancel{\frac{4}{3}\pi} r_i^3}\right)^{\gamma-1} = T_f\left(\frac{r_f}{r_i}\right)^{3\gamma-3}$$

or

$$T_i = 2.7\text{ K}\left(\frac{1.4 \times 10^{26}\text{ m}}{2.0 \times 10^{-2}\text{ m}}\right)^{3\ 5/3\ -3} = 2.7\text{ K}\quad 7.0 \times 10^{27}\ ^{2.0} = \boxed{1.3 \times 10^{56}\text{ K}}$$

12.28 (a) The number of atoms per mole in any monatomic gas is Avogadro's number $N_A = 6.02 \times 10^{23}\text{ atoms/mol}$. Thus, if the density of gas in the Universe is 1 hydrogen atom per cubic meter, the number of moles per unit volume is

$$\frac{n}{V} = \frac{1\text{ atom/m}^3}{N_A} == \frac{1\text{ atom/m}^3}{6.02 \times 10^{23}\text{ atoms/mol}} = \boxed{1.66 \times 10^{-24}\text{ mol/m}^3}$$

(b) With the density of gas found in part (a) and an absolute temperature of $T = 2.7\text{ K}$, the ideal gas law gives the pressure of the Universe as

$$P = \left(\frac{n}{V}\right) RT = 1.66 \times 10^{-24}\ \text{mol/m}^3 \quad 8.31\ \text{J/mol} \cdot \text{K} \quad 2.7\ \text{K} = \boxed{3.7 \times 10^{-23}\ \text{Pa}}$$

(c) For an adiabatic expansion, $P_i V_i^\gamma = P_f V_f^\gamma$ with $\gamma = 1.67 = 5/3$ for monatomic ideal gases, so the initial pressure of the Universe is estimated to be

$$P_i = P_f \left(\frac{V_f^\gamma}{V_i^\gamma}\right) = P_f \left(\frac{V_f}{V_i}\right)^\gamma = P_f \left(\frac{\cancel{\frac{4}{3}\pi} r_f^3}{\cancel{\frac{4}{3}\pi} r_f^3}\right)^\gamma = P_f \left(\frac{r_f}{r_i}\right)^{3\gamma}$$

or

$$P_i = 3.7 \times 10^{-23}\ \text{Pa} \left(\frac{1.4 \times 10^{26}\ \text{m}}{2.0 \times 10^{-2}\ \text{m}}\right)^{3\ 5/3} = 3.7 \times 10^{-23}\ \text{Pa} \quad 7.0 \times 10^{27}\ ^{5.0}$$

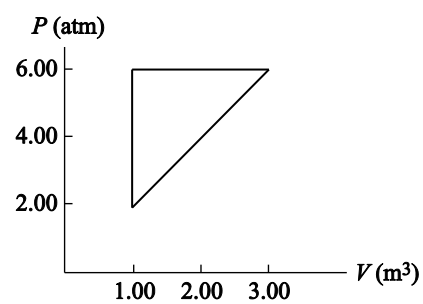
giving

$$P_i = \left[\ 3.7 \times 10^{-23}\ \text{Pa} \quad 7.0\ ^{5.0}\right] 10^{27}\ ^{5.0} = \left[\ 3.7 \times 10^{-23}\ \text{Pa} \quad 1.7 \times 10^4\ \right] 10^{135}$$

and

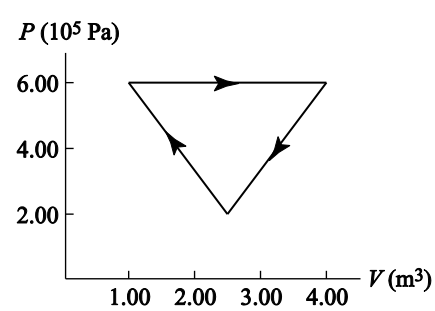
$$P_i = \left[6.3 \times 10^{-19}\ \text{Pa}\right] 10^{135} = \boxed{6.3 \times 10^{116}\ \text{Pa}}$$

12.29 The net work done by a heat engine operating on the cyclic process shown in Figure P12.29 equals the triangular area enclosed by this process curve. Thus,



$$W_{net} = \frac{1}{2}\ 6.00\ \text{atm} - 2.00\ \text{atm} \quad 3.00\ \text{m}^3 - 1.00\ \text{m}^3$$
$$= 4.00\ \cancel{\text{atm}} \cdot \text{m}^3 \left(\frac{1.013 \times 10^5\ \text{Pa}}{1\ \cancel{\text{atm}}}\right) = 4.05 \times 10^5\ \text{J}$$
$$= 405 \times 10^3\ \text{J} = \boxed{405\ \text{kJ}}$$

12.30 The net work done by a heat engine operating on the cyclic process shown in Figure P12.30 equals the triangular area enclosed



by this process curve. This is,

$$W_{net} = \frac{1}{2}\ base \quad altitude$$
$$= \frac{1}{2}\left[\ 4.00 - 1.00 \quad \text{m}^3\right]\left[\ 6.00 - 2.00 \ \times 10^5\ \text{Pa}\right]$$
$$= \frac{1}{2}\ 3.00\ \text{m}^3 \quad 4.00 \times 10^5\ \text{Pa} \ = \boxed{6.00 \times 10^5\ \text{J}}$$

12.31 The maximum possible efficiency for a heat engine operating between reservoirs with absolute temperatures of $T_c = 25° + 273 = 298\ \text{K}$ and $T_h = 375° + 273 = 648\ \text{K}$ is the Carnot efficiency

$$e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{298\ \text{K}}{648\ \text{K}} = \boxed{0.540 \text{ or } 54.0\%}$$

12.32 (a) The absolute temperature of the cold reservoir is $T_c = 20° + 273 = 293\ \text{K}$. If the Carnot efficiency is to be $e_C = 0.65$, it is necessary that

$$1 - \frac{T_c}{T_h} = 0.65 \qquad \text{or} \qquad \frac{T_c}{T_h} = 0.35 \qquad \text{and} \qquad T_h = \frac{T_c}{0.35}$$

Thus,

$$T_h = \frac{293\ \text{K}}{0.35} = 837\ \text{K} \qquad \text{or} \qquad T_h = 837 - 273 = \boxed{564°\text{C}}$$

(b) $\boxed{\text{No.}}$ Any real heat engine will have an efficiency less that the Carnot efficiency because it operates in an irreversible manner.

12.33 (a) The efficiency of a heat engine is $e = W_{\text{env}}/|Q_h|$, where W_{env} is the work done by the engine and $|Q_h|$ is the energy absorbed from the higher temperature reservoir. Thus, if $W_{\text{env}} = |Q_h|/4$, the efficiency is $e = 1/4 = \boxed{0.25 \text{ or } 25\%}$.

(b) From conservation of energy, the energy exhausted to the lower temperature reservoir is $|Q_c| = |Q_h| - W_{\text{env}}$. Therefore, if $W_{\text{env}} = |Q_h|/4$, we have $|Q_c| = 3|Q_h|/4$ or $\boxed{|Q_c|/|Q_h| = 3/4}$.

12.34 (a) From $e \equiv W_{\text{eng}}/|Q_h| = 1 - |Q_c|/|Q_h|$, the energy intake each cycle is

$$|Q_h| = \frac{|Q_c|}{1-e} = \frac{8\ 000\ \text{J}}{1-0.250} = 10\ 667\ \text{J} = \boxed{10.7\ \text{kJ}}$$

(b) From $\mathcal{P} = W_{\text{eng}}/t = e|Q_c|/t$, the time for one cycle is

$$t = \frac{e|Q_c|}{\mathcal{P}} = \frac{0.250\quad 10\ 667\ \text{J}}{5.00 \times 10^3\ \text{W}} = \boxed{0.533\ \text{s}}$$

12.35 (a) The maximum efficiency possible is that of a Carnot engine operating between the specified reservoirs.

$$e_C = \frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h} = 1 - \frac{703\ \text{K}}{2\ 143\ \text{K}} = \boxed{0.672 \quad \text{or } 67.2\%}$$

(b) From $e = \dfrac{W_{\text{eng}}}{|Q_h|}$, we find $W_{\text{eng}} = e|Q_h| = 0.420\ \ 1.40 \times 10^5\ \text{J} = 5.88 \times 10^4\ \text{J}$, so

$$\mathcal{P} = \frac{W_{\text{eng}}}{t} = \frac{5.88 \times 10^4\ \text{J}}{1.00\ \text{s}} = 5.88 \times 10^4\ \text{W} = \boxed{58.8\ \text{kW}}$$

12.36 The work done by the engine equals the change in the kinetic energy of the bullet, or

$$W_{\text{eng}} = \frac{1}{2} m_b v_f^2 - 0 = \frac{1}{2}\ 2.40 \times 10^{-3}\ \text{kg}\quad 320\ \text{m/s}^{\ 2} = 123\ \text{J}$$

Since the efficiency of an engine may be written as

$$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}}{W_{\text{eng}} + |Q_c|}$$

Where $|Q_c|$ is the exhaust energy from the engine, we find that

$$|Q_c| = W_{\text{eng}}\left(\frac{1}{e} - 1\right) = 123\ \text{J}\ \left(\frac{1}{0.110} - 1\right) = 1.10 \times 10^4\ \text{J}$$

This exhaust energy is absorbed by the 1.80-kg iron body of the gun, so the rise in temperature is

$$\Delta T = \frac{|Q_c|}{m_{gun}c_{iron}} = \frac{1.10 \times 10^4 \text{ J}}{1.80 \text{ kg} \quad 448 \text{ J/kg} \cdot {}^\circ\text{C}} = \boxed{13.7\,{}^\circ\text{C}}$$

12.37 (a) $e \equiv \dfrac{W_{\text{eng}}}{|Q_h|} = 1 - \dfrac{|Q_c|}{|Q_h|} = 1 - \dfrac{1\,200 \text{ J}}{1\,700 \text{ J}} = \boxed{0.294 \quad \text{or } 29.4\%}$

(b) $W_{\text{eng}} = |Q_h| - |Q_c| = 1\,700 \text{ J} - 1\,200 \text{ J} = \boxed{500 \text{ J}}$

(c) $\mathcal{P} = \dfrac{W_{\text{eng}}}{t} = \dfrac{500 \text{ J}}{0.300 \text{ s}} = 1.67 \times 10^3 \text{ W} = \boxed{1.67 \text{ kW}}$

12.38 (a) The coefficient of performance of a heat pump is $\text{COP} = |Q_h|/W$, where $|Q_h|$ is the thermal energy delivered to the warm space and W is the work input required to operate the heat pump. Therefore,

$$|Q_h| = W \cdot \text{COP} = \mathcal{P} \cdot \Delta t \cdot \text{COP} = \left[\left(7.03 \times 10^3 \ \frac{\text{J}}{\cancel{\text{s}}}\right)(8.00 \ \cancel{\text{h}})\left(\frac{3\,600 \ \cancel{\text{s}}}{1 \ \cancel{\text{h}}}\right)\right]3.80 = \boxed{7.69 \times 10^8 \text{ J}}$$

(b) The energy extracted from the cold space (outside air) is

$$|Q_c| = |Q_h| - W = |Q_h| - \frac{|Q_h|}{\text{COP}} = |Q_h|\left(1 - \frac{1}{\text{COP}}\right)$$

or

$$|Q_c| = 7.69 \times 10^8 \text{ J}\left(1 - \frac{1}{3.80}\right) = \boxed{5.67 \times 10^8 \text{ J}}$$

12.39 (a) $W = \mathcal{P} \cdot \Delta t = \left(457 \ \dfrac{\cancel{\text{kWh}}}{\cancel{\text{y}}}\right)\left(\dfrac{3.60 \times 10^6 \text{ J}}{1 \ \cancel{\text{kWh}}}\right)\left(\dfrac{1 \ \cancel{\text{y}}}{365.242 \text{ d}}\right) \cdot (1 \text{ d}) = \boxed{4.50 \times 10^6 \text{ J}}$

(b) From the definition of the coefficient of performance for a refrigerator, $\text{COP}_{\text{R}} = |Q_c|/W$, the thermal energy removed from the cold space each day is

$$|Q_c| = \text{COP}_{\text{R}} \cdot W = 6.30 \ \ 4.50 \times 10^6 \text{ J} = \boxed{2.84 \times 10^7 \text{ J}}$$

(c) The water must be cooled 20.0°C before it will start to freeze, so the thermal energy that must be removed

from mass m of water to freeze it is $|Q_c| = mc_w|\Delta T| + mL_f$. The mass of water that can be frozen each day is then

$$m = \frac{|Q_c|}{c_w|\Delta T| + L_f} = \frac{2.84 \times 10^7 \text{ J}}{4\,186 \text{ J/kg} \cdot °\text{C} \quad 20.0°\text{C} \quad + 3.33 \times 10^5 \text{ J/kg}} = \boxed{68.2 \text{ kg}}$$

12.40 (a) The coefficient of performance of a heat pump is defined as

$$\text{COP}_{\text{hp}} = \frac{|Q_h|}{W} = \frac{|Q_h|}{|Q_h| - |Q_c|} = \frac{1}{1 - |Q_c|/|Q_h|}$$

But when a device operates on the Carnot cycle, $|Q_c|/|Q_h| = T_c/T_h$. Thus, the coefficient of performance for a Carnot heat pump would be

$$\text{COP}_{\text{hp,C}} = \frac{1}{1 - T_c/T_h} = \boxed{\frac{T_h}{T_h - T_c}}$$

(b) From the result of part (a) above, we observe that the COP of a Carnot heat pump would increase if the temperature difference $T_h - T_c$ became $\boxed{\text{smaller.}}$

(c) If $T_c = 50° + 273 = 323 \text{ K}$ and $T_h = 70° + 273 = 343 \text{ K}$, the COP of a Carnot heat pump would be

$$\text{COP}_{\text{hp,C}} = \frac{T_h}{T_h - T_c} = \frac{343 \text{ K}}{343 \text{ K} - 323 \text{ K}} = \boxed{17.2}$$

12.41 The actual efficiency of the engine is

$$e = 1 - \frac{|Q_c|}{|Q_h|} = 1 - \frac{300 \text{ J}}{500 \text{ J}} = 0.400$$

If this is 60.0% of the Carnot efficiency, then

$$e_C = \frac{e}{0.600} = \frac{0.400}{0.600} = \frac{2}{3}$$

Thus, from $e_C = 1 - T_c/T_h$, we find

$$\frac{T_c}{T_h} = 1 - e_C = 1 - \frac{2}{3} = \boxed{\frac{1}{3}}$$

12.42 (a) The Carnot efficiency represents the maximum possible efficiency. With $T_h = 20.0°\text{C} = 293\text{ K}$ and $T_c = 5.00°\text{C} = 278\text{ K}$, this efficiency is given by

$$e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{278\text{ K}}{293\text{ K}} = \boxed{0.0512 \quad \text{or } 5.12\%}$$

(b) The efficiency of an engine is $e = W_{\text{eng}}/|Q_h|$, so the minimum energy input by heat each hour is

$$|Q_h|_{\min} = \frac{W_{\text{eng}}}{e_{\max}} = \frac{\mathcal{P} \cdot \Delta t}{e_{\max}} = \frac{75.0 \times 10^6\ \text{J/s} \quad 3\ 600\text{ s}}{0.0512} = \boxed{5.27 \times 10^{12}\text{ J}}$$

(c) As fossil-fuel prices rise, this could be an attractive way to use solar energy. However, the potential environmental impact of such an engine would require serious study. The energy output, $|Q_c| = |Q_h| - W_{\text{eng}} = |Q_h|\ 1 - e$, to the low temperature reservoir (cool water deep in the ocean) could raise the temperature of over a million cubic meters of water by 1°C every hour.

12.43 From $\mathcal{P} = W_{\text{eng}}/t = e|Q_h|/t$, the energy input by heat in time t is $|Q_h| = \dfrac{\mathcal{P} \cdot t}{e}$

Thus, from $e = |Q_h| - |Q_c|/|Q_h|$, the energy expelled in time t is

$$|Q_c| = |Q_h|\ 1 - e = \left(\frac{\mathcal{P} \cdot t}{e}\right)\ 1 - e = \mathcal{P} \cdot t\left(\frac{1}{e} - 1\right)$$

In time t, the mass of cooling water used is $m = 1.0 \times 10^6\ \text{kg/s} \cdot t$, and its rise in temperature is

$$\Delta T = \frac{|Q_c|}{mc} = \frac{\mathcal{P} \cdot t}{1.0 \times 10^6 \text{ kg/s} \cdot t \cdot c}\left(\frac{1}{e} - 1\right)$$

$$= \frac{1\,000 \times 10^6 \text{ J/s}}{1.0 \times 10^6 \text{ kg/s} \quad 4\,186 \text{ J/kg} \cdot °\text{C}}\left(\frac{1}{0.33} - 1\right)$$

or

$$\Delta T = \boxed{0.49°\text{C}}$$

12.44 (a) The Carnot efficiency is

$$e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{353 \text{ K}}{623 \text{ K}} = 0.433$$

so the maximum power output is

$$\mathcal{P}_{\max} = \frac{W_{\text{eng max}}}{t} = \frac{e_C |Q_h|}{t} = \frac{0.433 \quad 21.0 \text{ kJ}}{1.00 \text{ s}} = \boxed{9.10 \text{ kW}}$$

(b) From $e = 1 - |Q_c|/|Q_h|$, the energy expelled by heat each cycle is

$$|Q_c| = |Q_h| \quad 1 - e \quad = \quad 21.0 \text{ kJ} \quad 1 - 0.433 \quad = \boxed{11.9 \text{ kJ}}$$

12.45 The thermal energy transferred to the room by the water as the water cools from 1.00×10^2 °C to 20.0°C is

$$Q = mc_w |\Delta T| = \quad 0.120 \text{ kg} \quad 4\,186 \text{ J/kg} \cdot °\text{C} \quad 80°\text{C} \quad = 4.0 \times 10^4 \text{ J}$$

If the room has a constant absolute temperature of $T = 20.0° + 273 = 293 \text{ K}$, the increase in the entropy of the room is

$$\Delta S = \frac{Q}{T} = \frac{4.0 \times 10^4 \text{ J}}{293 \text{ K}} = \boxed{140 \text{ J/K}}$$

12.46 The total momentum before collision is zero, so the combined mass must be at rest after the collision. The energy dissipated by heat equals the total initial kinetic energy,

$$Q = 2\left(\frac{1}{2}mv^2\right) = \ 2\ 000\ \text{kg}\ \ 20\ \text{m/s}^{\ 2} = 8.0 \times 10^5\ \text{J} = 800\ \text{kJ}$$

With the environment at an absolute temperature of $T = 23° + 273 = 296\ \text{K}$, the change in entropy is

$$\Delta S = \frac{\Delta Q_r}{T} = \frac{800\ \text{kJ}}{296\ \text{K}} = \boxed{2.7\ \text{kJ/K}}$$

12.47 The energy transferred from the water by heat, and absorbed by the freezer, is

$$Q = mL_f = \ \rho V\ \ L_f = \left[\ 10^3\ \ \text{kg/m}^3\ \ \ 1.0 \times 10^{-3}\ \text{m}^3\ \right]\left(3.33 \times 10^5\ \ \frac{\text{J}}{\text{kg}}\right) = 3.3 \times 10^5\ \text{J}$$

Thus, the change in entropy of the water is

(a) $$\Delta S_{water} = \frac{\Delta Q_{r\ \ water}}{T} = \frac{-3.3 \times 10^5\ \text{J}}{273\ \text{K}} = -1.2 \times 10^3\ \ \frac{\text{J}}{\text{K}} = \boxed{-1.2\ \ \text{kJ/K}}$$

and that of the freezer is

(b) $$\Delta S_{freezer} = \frac{\Delta Q_{r\ \ freezer}}{T} = \frac{+3.3 \times 10^5\ \text{J}}{273\ \text{K}} = \boxed{+1.2\ \ \text{kJ/K}}$$

12.48 The energy added to the water by heat is

$$\Delta Q_r = mL_v = \ 1.00\ \text{kg}\ \ \ 2.26 \times 10^6\ \ \text{J/kg}\ \ = 2.26 \times 10^6\ \text{J}$$

so the change in entropy is

$$\Delta S = \frac{\Delta Q_r}{T} = \frac{2.26 \times 10^6\ \text{J}}{373\ \text{K}} = 6.06 \times 10^3\ \ \frac{\text{J}}{\text{K}} = \boxed{6.06\ \ \text{kJ/K}}$$

12.49 The potential energy lost by the log is transferred away by heat, so

$$Q = mgh = \ 70\ \text{kg}\ \ \ 9.80\ \ \text{m/s}^2\ \ \ 25\ \text{m}\ \ = 1.7 \times 10^4\ \text{J}$$

and the change in entropy is $$\Delta S = \frac{\Delta Q_r}{T} = \frac{1.7 \times 10^4\ \text{J}}{300\ \text{K}} = \boxed{57\ \ \text{J/K}}$$

12.50 (a) In a game of dice, there is only $\boxed{\text{one way}}$ you can roll a 12. You must have a 6 on each die.

(b) There are $\boxed{\text{six ways}}$ to obtain a 7 with a pair of dice. The combinations that yield a 7 are 1 + 6, 2 + 5, 3 + 4, 4 + 3, 5 + 2, and 6 + 1. Note that 1 + 6 and 6 + 1 are different combinations in that the 6 occurs on different members of the pair of dice in the two combinations.

12.51 A quantity of energy, of magnitude Q, is transferred from the Sun and added to Earth. Thus,

$$\Delta S_{Sun} = \frac{-Q}{T_{Sun}} \quad \text{and} \quad \Delta S_{Earth} = \frac{+Q}{T_{Earth}}$$

so the total change in entropy is

$$\Delta S_{total} = \Delta S_{Earth} + \Delta S_{Sun} = \frac{Q}{T_{Earth}} - \frac{Q}{T_{Sun}}$$

$$= 1\,000 \text{ J} \left(\frac{1}{290 \text{ K}} - \frac{1}{5\,700 \text{ K}} \right) = \boxed{3.27 \text{ J/K}}$$

12.52 The change in entropy of a reservoir is $\Delta S = Q_r / T$, where Q_r is the energy absorbed $Q_r > 0$ or $Q_r < 0$ expelled by the reservoir, and T is the absolute temperature of the reservoir.

(a) For the hot reservoir:

$$\Delta S_h = \frac{-2.50 \times 10^3 \text{ J}}{725 \text{ K}} = \boxed{-3.45 \text{ J/K}}$$

(b) For the cold reservoir:

$$\Delta S_c = \frac{+2.50 \times 10^3 \text{ J}}{310 \text{ K}} = \boxed{+8.06 \text{ J/K}}$$

(c) For the Universe:

$$\Delta S_U = \Delta S_h + \Delta S_c = -3.45 \text{ J/K} + 8.06 \text{ J/K} = \boxed{+4.61 \text{ J/K}}$$

(d) The magnitudes of the thermal energy transfers, appearing in the numerators, are the same for the two reservoirs, but the cold reservoir necessarily has a smaller denominator. Hence, its positive change dominates.

12.53 (a) The table is shown below. On the basis of the table, the most probable result of a toss is $\boxed{\text{2 H and 2 T}}$

End Result	Possible Tosses	Total Number of Same Result
All H	HHHH	1
1T, 3H	HHHT, HHTH, HTHH, THHH	4
2T, 2H	HHTT, HTHT, THHT, HTTH, THTH, TTHH	6
3T, 1H	TTTH, TTHT, THTT, HTTT	4
All T	TTTT	1

(b) The most ordered state is the least likely. This is seen to be all H or all T .

(c) The least ordered state is the most likely. This is seen to be $\boxed{\text{2 H and 2 T}}$.

12.54 The change in entropy of a reservoir is $\Delta S = Q_r / T$, where Q_r is the energy absorbed $Q_r > 0$ or expelled $Q_r < 0$ by the reservoir, and T is the absolute temperature of the reservoir.

(a) For the hot reservoir, $Q_r = -|Q_h|$, and

$$\boxed{\Delta S_h = \frac{-|Q_h|}{T_h}}$$

(b) For the cold reservoir, $Q_r = +|Q_h|$, and

$$\boxed{\Delta S_c = \frac{+|Q_h|}{T_c}}$$

(c) For the Universe,

$$\Delta S_U = \Delta S_h + \Delta S_c = \boxed{-\frac{|Q_h|}{T_h} + \frac{|Q_h|}{T_c}}$$

12.55 By substituting 1 hour of study for 1 hour of sleep each day for a year, energy consumed while sleeping would change by $\Delta E_{sleep} = -\ 80\ \text{W}\ \cdot \Delta t$, and the energy consumed while studying would change by $\Delta E_{study} = +\ 230\ \text{W}\ \cdot \Delta t$. The net change in energy consumed would be

$$\Delta E_{net} = \Delta E_{sleep} + \Delta E_{study} = -\ 80\ \text{W}\ \cdot \Delta t + \ 230\ \text{W}\ \cdot \Delta t = \ +1.5 \times 10^2\ \text{W}\ \cdot \Delta t$$

The total time during which this change occurs is 1 hour per day for a year or a total of 365 hours. Thus,

$$\Delta E_{net} = \left(+1.5 \times 10^2\ \frac{\text{J}}{\cancel{\text{s}}}\right)\left(365\ \cancel{\text{h}}\right)\left(\frac{3\ 600\ \cancel{\text{s}}}{1\ \cancel{\text{h}}}\right) = +2.0 \times 10^8\ \text{J}$$

The additional fat burned to yield this increased consumption will be

$$\textit{Fat Burned} = \frac{\Delta E_{net}}{\textit{Energy Equivalent}} = \frac{+2.0 \times 10^8\ \text{J}}{1.7 \times 10^7\ \text{J/lb}} = \boxed{12\ \text{lb}}$$

12.56 (a) At the sleeping rate of 80 W, the time required for the body to use the 450 Cal of energy supplied by the bagel is

$$\Delta t = \frac{\Delta U}{\mathcal{P}} = \frac{450\ \text{Cal}}{80\ \text{J/s}}\left(\frac{4\ 186\ \text{J}}{1\ \text{Cal}}\right)\left(\frac{1\ \text{h}}{3\ 600\ \text{s}}\right) = \boxed{6.5\ \text{h}}$$

(b) The increased metabolic rate while working out is $\mathcal{P} = 80\ \text{W} + 650\ \text{W} = 7.3 \times 10^2\ \text{W}$, and the time to use the energy from the bagel at this rate is

$$\Delta t = \frac{\Delta U}{\mathcal{P}} = \frac{450\ \text{Cal}}{7.3 \times 10^2\ \text{J/s}}\left(\frac{4\ 186\ \text{J}}{1\ \text{Cal}}\right)\left(\frac{1\ \text{h}}{3\ 600\ \text{s}}\right) = \boxed{0.72\ \text{h}}$$

(c) $W_{\substack{\text{per}\\ \text{lift}}} = F \cdot \Delta x = mgh = \ 120\ \text{kg}\ \ 9.8\ \text{m/s}^2\ \ 2.0\ \text{m}\ = \boxed{2.4 \times 10^3\ \text{J}}$

(d) The *additional* energy consumed in 1 minute while working out (instead of sleeping) is $\Delta U = \mathcal{P}_{\text{increase}}\ 1\ \text{min}\ = \ 650\ \text{J/s}\ \ 60\ \text{s}\ = 3.9 \times 10^4\ \text{J}$. The number of barbell lifts this should allow in

1 minute is

$$N = \frac{\Delta U}{W_{\substack{\text{per}\\\text{lift}}}} = \frac{3.9 \times 10^4 \text{ J}}{2.4 \times 10^3 \text{ J}} = \boxed{16}$$

(e) $\boxed{\text{No.}}$ The body is only about 25% efficient in converting chemical energy to mechanical energy.

12.57 The maximum rate at which the body can dissipate waste heat by sweating is

$$\frac{\Delta Q}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right) L_v = \left(1.5 \ \frac{\cancel{\text{kg}}}{\cancel{\text{h}}}\right)\left(2\ 430 \times 10^3 \ \frac{\text{J}}{\cancel{\text{kg}}}\right)\left(\frac{1 \ \cancel{\text{h}}}{3\ 600 \text{ s}}\right) = 1.0 \times 10^3 \text{ W}$$

If this represents 80% of the maximum sustainable metabolic rate $\left[\text{i.e., } \Delta Q/\Delta t = 0.80 \ \Delta U/\Delta t_{\text{max}}\right]$, then that maximum rate is

$$\left(\frac{\Delta U}{\Delta t}\right)_{\text{max}} = \frac{\Delta Q/\Delta t}{0.80} = \frac{1.0 \times 10^3 \text{ W}}{0.80} = \boxed{1.3 \times 10^3 \text{ W}}$$

12.58 Operating between reservoirs having temperatures of $T_h = 100°\text{C} = 373 \text{ K}$ and $T_c = 20°\text{C} = 293 \text{ K}$, the theoretical efficiency of a Carnot engine is

$$e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{293 \text{ K}}{373 \text{ K}} = 0.21$$

If the temperature of the hotter reservoir is changed to $T_h' = 550°\text{C} = 823 \text{ K}$, the theoretical efficiency of the Carnot engine increases to

$$e_C' = 1 - \frac{T_c}{T_h'} = 1 - \frac{293 \text{ K}}{823 \text{ K}} = 0.64$$

The factor by which the efficiency has increased is

$$\frac{e_C'}{e_C} = \frac{0.64}{0.21} = \boxed{3.0}$$

12.59 The work output from the engine in an interval of one second is $W_{\text{eng}} = 1\ 500 \text{ kJ}$. Since the efficiency of an engine may be expressed as

$$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}}{W_{\text{eng}} + |Q_c|}$$

the exhaust energy each second is $|Q_c| = W_{\text{eng}}\left(\frac{1}{e} - 1\right) = 1\ 500\ \text{kJ}\ \left(\frac{1}{0.25} - 1\right) = 4.5 \times 10^3\ \text{kJ}$

The mass of water flowing through the cooling coils each second is

$$m = \rho V = 10^3\ \text{kg/m}^3\ \ 60\ \text{L}\ \ 10^{-3}\ \text{m}^3/1\ \text{L} = 60\ \text{kg}$$

so the rise in the temperature of the water is

$$\Delta T = \frac{|Q_c|}{mc_{water}} = \frac{4.5 \times 10^6\ \text{J}}{60\ \text{kg}\ \ 4\ 186\ \text{J/kg} \cdot °\text{C}} = \boxed{18°\text{C}}$$

12.60 The energy exhausted from a heat engine is

$$Q_c = Q_h - W_{\text{eng}} = \frac{W_{\text{eng}}}{e} - W_{\text{eng}} = W_{\text{eng}}\left(\frac{1}{e} - 1\right)$$

where Q_h is the energy input from the high temperature reservoir, W_{eng} is the useful work done, and $e = W_{\text{eng}}/Q_h$ is the efficiency of the engine.

For a Carnot engine, the efficiency is $e_C = 1 - T_c/T_h = T_h - T_c\ /T_h$, so we now have

$$Q_c = W_{\text{eng}}\left(\frac{T_h}{T_h - T_c} - 1\right) = W_{\text{eng}}\left(\frac{T_c}{T_h - T_c}\right)$$

Thus, if $T_h = 100°\text{C} = 373\ \text{K}$ and $T_c = 20°\text{C}{=}293\ \text{K}$, the energy exhausted when the engine has done $5.0 \times 10^4\ \text{J}$ of work is

$$Q_c = 5.0 \times 10^4\ \text{J}\ \left(\frac{293\ \text{K}}{373\ \text{K} - 293\ \text{K}}\right) = 1.83 \times 10^5\ \text{J}$$

The mass of ice (at 0°C) this exhaust energy could melt is

$$m = \frac{Q_c}{L_{f,\text{ water}}} = \frac{1.83 \times 10^5 \text{ J}}{3.33 \times 10^5 \text{ J/kg}} = \boxed{0.55 \text{ kg}}$$

12.61 (a) The work done *by* the system in process *AB* equals the area under this curve on the *PV* diagram. Thus,

$$W_{\text{by system}} = \text{triangular area} + \text{rectangular area} \quad \text{or}$$

$$W_{\substack{\text{by}\\ \text{system}}} = \left[\frac{1}{2}\ 4.00 \text{ atm} \quad 40.0 \text{ L} + 1.00 \text{ atm} \quad 40.0 \text{ L}\right]\left(1.013 \times 10^5 \ \frac{\text{Pa}}{\text{atm}}\right)\left(\frac{10^{-3} \text{ m}^3}{\text{L}}\right)$$

$$= 1.22 \times 10^4 \text{ J} = \boxed{12.2 \text{ kJ}}$$

Note that the work done on the system is $W_{AB} = -W_{\substack{\text{by}\\ \text{system}}} = -12.2 \text{ kJ}$ for this process.

(b) The work done on the system (that is, the work input) for process *BC* is the negative of the area under the curve on the *PV* diagram, or

$$W_{BC} = -\left[1.00 \text{ atm} \quad 10.0 \text{ L} - 50.0 \text{ L}\right]\left(1.013 \times 10^5 \ \frac{\text{Pa}}{\text{atm}}\right)\left(\frac{10^{-3} \text{ m}^3}{1 \text{ L}}\right)$$

$$= \boxed{4.05 \text{ kJ}}$$

(c) The change in internal energy is zero for any full cycle, so the first law gives

$$Q_{cycle} = \Delta U_{cycle} - W_{cycle} = 0 - W_{AB} + W_{BC} + W_{CA}$$

$$= 0 - -12.2 \text{ kJ} + 4.05 \text{ kJ} + 0 = \boxed{8.15 \text{ kJ}}$$

12.62 (a) From the first law, $\Delta U_{1\to 3} = Q_{123} + W_{123} = +418 \text{ J} + -167 \text{ J} = \boxed{251 \text{ J}}$.

(b) The difference in internal energy between states 1 and 3 is independent of the path used to get from state 1 to state 3.

Thus, $\Delta U_{1\to 3} = Q_{143} + W_{143} = 251 \text{ J}$, and

$$Q_{143} = 251 \text{ J} - W_{143} = 251 \text{ J} - -63.0 \text{ J} = \boxed{314 \text{ J}}$$

(c) $W_{12341} = W_{123} + W_{341} = W_{123} + -W_{143} = -167\text{ J} - -63.0\text{ J} = -104\text{ J}$

or $\boxed{\text{104 J of work is done } \textit{by} \text{ the gas}}$ in the cyclic process 12341.

(d) $W_{14321} = W_{143} + W_{321} = W_{143} + -W_{123} = -63.0\text{ J} - -167\text{ J} = +104\text{ J}$

or $\boxed{\text{104 J of work is done } \textit{on} \text{ the gas}}$ in the cyclic process 14321.

(e) The change in internal energy is $\boxed{\text{zero}}$ for both parts (c) and (d) since both are cyclic processes.

12.63 (a) The change in length, due to linear expansion, of the rod is

$$\Delta L = \alpha L_0 \ \Delta T = \left[11 \times 10^{-6} \ \ ^\circ\text{C}^{-1}\right] 2.0\text{ m} \ \ 40^\circ\text{C} - 20^\circ\text{C} = 4.4 \times 10^{-4}\text{ m}$$

The load exerts a force $F = mg = 6\ 000\text{ kg} \ \ 9.80\text{ m/s}^2 = 5.88 \times 10^4\text{ N}$ on the end of the rod in the direction of movement of that end. Thus, the work done on the rod is

$$W = F \cdot \Delta L = 5.88 \times 10^4\text{ N} \ \ 4.4 \times 10^{-4}\text{ m} = \boxed{26\text{ J}}$$

(b) The energy added by heat is

$$Q = mc \ \Delta T = 100\text{ kg}\left(448 \ \frac{\text{J}}{\text{kg} \cdot {}^\circ\text{C}}\right) 20\ ^\circ\text{C} = \boxed{9.0 \times 10^5\text{ J}}$$

(c) From the first law, $\Delta U = Q + W = 9.0 \times 10^5\text{ J} + 26\text{ J} = \boxed{9.0 \times 10^5\text{ J}}$.

12.64 (a) The work done *by* the gas during each full cycle equals the area enclosed by the cycle on the PV diagram. Thus

$$W_{\text{by gas}} = 3P_0 - P_0 \ \ 3V_0 - V_0 = \boxed{4P_0V_0}$$

(b) Since the work done on the gas is $W = -W_{\text{by gas}} = -4P_0V_0$ and $\Delta U = 0$ for any cyclic process, the first law gives

$$Q = \Delta U - W = 0 - \ -4P_0V_0 \ = \boxed{4P_0V_0}$$

(c) From the ideal gas law, $P_0V_0 = nRT_0$, so the work done by the gas each cycle is

$$W_{\text{by gas}} = 4nRT_0 = 4 \ 1.00 \text{ mol} \left(8.31 \ \frac{\text{J}}{\text{mol} \cdot \text{K}}\right) 273 \text{ K}$$

$$= 9.07 \times 10^3 \text{ J} = \boxed{9.07 \text{ kJ}}$$

12.65 (a) The energy transferred to the gas by heat is

$$Q = mc \ \Delta T = 1.00 \text{ mol} \left(20.79 \ \frac{\text{J}}{\text{mol} \cdot \text{K}}\right) 120 \text{ K}$$

$$= 2.49 \times 10^3 \text{ J} = \boxed{2.49 \text{ kJ}}$$

(b) Treating the neon as an ideal gas, the result of problem 11 gives the change in internal energy as

$$\Delta U = \frac{3}{2} \ P_fV_f - P_iV_i = \frac{3}{2} \ nRT_f - nRT_i = \frac{3}{2} nR \ \Delta T$$

or

$$\Delta U = \frac{3}{2} \ 1.00 \text{ mol} \left(8.31 \ \frac{\text{J}}{\text{mol} \cdot \text{K}}\right) 120 \text{ K} = 1.50 \times 10^3 \text{ J} = \boxed{1.50 \text{ kJ}}$$

(c) From the first law, the work done *on* the gas is

$$W = \Delta U - Q = 1.50 \times 10^3 \text{ J} - 2.49 \times 10^3 \text{ J} = \boxed{-990 \text{ J}}$$

12.66 Assuming the gravitational potential energy given up by the falling water is transformed into thermal energy when the water hits the bottom of the falls, the rate of thermal energy production is

$$\frac{\Delta Q}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right) gh = \rho_w \left(\frac{\Delta V}{\Delta t}\right) gh$$

Then, if the absolute temperature of the environment is $T_{\text{K}} = 20.0° + 273 = 293 \text{ K}$, the rate of entropy production is

$$\frac{\Delta S}{\Delta t} = \frac{\Delta Q/\Delta t}{T_K} = \frac{\rho_w \ \Delta V/\Delta t \ gh}{T_K}$$

or

$$\frac{\Delta S}{\Delta t} = \frac{1}{293\ \text{K}}\left(10^3\ \frac{\text{kg}}{\cancel{\text{m}^3}}\right)\left(5\times10^3\ \frac{\cancel{\text{m}^3}}{\text{s}}\right)\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\left(50.0\ \text{m}\right) = 8\times10^6\ \frac{\left(\text{kg}\cdot\text{m/s}^2\right)\text{m}}{\text{K}\cdot\text{s}}$$

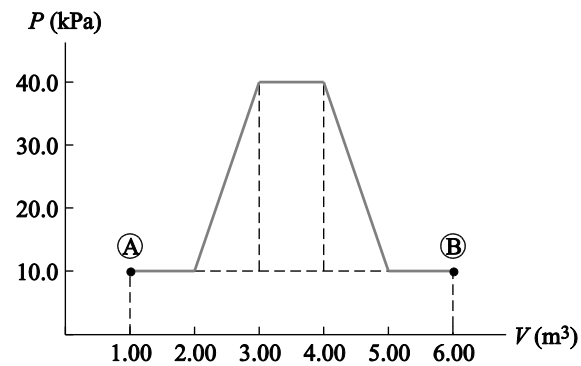
and

$$\frac{\Delta S}{\Delta t} = 8\times10^6\ \frac{\text{N}\cdot\text{m}}{\text{K}\cdot\text{s}} = \boxed{8\times10^6\ \text{J/K}\cdot\text{s}}$$

12.67 (a)

$$T_A = \frac{P_A V_A}{nR} = \frac{10.0\times10^3\ \text{Pa}\quad 1.00\ \text{m}^3}{10.0\ \text{mol}\quad 8.31\ \text{J/mol}\cdot\text{K}} = \boxed{1.20\times10^2\ \text{K}}$$

$$T_B = \frac{P_B V_B}{nR} = \frac{10.0\times10^3\ \text{Pa}\quad 6.00\ \text{m}^3}{10.0\ \text{mol}\quad 8.31\ \text{J/mol}\cdot\text{K}} = \boxed{722\ \text{K}}$$



(b) As it goes from A to B, the gas is expanding and hence, doing work on the environment. The magnitude of the work done equals the area under the process curve from A to B. We subdivide this area into 2 rectangular and 2 triangular parts:

$$W_{\text{env}} = \left[\ 10.0\times10^3\ \text{Pa}\quad 6.00-1.00\quad \text{m}^3\right] + \left[\ 40.0-10.0\ \ \times10^3\ \text{Pa}\right]\ 1.00\ \text{m}^3 + 2\left[\frac{1}{2}\ 1.00\ \text{m}^3\quad 40.0-10.0\ \ \times10^3\ \text{Pa}\right] = \boxed{1.10\times10^5\ \text{J}}$$

(c) The change in the internal energy of a monatomic, ideal gas is $\Delta U = \frac{3}{2}nR\ \ \Delta T$, so

$$\Delta U_{A\to B} = \frac{3}{2}nR\ \ T_B - T_A\ \ = \frac{3}{2}\ 10.0\ \text{mol}\quad 8.31\ \text{J/mol}\cdot\text{K}\quad 722\ \text{K}-120\ \text{K}\ \ = \boxed{7.50\times10^4\ \text{J}}$$

(d) From the first law of thermodynamics, $Q = \Delta U - W$, where W is the work done *on the gas*. In this case, $W = -W_{\text{env}} = -1.10 \times 10^5 \text{ J}$, and

$$Q = \Delta U - W = 7.50 \times 10^4 \text{ J} - -1.10 \times 10^5 \text{ J} = \boxed{1.85 \times 10^5 \text{ J}}$$

12.68 (a) The constant volume occupied by the gases is $V = \frac{4}{3}\pi r^3 = 4\pi \ 0.500 \text{ m}^3/3 = 0.524 \text{ m}^3$ and the initial absolute temperature is $T_i = 20.0° + 273 = 293 \text{ K}$.

To determine the initial pressure, we treat each component of the mixture as an ideal gas and compute the pressure it would exert if it occupied the entire volume of the container. The total pressure exerted by the mixture is then the sum of the partial pressures exerted by the components of the mixture. This gives

$$P_{H_2} = \frac{n_{H_2}RT}{V} \qquad P_{O_2} = \frac{n_{O_2}RT}{V}$$

and

$$P_i = P_{H_2} + P_{O_2} = \frac{n_{H_2}RT}{V} + \frac{n_{O_2}RT}{V} = \frac{n_{H_2} + n_{O_2} \ RT}{V}$$

or

$$P_i = \frac{14.4 \text{ mol} + 7.2 \text{ mol} \quad 8.31 \text{ J/mol} \cdot \text{K} \quad 293 \text{ K}}{0.524 \text{ m}^3} = \boxed{1.00 \times 10^5 \text{ Pa}}$$

(b) Treating both the hydrogen and oxygen as ideal gases, each with internal energy $U = nC_vT$ where C_v is the molar specific heat at constant volume, we use Table 12.1 and find the initial internal energy of the mixture as

$$U_i = U_{H_2,i} + U_{O_2,i} = n_{H_2}C_{v,H_2} + n_{O_2}C_{v,O_2} \ T_i$$

or

$$U_i = \left[14.4 \text{ mol} \quad 20.4 \text{ J/mol} \cdot \text{K} + 7.2 \text{ mol} \quad 21.1 \text{ J/mol} \cdot \text{K} \right] 293 \text{ K} = \boxed{1.31 \times 10^5 \text{ J}}$$

(c) During combustion this mixture produces 14.4 moles of water (1 mole of water for each mole of hydrogen used) with a conversion of 241.8 kJ of chemical potential energy per mole. Since the volume is constant, no work is done and the additional internal energy generated in the combustion is

$$\Delta U = 14.4 \text{ mol} \quad 241.8 \text{ kJ/mol} = 3.48 \times 10^3 \text{ kJ} = \boxed{3.48 \times 10^6 \text{ J}}$$

(d) After combustion, the internal energy of the system is

$$U_f = U_i + \Delta U = 1.31 \times 10^5 \text{ J} + 3.48 \times 10^6 \text{ J} = 3.61 \times 10^6 \text{ J}$$

Treating the steam as an ideal gas, so $U = nC_\upsilon T_K$, and obtaining the molar heat capacity for water vapor (a polyatomic gas) from Table 12.1, we find

$$T_f = \frac{U_f}{n_{water} C_{\upsilon,water}} = \frac{3.61 \times 10^6 \text{ J}}{14.4 \text{ mol} \quad 27.0 \text{ J/mol} \cdot \text{K}} = \boxed{9.28 \times 10^3 \text{ K}}$$

and the final pressure is

$$P_f = \frac{n_{water} R T_f}{V} = \frac{14.4 \text{ mol} \quad 8.31 \text{ J/mol} \cdot \text{K} \quad 9.28 \times 10^3 \text{ K}}{0.524 \text{ m}^3} = \boxed{2.12 \times 10^6 \text{ Pa}}$$

(e) The total mass of steam present after combustion is

$$m_{steam} = n_{steam} M_{water} = 14.4 \text{ mol} \quad 18.0 \times 10^{-3} \text{ kg/mol} = \boxed{0.259 \text{ kg}}$$

and its density is

$$\rho_s = \frac{m_{steam}}{V} = \frac{0.259 \text{ kg}}{0.524 \text{ m}^3} = \boxed{0.494 \text{ kg/m}^3}$$

(f) Assuming the steam is essentially at rest within the container $\upsilon_1 \approx 0$, $P_2 = 0$ (since the steam spews into a vacuum), and $y_2 = y_1$, we use the pressure from part (d) above and Bernoulli's equation $P_2 + \frac{1}{2}\rho_s \upsilon_2^2 + \rho_s g y_2 = P_1 + \frac{1}{2}\rho_s \upsilon_1^2 + \rho_s g y_1$ to find the exhaust speed as

$$v_2 = \sqrt{\frac{2P_1}{\rho_s}} = \sqrt{\frac{2 \quad 2.12 \times 10^6 \text{ Pa}}{0.494 \text{ kg/m}^3}} = \boxed{2.93 \times 10^3 \text{ m/s}}$$

12.69 The work that you have done is

$$W_{\text{eng}} = mg\ \Delta h = \left[150\text{ lb}\left(\frac{4.448\text{N}}{1\text{ lb}}\right)\right]\left[\left(90.0\ \frac{\text{step}}{\text{min}}\right)30.0\text{ min}\left(8.00\ \frac{\text{in}}{\text{step}}\right)\left(\frac{2.54\times10^{-2}\text{ m}}{1\text{ in}}\right)\right]$$

or $W_{\text{eng}} = 3.66\times10^{5}\text{ J}$.

If the energy input by heat was $|Q_h| = 600\text{ kcal}\left(\frac{4\,186\text{ J}}{1\text{ kcal}}\right) = 2.51\times10^{6}\text{ J}$, your efficiency has been

$$e = \frac{W_{eng}}{|Q_h|} = \frac{3.66\times10^{5}\text{ J}}{2.51\times10^{6}\text{ J}} = \boxed{0.146\text{ or }14.6\%}$$

If the actual efficiency was $e = 0.180$ or 18.0%, the actual energy input was

$$|Q_h|_{actual} = \frac{W_{eng}}{e_{actual}} = \frac{3.66\times10^{5}\text{ J}}{0.180} = 2.03\times10^{6}\text{ J}\left(\frac{1\text{ kcal}}{4\,186\text{ J}}\right) = \boxed{486\text{ kcal}}$$

12.70 (a) The energy transferred from the water by heat as it cools is

$$|Q_h| = mc|\Delta T| = \rho V\ c|\Delta T|$$

$$= \left[\left(1.0\ \frac{\text{g}}{\text{cm}^3}\right)1.0\text{ L}\left(\frac{10^3\text{ cm}^3}{1\text{ L}}\right)\right]\left(1.0\ \frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}}\right)\left(\frac{4.186\text{ J}}{1\text{ cal}}\right)570^\circ\text{C} - 4.0^\circ\text{C}$$

or $|Q_h| = \boxed{2.4\times10^{6}\text{ J}}$

(b) The maximum efficiency of a heat engine is the Carnot efficiency. Thus,

$$e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{4.0 + 273\ \text{K}}{570+273\ \text{K}} = 1 - \frac{277\text{ K}}{843\text{ K}} = 0.67$$

The maximum useful work output is then

$$W_{\text{eng}\ \max} = e_C|Q_h| = 0.67\ \ 2.4\times10^{6}\text{ J} = \boxed{1.6\times10^{6}\text{ J}}$$

(c) The energy available from oxidation of the hydrogen sulfide in 1.0 L of this water is

$$U = n\ 310\ \text{kJ/mol} = \left[\left(0.90 \times 10^{-3}\ \frac{\text{mol}}{\text{L}}\right)\ 1.0\ \text{L}\right]\left(310 \times 10^{3}\ \frac{\text{J}}{\text{mol}}\right) = \boxed{2.8 \times 10^{2}\ \text{J}}$$

12.71 (a) With an overall efficiency of $e = 0.15$ and a power output of $\mathcal{P}_{\text{out}} = 150\ \text{MW}$, the required power input (from burning coal) is

$$\mathcal{P}_{\text{in}} = \frac{\mathcal{P}_{\text{out}}}{e} = \frac{150 \times 10^{6}\ \text{W}}{0.15} = 1.0 \times 10^{9}\ \text{J/s}$$

The coal used each day is

$$\frac{\Delta m}{\Delta t} = \frac{\mathcal{P}_{\text{in}}}{\textit{heat of combustion}} = \frac{1.0 \times 10^{9}\ \text{J/s}\ \ 8.64 \times 10^{4}\ \text{s/d}}{\left(7.8 \times 10^{3}\ \frac{\text{cal}}{\text{g}}\right)\left(\frac{10^{3}\ \text{g}}{1\ \text{kg}}\right)\left(\frac{4.186\ \text{J}}{1\ \text{cal}}\right)} = 2.64 \times 10^{6}\ \frac{\text{kg}}{\text{d}}$$

or

$$\frac{\Delta m}{\Delta t} = \left(2.64 \times 10^{6}\ \frac{\text{kg}}{\text{d}}\right)\left(\frac{1\ \text{metric ton}}{10^{3}\ \text{kg}}\right) = \boxed{2.6 \times 10^{3}\ \text{metric ton/d}}$$

(b) The annual fuel cost is: $\textit{cost} = \textit{coal used yearly} \cdot \textit{rate}$, or

$$\textit{cost} = 2.64 \times 10^{3}\ \text{ton/d}\ \ 365\ \text{d/y}\ \ \$8.0/\text{ton} = \boxed{\$7.7 \times 10^{6}/\text{y}}$$

(c) The rate of energy transfer to the river by heat is

$$\mathcal{P}_{\text{exhaust}} = \mathcal{P}_{\text{in}} - \mathcal{P}_{\text{out}} = 1.0 \times 10^{9}\ \text{W} - 150 \times 10^{6}\ \text{W} = 8.5 \times 10^{8}\ \text{W}$$

Thus, the flow required if the maximum rise in temperature is 5.0°C is

$$\textit{flow rate} = \frac{\Delta m_{\text{water}}}{\Delta t} = \frac{\mathcal{P}_{\text{exhaust}}}{c_{\text{water}}\ \Delta T} = \frac{8.5 \times 10^{8}\ \text{J/s}}{4\ 186\ \text{J/kg} \cdot °\text{C}\ \ 5.0°\text{C}} = \boxed{4.1 \times 10^{4}\ \text{kg/s}}$$